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Electric Circuits • EM Fields • Signals & Systems • Electrical Machines • Power Systems
Control Systems • Measurements • Analog & Digital Electronics • 8085 • Power Electronics







UNDER THE GUIDANCE OF

G. V. SATISH

SBI PO – 2005 Batch

INTERVIEW PANEL MEMBER

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Professional Knowledge – Engineering (Electrical)

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Prepared by

G V Satish

Chairman and Managing Director

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About the Exam & How to Use This Book

Online examination – Assistant Manager (Engineering-Electrical)

#	Test (not by sequence)	Questions	Marks	Time
1	**Professional / Subject Knowledge (this book)**	60	60	55 min
2	General Awareness	40	40	20 min
3	English Language	40	40	25 min
4	Reasoning & Computer Aptitude	30	30	25 min
5	Data Analysis & Interpretation and Quantitative Aptitude	30	30	25 min
6	Language & Comprehension Abilities – Descriptive	2	30	30 min
	Total	**202**	**230**	**3 hours**

- **Negative marking**: ¼ of the marks of the question for each wrong answer; no penalty for a blank.
- **Qualifying marks** in Professional Knowledge: **40%** (UR), **35%** (OBC/EWS/SC/ST/PwBD/ExSM). Same for the Descriptive test.
- Shortlisting for interview / DV up to **1:5**. Eligibility: **B.E./B.Tech in Electrical or Electrical & Electronics Engineering**; age 18–28 years (as on 15.11.2026). Pay scale **₹40,000–1,40,000 (E-1, IDA)**.
- Roughly **1 minute per question**: these notes are built for recall. Each section ends with a **Previously Asked / Likely Questions** box; stars show weightage (★★★ = must-do).

Priority order of subjects (by usual weightage)

#	Subject	Syllabus part
1	Electrical Machines (Transformers, DC, Induction, Synchronous)	D
2	Power Systems (Generation, Transmission, Faults, Protection, Stability)	E
3	Electric Circuits	A
4	Control Systems	F
5	Electrical & Electronic Measurements	G
6	Power Electronics	I
7	Analog Electronics	H (i–vi)
8	Digital Electronics & 8085 Microprocessor	H (vi–vii)
9	Electromagnetic Fields	B
10	Signals & Systems	C
11	Last-Day Revision (formula & value sheet)	All
12	500 Most Expected Questions & Answers	All

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1. Electric Circuits

NOTE

Network theory supplies about **6–9 of the 60** Professional Knowledge MCQs. It also supports Machines, Power Systems and Measurements. Questions are mostly 30-second numericals: Thevenin/maximum power, time constants, form factor, resonance Q and bandwidth, two-port conditions, two-wattmeter readings and series aiding/opposing inductance. Learn the conditions and the numbers. Derivations are not needed.

1.1 Basics, Sources & Network Graph ★★☆

- **Charge** in coulomb (C). **Current** i = dq/dt (A). **Voltage** = work per unit charge, dW/dq (V). **Power** p = vi (W). **Energy** W = ∫p dt (J). 1 kWh = **3.6 MJ**.
- Charge of an electron = **1.602×10^{-19} C**. 1 C ≈ **6.24×10^{18}** electrons.
- **Passive sign convention**: if current enters the + terminal, p = vi is power **absorbed**. A negative value means power is delivered.
- **Linear** element: obeys homogeneity and additivity (R, L, C with constant values). **Bilateral** element: same V–I relation for both directions of current (R, L, C). A **diode** is non-linear and **unilateral**.
- **Active** elements deliver net energy (sources, transistors with bias). **Passive** elements (R, L, C) do not. R **dissipates** energy. L and C **store** it and return it.
- **Lumped** network: physical size ≪ wavelength. **Distributed** network: transmission lines, waveguides.
- An inductor **opposes sudden change of current**, so its current cannot change instantaneously. A capacitor **opposes sudden change of voltage**, so its voltage cannot change instantaneously.
- At DC steady state an **inductor = short circuit** and a **capacitor = open circuit**. At very high frequency (f → ∞) L is **open** and C is **short**.

Element	v–i relation	Energy	Series combination	Parallel combination
Resistor R (Ω)	v = Ri	Dissipated: $i^2R\,t$	**$R_1 + R_2$**	$R_1R_2/(R_1 + R_2)$
Inductor L (H)	v = L di/dt; i = (1/L)∫v dt	Stored: **½ L I^2** (magnetic field)	**$L_1 + L_2$** (no mutual)	$L_1L_2/(L_1 + L_2)$
Capacitor C (F)	i = C dv/dt; v = (1/C)∫i dt	Stored: **½ C V^2** = $Q^2/2C$ (electric field)	$C_1C_2/(C_1 + C_2)$	**$C_1 + C_2$**

- **Ideal voltage source**: internal resistance **zero**. Terminal voltage is independent of load current. Its V–I graph is a horizontal line (V vs I).
- **Ideal current source**: internal resistance **infinite**. Current is independent of terminal voltage.
- **Practical voltage source** = V_s with R_s in **series**. Terminal voltage $V = V_s - IR_s$, so it falls as load increases. **Practical current source** = I_s with R_s in **parallel**.
- A good voltage source has **low** R_s. A good current source has **high** R_s.
- **Source transformation**: V_s in series with $R \Leftrightarrow I_s = V_s/R$ in parallel with the same R. It is valid only for **practical** sources. An ideal source has no equivalent of the other type. The equivalence holds only at the terminals and does not hold for internal power.
- Ideal voltage sources of unequal value must not be connected in parallel, and unequal ideal current sources must not be connected in series, because KVL/KCL would be violated.
- An element in parallel with an ideal voltage source is **redundant** for the external circuit. An element in series with an ideal current source is also redundant.
- **Dependent (controlled) sources**: VCVS, VCCS, CCVS, CCCS. A BJT is modelled as a **CCCS** ($\beta\, i_b$). A FET is modelled as a **VCCS** ($g_m v_{gs}$). An op-amp is modelled as a **VCVS**.

KEY FORMULAE – STAR ⇔ DELTA

- Δ → Y: **$R_A = R_{AB}R_{CA} / (R_{AB} + R_{BC} + R_{CA})$**, i.e. product of the two adjacent Δ resistors ÷ sum of all three.
- Y → Δ: **$R_{AB} = R_A + R_B + R_AR_B/R_C$** = $(R_AR_B + R_BR_C + R_CR_A)/R_C$ (the opposite star arm).
- Balanced: **$R_Y = R_\Delta/3$** and $R_\Delta = 3R_Y$. For capacitors $C_Y = 3C_\Delta$, because C behaves like conductance.
- Δ of three 9 Ω resistors gives a star of **3 Ω** each.

- **Graph**: each element is replaced by a line segment (branch). **Oriented** (directed) graph: each branch carries an arrow for current direction. Voltage sources are replaced by short circuits and current sources by open circuits before the graph is drawn.
- **Tree**: connected sub-graph containing **all nodes** and **no closed loop**. Its branches are **twigs** = **n − 1**. The remaining branches are **links (chords)** = **b − n + 1**. The links form the **co-tree**.
- Number of **fundamental loops (tie-sets)** = number of links = **b − n + 1**. Number of **fundamental cut-sets** = number of twigs = **n − 1**.
- For a graph with **p separate parts**, twigs = n − p and links = b − n + p.
- **Complete incidence matrix A_a**: order **n × b**. Entry is +1 if the branch leaves the node, −1 if it enters, 0 otherwise. Every column has exactly one +1 and one −1, so **column sum = 0**. Its rank is **n − 1**.
- **Reduced incidence matrix A**: delete the reference-node row to get order **(n − 1) × b**. KCL: **$A\, I_b = 0$**.
- **Tie-set matrix B**: order **(b − n + 1) × b**. KVL: **$B\, V_b = 0$**. It is used for loop (mesh) analysis.
- **Cut-set matrix Q**: order **(n − 1) × b**. KCL: **$Q\, I_b = 0$**. It is used for node-pair analysis.
- Number of possible trees = **det(A A^T)**, where A is the reduced incidence matrix. A **complete graph** of n nodes has **n^{n-2}** trees (Cayley). For example, n = 4 gives 16 trees.
- **Planar** graph: can be drawn on a plane without branches crossing. Meshes = b − n + 1. Mesh analysis and the dual network apply **only to planar** networks.

Quantity	Number / Order
Twigs (tree branches)	**n − 1**
Links / independent loops / mesh equations	**b − n + 1**
Independent node (KCL) equations	**n − 1**
Complete incidence matrix	n × b
Reduced incidence matrix / cut-set matrix	(n − 1) × b
Tie-set (fundamental loop) matrix	(b − n + 1) × b
Trees of complete graph	n^{n-2}

PREVIOUSLY ASKED / LIKELY QUESTIONS

- A graph has 6 nodes and 10 branches. Number of links? → **5** (b − n + 1).
- Internal resistance of an ideal current source? → **Infinite**.
- Energy stored in a 2 H inductor carrying 3 A? → **9 J**.
- Three equal 30 Ω resistors in delta. Equivalent star arm? → **10 Ω**.
- Sum of the elements of any column of a complete incidence matrix? → **Zero**.
- Element whose current cannot change instantaneously? → **Inductor**.

1.2 KCL, KVL, Nodal & Mesh Analysis ★★★

- **KCL**: the algebraic sum of currents at a node = 0. It is based on **conservation of charge**.
- **KVL**: the algebraic sum of voltages around any closed loop = 0. It is based on **conservation of energy**.
- KCL and KVL apply to **all** lumped networks: linear or non-linear, active or passive, time-variant or invariant, AC or DC. They do **not** apply to distributed-parameter circuits at high frequency.
- **Voltage divider** (series): $V_1 = V \cdot R_1/(R_1 + R_2)$. **Current divider** (parallel): $I_1 = I \cdot R_2/(R_1 + R_2)$. Note that the current in one branch uses the *other* branch's resistance.

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- **Nodal analysis**: based on KCL, with node voltages as unknowns. It needs **n − 1** equations, minus one for each voltage source between two nodes (or between a node and the reference). Prefer it when there are fewer nodes than meshes or when the circuit has many current sources.
- **Mesh analysis**: based on KVL, with mesh currents as unknowns. It needs **b − n + 1** equations, minus one for each current source. It applies to **planar circuits only**. Prefer it when the circuit has many voltage sources.
- **Supernode**: formed when a **voltage source** (dependent or independent) is connected between two non-reference nodes. Write one KCL for the enclosed region and add the constraint equation $V_a - V_b = V_s$.
- **Supermesh**: formed when a **current source** is common to two meshes. Write KVL around the outer loop, avoiding the current source, and add the constraint $I_1 - I_2 = I_s$.
- A **current source in an outer branch** (in one mesh only) fixes that mesh current directly. A **voltage source from a node to reference** fixes that node voltage directly.
- **Dependent sources**: write the circuit equations as for independent sources, then express the controlling variable in terms of the unknowns. A dependent source **is never switched off** during superposition or Thevenin analysis.
- The nodal admittance matrix of a network with only passive elements and independent sources is **symmetric** ($Y_{ij} = Y_{ji}$), with diagonal = sum of admittances at the node and off-diagonal = −(mutual admittance). **Dependent sources destroy the symmetry**.

Quantity	Dual
Voltage (V)	Current (I)
Resistance R	Conductance G
Inductance L	Capacitance C
Series connection	Parallel connection
KVL / mesh / loop	KCL / node
Short circuit	Open circuit
Thevenin	Norton
Voltage source	Current source
Star (T)	Delta (Π)
Switch closing	Switch opening

- In the dual network, each **mesh** of the original becomes a **node**, plus one reference node for the outside region. Dual networks exist **only for planar** networks.
- The dual of a series RLC circuit driven by a voltage source is a **parallel GCL** circuit driven by a current source, with numerically equal values.

MEMORY TRICK

"Voltage source → Supernode, Current source → Supermesh": each source type blocks the method that would otherwise use it as a natural unknown. A **V**oltage source sits between **nodes** (nodal analysis problem). A **C**urrent source sits in a shared **mesh** branch (mesh analysis problem).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- KVL is based on conservation of? → **Energy**. KCL is based on conservation of? → **Charge**.
- A circuit has 5 nodes and 8 branches. Number of nodal equations? → **4**. Mesh equations? → **4**.
- Concept used when a voltage source lies between two non-reference nodes? → **Supernode**.
- Mesh analysis is applicable to which networks? → **Planar networks only**.
- Dual of an inductor of 2 H? → **Capacitor of 2 F**.
- 10 A divides between 2 Ω and 3 Ω in parallel. Current in 2 Ω? → **6 A**.

1.3 Network Theorems ★★★

- **Thevenin's theorem** (Léon Thévenin, 1883): any linear bilateral two-terminal network can be replaced by $\mathbf{V_{th}}$ (the open-circuit voltage) in **series** with $\mathbf{R_{th}}$ (Z_{th}).
- R_{th} is the resistance seen from the terminals with **independent** voltage sources **shorted** and independent current sources **opened**, internal resistances retained. Alternatively, $\mathbf{R_{th} = V_{oc}/I_{sc}}$.
- With **dependent sources only** (no independent source), $\mathbf{V_{th} = 0}$. Find R_{th} by applying a test source: $R_{th} = V_{test}/I_{test}$. R_{th} can then be **negative**.
- **Norton's theorem** (E.L. Norton, 1926): the network is replaced by $\mathbf{I_N = I_{sc}}$ in **parallel** with $\mathbf{R_N = R_{th}}$. Also $I_N = V_{th}/R_{th}$. Thevenin and Norton equivalents are **duals**.
- The Thevenin equivalent gives the correct **terminal** behaviour only. Power dissipated inside the equivalent ≠ power inside the original network.
- **Superposition theorem**: in a **linear** network with several **independent** sources, a response equals the sum of the responses to each source acting alone. Others are deactivated: voltage source → **short**, current source → **open**. Dependent sources stay active.
- Superposition is **not applicable to power**, because $P \propto I^2$ is non-linear. It also does not apply to non-linear elements (diode). It applies to voltage and current only.
- When sources have **different frequencies** (e.g. DC + AC), superposition is the **only** method. Powers due to different-frequency components may then be added (orthogonality): $P = \Sigma I_k^2 R$.
- **Reciprocity theorem**: in a linear, bilateral, passive network with a **single source**, the ratio of excitation to response stays the same when the positions of source and response are interchanged. The voltage source must be replaced by a short circuit and the ammeter by the source. It is **not** applicable to networks with **dependent sources** or more than one source.
- **Millman's theorem**: several parallel branches, each V_k in series with Z_k, reduce to one source $\mathbf{V = \Sigma(V_k Y_k)/\Sigma Y_k}$ with $\mathbf{Z = 1/\Sigma Y_k}$. It is used for neutral shift in unbalanced star loads.
- **Tellegen's theorem**: $\mathbf{\Sigma\, v_k\, i_k = 0}$ over all branches at any instant (conservation of power). It depends only on **KCL, KVL and topology**, not on element nature. It applies to **any lumped network**: linear or non-linear, active or passive, time-variant or invariant. It also holds for two networks with the same graph (v of one, i of the other).
- **Substitution theorem**: any branch carrying I with voltage V may be replaced by an ideal source of V (or I) without changing the other currents and voltages.
- **Compensation theorem**: if impedance Z in a branch carrying I changes by ΔZ, the changes elsewhere equal those produced by a **compensating voltage source I·ΔZ** inserted in that branch (opposing the original current), with all other sources deactivated. It is used for sensitivity and bridge-unbalance calculations.

KEY FORMULAE – MAXIMUM POWER TRANSFER

- DC (resistive): $\mathbf{R_L = R_{th}}$ gives $\mathbf{P_{max} = V_{th}^2/(4R_{th})} = I_N^2 R_{th}/4$.
- AC, complex load (both R_L and X_L variable): $\mathbf{Z_L = Z_{th}^*}$ (conjugate match) gives $P_{max} = |V_{th}|^2/(4R_{th})$. V_{th} is RMS.
- AC, purely resistive load: $\mathbf{R_L = |Z_{th}| = \sqrt{(R_{th}^2 + X_{th}^2)}}$.
- Fixed load reactance X_L, variable R_L: $R_L = \sqrt{(R_{th}^2 + (X_{th} + X_L)^2)}$.
- Efficiency at maximum power transfer = **50%**, since half the power is lost in R_{th}. For this reason power systems are **not** run at the maximum power transfer condition. Communication and electronic circuits use it for impedance matching.
- If the **load is fixed** and the source resistance R_s is variable, maximum power to the load occurs at $\mathbf{R_s = 0}$ (not $R_s = R_L$).

Theorem	Linear only?	Bilateral needed?	Dependent sources allowed?	Key remark
Thevenin / Norton	Yes (network side)	Yes	Yes (kept active)	Load may be non-linear
Superposition	**Yes**	No	Yes (kept active)	**Not for power**
Reciprocity	Yes	**Yes**	**No**	Single source only
Maximum power	Yes	Yes	Yes	η = 50%
Millman	Yes	Yes	—	Parallel sources
Tellegen	**No** (any network)	No	Yes	Power conservation
Substitution	No (any network with unique solution)	No	Yes	Branch → source

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Compensation	Yes	Yes	—	Source = I·ΔZ

MEMORY TRICK

"Tellegen Tolerates everything": it needs only topology. **"Reciprocity Rejects dependent sources"**. **"Superposition Skips power"**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- V_{th} = 20 V, R_{th} = 5 Ω. Maximum power to load? → **20 W** (400/20).
- Z_{th} = (3 + j4) Ω. Load for maximum power if the load is purely resistive? → **5 Ω**. If the load is complex? → **(3 − j4) Ω**.
- Efficiency at maximum power transfer? → **50%**.
- Theorem applicable to both linear and non-linear networks? → **Tellegen's**.
- Superposition theorem cannot be used to calculate? → **Power**.
- V_{oc} = 12 V, I_{sc} = 3 A. R_{th}? → **4 Ω**.

1.4 Transient Response ★★★

- **Transient**: the response while the circuit moves from one steady state to another after switching. It arises only because of energy-storing elements (L, C). A purely resistive circuit has **no transient**.
- **Continuity rules**: $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$, because energy cannot change in zero time. Inductor voltage and capacitor current **can** jump.

Element	At $t = 0^+$ (initially de-energised)	At $t = 0^+$ (initially energised)	At $t = \infty$ (DC steady state)
Inductor	**Open circuit**	Current source I_0	**Short circuit**
Capacitor	**Short circuit**	Voltage source V_0	**Open circuit**
Resistor	R	R	R

KEY FORMULAE – FIRST-ORDER CIRCUITS

- General: $\mathbf{x(t) = x(\infty) + [x(0^+) - x(\infty)]\, e^{-t/\tau}}$.
- RC: $\tau = RC$. Charging: $v_C = V(1 - e^{-t/RC})$, $i = (V/R)e^{-t/RC}$. Discharging: $v_C = V_0 e^{-t/RC}$.
- RL: $\tau = L/R$. Rise: $i = (V/R)(1 - e^{-Rt/L})$, $v_L = Ve^{-Rt/L}$. Decay: $i = I_0 e^{-Rt/L}$.
- R in τ is the **Thevenin resistance seen by L or C** with sources deactivated.
- Initial slope: $di/dt\,(0^+) = V/L$ (RL). $dv_C/dt\,(0^+) = i(0^+)/C = V/(RC)$ (RC). If the initial rate were maintained, the final value would be reached in exactly **one τ**.
- Time for 10% to 90% rise (rise time) = **2.2 τ**. Half-value time = 0.693 τ.

Time	Rising quantity (% of final)	Decaying quantity (% of initial)
1 τ	**63.2%**	**36.8%**
2 τ	86.5%	13.5%
3 τ	95.0%	5.0%
4 τ	98.2%	1.8%
5 τ	**99.3%** (taken as steady state)	0.7%

- When a capacitor is charged from a DC source V through any resistance R, energy lost in R = $\mathbf{\frac{1}{2}CV^2}$ = energy stored. Charging efficiency is therefore **50%, independent of R**.
- **Laplace models** (s-domain): R → R. L → **sL** in series with a voltage source **L i(0⁻)**, or in parallel with a current source i(0⁻)/s. C → **1/(sC)** in series with a voltage source **v(0⁻)/s**, or in parallel with a current source C v(0⁻).
- **Initial value theorem**: $f(0^+) = \lim_{s\to\infty} sF(s)$. **Final value theorem**: $f(\infty) = \lim_{s\to 0} sF(s)$. The FVT is valid only if all poles of sF(s) lie in the left half-plane.
- **Step** input transforms to 1/s, **impulse** to 1, **ramp** to $1/s^2$. An impulse current into C produces an instantaneous voltage step. An impulse voltage across L produces a current step.
- **Series RLC** (DC step): $L\, d^2i/dt^2 + R\, di/dt + i/C = 0$. Characteristic equation: $\mathbf{s^2 + (R/L)s + 1/(LC) = 0}$.
- Natural frequency $\omega_n = 1/\sqrt{LC}$. Damping coefficient (neper frequency) $\alpha = R/(2L)$ for series and $\alpha = 1/(2RC)$ for parallel RLC. Roots: $s = -\alpha \pm \sqrt{(\alpha^2 - \omega_n^2)}$.
- Damped frequency $\omega_d = \omega_n\sqrt{(1 - \zeta^2)} = \sqrt{(\omega_n^2 - \alpha^2)}$.

KEY FORMULAE – DAMPING RATIO

- Series RLC: $\zeta = (R/2)\sqrt{(C/L)} = \alpha/\omega_n$. Critical resistance $R_{cr} = 2\sqrt{(L/C)}$.
- Parallel RLC: $\zeta = (1/2R)\sqrt{(L/C)}$. Critical R = ½√(L/C). Here a **larger R gives less damping**, the opposite of the series case.
- Series RLC: ζ = 1/(2Q). Q = (1/R)√(L/C).

Condition	ζ	Roots	Response
Overdamped	**ζ > 1** (R > 2√(L/C) series)	Real, unequal, negative	Sluggish, no oscillation
Critically damped	**ζ = 1**	Real, equal (−α, −α)	**Fastest without overshoot**
Underdamped	**0 < ζ < 1**	Complex conjugate, LHP	Decaying oscillation at ω_d
Undamped	**ζ = 0** (R = 0 series)	Purely imaginary $\pm j\omega_n$	Sustained oscillation (LC)
Negative damping	ζ < 0	RHP	Growing oscillation (unstable)

MEMORY TRICK

"L Lags change of current, C Clamps voltage": at the switching instant an uncharged C is a short and an uncharged L is an **open**. At steady DC they swap: C is **open** and L is **short**. Time constants: **RC** (multiply) and **L/R** (divide). "LR = L over R".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Time constant of a series RL circuit with R = 10 Ω and L = 0.5 H? → **50 ms**.
- Percentage of final voltage reached by a charging capacitor in one time constant? → **63.2%**.
- Series RLC with L = 1 H, C = 1 F. Value of R for critical damping? → **2 Ω**.
- Behaviour of an uncharged capacitor at $t = 0^+$? → **Short circuit**.
- Roots of an underdamped system? → **Complex conjugate with negative real part**.
- Energy dissipated in R while charging C to V from a DC source? → $\mathbf{\frac{1}{2}CV^2}$, independent of R.

1.5 Sinusoidal Steady State & AC Power ★★★

- $v = V_m \sin(\omega t + \theta)$. $\omega = 2\pi f$ (rad/s). T = 1/f. India's supply frequency is **50 Hz** (ω = 314 rad/s, T = 20 ms).
- **RMS (effective) value**: the DC value that produces the same heat in the same resistor in the same time. $V_{rms} = \sqrt{[(1/T)\int v^2 dt]}$. AC meters read RMS, and nameplate voltages such as 230 V are RMS.
- **Average value**: the average of a full cycle of a symmetrical AC wave is **zero**, so for AC it is taken over a half cycle. A PMMC meter reads the average value.
- **Form factor** = RMS ÷ average. **Peak (crest) factor** = peak ÷ RMS.
- RMS of a composite wave: $V = \sqrt{(V_0^2 + V_1^2 + V_2^2 + \ldots)}$, where V_0 is DC and V_k are the RMS values of harmonics. For example, $v = 10 + 10\sqrt{2}\sin\omega t$ gives $V_{rms} = \sqrt{(100 + 100)} = \mathbf{14.14\ V}$.

Waveform (peak V_m)	Average	RMS	Form factor	Peak factor
Sine (half-cycle avg) / full-wave rectified	$\mathbf{2V_m/\pi = 0.637V_m}$	$\mathbf{V_m/\sqrt{2} = 0.707V_m}$	**1.11**	**1.414**
Half-wave rectified sine	$\mathbf{V_m/\pi = 0.318V_m}$	$\mathbf{V_m/2}$	**1.57**	**2**
Square (symmetrical)	V_m	V_m	**1**	**1**
Triangular / sawtooth	$V_m/2$	$\mathbf{V_m/\sqrt{3} = 0.577V_m}$	**1.155**	**1.732**
DC	V	V	1	1

- The **square wave** has the lowest form factor and peak factor (both 1). A peakier wave has a higher form factor.
- **Phasor**: a rotating vector of RMS magnitude and phase angle. It is valid only for sinusoids of a **single frequency**. Phasors rotate **anticlockwise** at ω.
- Differentiation in the phasor domain is multiplication by **jω**, a 90° lead. Integration is division by jω. The operator $j = 1\angle 90°$.

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- In pure R, V and I are **in phase**. In pure L, I **lags** V by **90°**. In pure C, I **leads** V by **90°**.
- **Impedance** Z = R + jX (Ω). $\mathbf{X_L = \omega L}$ (∝ f). $\mathbf{X_C = 1/(\omega C)}$ (∝ 1/f). $|Z| = \sqrt{(R^2 + X^2)}$, $\varphi = \tan^{-1}(X/R)$.
- **Admittance** Y = 1/Z = G + jB (siemens). For Z = R + jX: $\mathbf{G = R/(R^2 + X^2)}$ and $\mathbf{B = -X/(R^2 + X^2)}$. G ≠ 1/R unless X = 0.

Circuit	Z	Phase of I w.r.t. V	Power factor
Pure R	R	0°	1 (unity)
Pure L	jωL	Lags 90°	0 lagging
Pure C	−j/(ωC)	Leads 90°	0 leading
Series RL	R + jωL	Lags $\varphi = \tan^{-1}(\omega L/R)$	R/|Z| lagging
Series RC	R − j/(ωC)	Leads $\varphi = \tan^{-1}(1/\omega CR)$	R/|Z| leading

KEY FORMULAE – AC POWER

- Instantaneous power p(t) = VI cos φ − VI cos(2ωt − φ). It pulsates at **twice the supply frequency**.
- Active (real) power $\mathbf{P = VI \cos\varphi = I^2R}$ (W).
- Reactive power $\mathbf{Q = VI \sin\varphi = I^2X}$ (VAR). By convention **inductive Q is positive** (absorbed) and capacitive Q is negative (supplied).
- Apparent power $\mathbf{S = VI = \sqrt{(P^2 + Q^2)}}$ (VA).
- Complex power $\mathbf{S = V I^* = P + jQ}$, where I* is the conjugate of the current phasor. For a lagging load, S = P + jQ with Q > 0.
- Power factor $\mathbf{\cos\varphi = P/S = R/|Z|}$. φ is the angle between V and I, equal to the impedance angle.
- PF correction capacitor kVAR: $\mathbf{Q_C = P(\tan\varphi_1 - \tan\varphi_2)}$.
- Single-phase: $C = Q_C/(\omega V^2)$. Three-phase bank: $\mathbf{C_\Delta = Q_C/(3\omega V_L^2)}$ per phase and $C_Y = Q_C/(\omega V_L^2) = \mathbf{3\,C_\Delta}$.

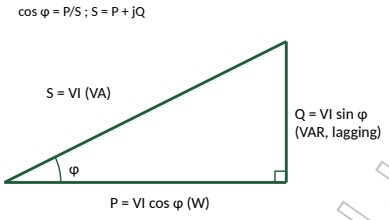

Fig 1.1 Power triangle for an inductive (lagging) load. For a capacitive load Q is drawn downward.

- Average power absorbed by pure L or pure C = **zero**. Energy only oscillates between source and element.
- In any network, total complex power supplied = sum of complex powers absorbed. P and Q are each conserved separately (Tellegen), but **apparent power S (magnitude) is not conserved**.
- **Causes of low pf**: induction motors at light load, transformers on no load, arc and induction furnaces, fluorescent tube chokes.
- **Disadvantages of low pf**: larger current for the same P, so higher I^2R loss, larger conductor and kVA rating, poor voltage regulation and lower efficiency.
- PF improvement devices: **static capacitors** (most common), **synchronous condensers** (over-excited synchronous motor on no load), and phase advancers for induction motors.
- For a fixed P, the line current is **proportional to 1/cos φ**. Line loss ∝ $1/\cos^2\varphi$.
- The most economical pf is usually **0.95 lagging** rather than unity, because capacitor cost rises steeply near unity.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Form factor of a sine wave? → **1.11**. Peak factor? → **1.414**.
- Form factor of a triangular wave? → **1.155**.
- RMS value of a half-wave rectified sine of peak V_m? → $\mathbf{V_m/2}$.
- A load draws 10 kW at 0.8 pf lagging. Reactive power? → **7.5 kVAR** (S = 12.5 kVA).
- Average power consumed by a pure inductor? → **Zero**.
- Frequency at which instantaneous power pulsates in a 50 Hz circuit? → **100 Hz**.

1.6 Resonance & Filters ★★★

- **Resonance**: the condition where the circuit's net reactance (or susceptance) is zero, so V and I are **in phase** and pf = **unity**. It requires both L and C.
- **Series resonance** (acceptor circuit): $X_L = X_C$, giving $\mathbf{f_0 = 1/(2\pi\sqrt{(LC)})}$. Impedance is **minimum = R**, current is **maximum = V/R**, power is maximum.
- Below f_0 the series RLC is **capacitive** (leading). Above f_0 it is **inductive** (lagging).
- **Voltage magnification**: $V_L = V_C = \mathbf{Q \times V}$ at resonance. The voltages across L and C may be far greater than the supply voltage.
- V_C is maximum slightly **below** f_0 and V_L is maximum slightly **above** f_0. Both peaks coincide at f_0 only for high Q.
- **Half-power frequencies** f_1, f_2: the current falls to $\mathbf{I_{max}/\sqrt{2}}$ (0.707), the power to half, i.e. **−3 dB**. At these points |X| = R and the phase angle is ±45°.
- $\mathbf{f_0 = \sqrt{(f_1 f_2)}}$ (geometric mean). For high Q, $f_0 \approx (f_1 + f_2)/2$ and $f_{1,2} \approx f_0 \mp BW/2$.
- **Selectivity** ∝ Q. Higher Q gives narrower bandwidth and a sharper peak.

KEY FORMULAE – SERIES RLC RESONANCE

- $\omega_0 = 1/\sqrt{(LC)}$. $f_0 = 1/(2\pi\sqrt{(LC)})$.
- $\mathbf{Q = \omega_0 L/R = 1/(\omega_0 CR) = (1/R)\sqrt{(L/C)}}$. Q = 2π × (maximum energy stored ÷ energy dissipated per cycle).
- **Bandwidth** = $\mathbf{f_0/Q = R/(2\pi L)}$ Hz, or R/L rad/s. BW is independent of C.
- $f_{1,2} = \mp R/(4\pi L) + \sqrt{[(R/4\pi L)^2 + f_0^2]}$.

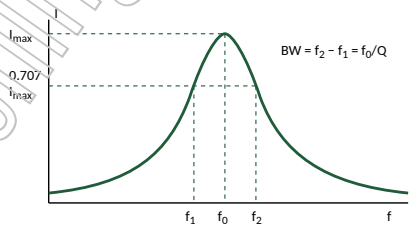

Fig 1.2 Series resonance curve: current is maximum at f_0, and the half-power points lie at 0.707 I_{max}.

- **Parallel resonance** (rejector / anti-resonant circuit), ideal G‖L‖C: $f_0 = 1/(2\pi\sqrt{(LC)})$. Impedance is **maximum = R**, line current is **minimum**.
- **Current magnification**: $I_L = I_C = Q \times I$. For an ideal parallel RLC, $\mathbf{Q = R/(\omega_0 L) = \omega_0 CR = R\sqrt{(C/L)}}$, the reciprocal form of the series case.
- Below f_0 the parallel circuit is **inductive**. Above f_0 it is **capacitive**, the opposite of series.
- **Tank circuit** (practical coil R–L in parallel with C): $\mathbf{f_0 = (1/2\pi)\sqrt{(1/(LC) - R^2/L^2)}}$. If R = 0 it reduces to $1/(2\pi\sqrt{(LC)})$.
- Resonance in a tank circuit is **not possible if** $\mathbf{R > \sqrt{(L/C)}}$, i.e. $R^2 > L/C$.
- **Dynamic impedance** $\mathbf{Z_D = L/(CR)}$, purely resistive. A smaller coil resistance gives a **larger** $\mathbf{Z_D}$. Q of a tank circuit = $\omega_0 L/R$.
- Applications: the series resonant circuit is used for tuning and for harmonic (filter) traps. The parallel tank is used in oscillators and in IF/RF amplifiers as a band-pass load.

Property	Series RLC	Parallel (tank)
Also called	**Acceptor**	**Rejector**
Impedance at f_0	**Minimum (R)**	**Maximum (L/CR)**
Current at f_0	Maximum	Minimum
Magnification	**Voltage** ($V_L = V_C = QV$)	**Current** ($I_C \approx QI$)
Q (ideal)	$\omega_0 L/R$	$R/(\omega_0 L)$ (ideal parallel R) or $\omega_0 L/R$ (coil R)
Below f_0	Capacitive	Inductive
Above f_0	Inductive	Capacitive
Power factor at f_0	Unity	Unity

- **Ideal filter**: a perfectly flat pass band (gain 1), zero gain in the stop band and a vertical transition. It is **physically unrealisable** (non-causal).
- **Cut-off frequency** f_c: the −3 dB point, where gain = $1/\sqrt{2} = 0.707$ and power is half.
- First-order roll-off = **−20 dB/decade** (−6 dB/octave). Second-order = −40 dB/decade. An n-th order filter rolls off at **−20n dB/decade**.

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- RC low-pass (output across C) and RC high-pass (output across R): **$f_c = 1/(2\pi RC)$**. RL low-pass (output across R) and RL high-pass (output across L): **$f_c = R/(2\pi L)$**.
- At f_c the phase shift of a first-order LPF is **−45°**, tending to −90° as $f \to \infty$. For an HPF it is +45° at f_c.
- A series RLC with output across R is a **band-pass** filter. With output across the L–C combination it is a **band-stop (notch)** filter. Band-pass BW = $f_H - f_L$.
- **Constant-k filter**: $Z_1Z_2 = R_0^2 = L/C$. LPF $f_c = 1/(\pi\sqrt{(LC)})$. HPF $f_c = 1/(4\pi\sqrt{(LC)})$. Its attenuation rises slowly beyond cut-off.
- **m-derived filter**: gives a sharper cut-off (infinite attenuation at f_∞). For an LPF, $m = \sqrt{(1 - (f_c/f_\infty)^2)}$, with $0 < m \le 1$. A **composite filter** combines constant-k and m-derived sections.
- Attenuation unit: **1 neper = 8.686 dB**. dB = $20 \log_{10}(V_2/V_1) = 10 \log_{10}(P_2/P_1)$.

Filter	Passes	Simple circuit (output across)
Low-pass	0 to f_c	Series R, shunt **C** / series L, shunt R
High-pass	f_c to ∞	Series C, shunt **R** / series R, shunt L
Band-pass	f_1 to f_2	Series RLC, output across R
Band-stop (notch)	All except f_1–f_2	Series RLC, output across LC
All-pass	All (only phase changes)	Lattice; used as a phase equaliser

MEMORY TRICK

"Series Sucks current in, Parallel Pushes it away": series gives minimum Z and maximum I (acceptor), parallel gives maximum Z and minimum I (rejector). For a **LPF the C goes down to ground**, because it shorts high frequencies to ground.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Series RLC: f_0 = 10 kHz, Q = 50. Bandwidth? → **200 Hz**.
- Series RLC: R = 10 Ω, L = 0.1 H, C = 10 µF. Q? → **10** ($(1/R)\sqrt{(L/C)} = (1/10)\sqrt{10^4}$).
- Dynamic impedance of a tank circuit? → **L/(CR)**.
- Parallel resonant circuit is also called? → **Rejector circuit**.
- Roll-off of a first-order filter? → **20 dB/decade**.
- Half-power frequencies 90 kHz and 110 kHz. Approximate Q? → **5** ($f_0 \approx 100$ kHz, BW = 20 kHz).

1.7 Two-Port Networks ★★★

- **Two-port**: a network with two pairs of terminals (input 1–1′ and output 2–2′). The current entering one terminal of a port leaves the other terminal of the same port. Convention: I_1 and I_2 are both taken as **entering** the network.
- There are 4 variables (V_1, I_1, V_2, I_2). Any 2 can be expressed in terms of the other 2, giving **6 possible parameter sets** (Z, Y, h, g, ABCD, inverse ABCD).

Parameter	Equations	Obtained by	Main use
Z (open-circuit impedance)	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$	Port **open** ($I = 0$)	Series-connected networks, T-network
Y (short-circuit admittance)	$I_1 = Y_{11}V_1 + Y_{12}V_2$ $I_2 = Y_{21}V_1 + Y_{22}V_2$	Port **shorted** ($V = 0$)	Π-network, high-frequency circuits
h (hybrid)	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$	Output short ($V_2 = 0$) / input open ($I_1 = 0$)	**Transistor (BJT)** models
g (inverse hybrid)	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$	Mixed	Parallel–series connection
ABCD (transmission, T)	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$	Output open / output shorted	**Transmission lines**, cascade

- **h-parameter meanings**: h_{11} = input impedance with output shorted (Ω). h_{12} = reverse voltage ratio with input open (dimensionless). h_{21} = forward current gain with output shorted (dimensionless). h_{22} = output admittance with input open (S).
- **ABCD meanings**: $A = V_1/V_2$ ($I_2 = 0$), dimensionless, the reverse voltage ratio. $B = -V_1/I_2$ ($V_2 = 0$), in Ω, the transfer impedance. $C = I_1/V_2$ ($I_2 = 0$), in S, the transfer admittance. $D = -I_1/I_2$ ($V_2 = 0$), dimensionless, the reverse current ratio.
- For a transmission line, A = D, and AD − BC = 1. Short line: A = D = 1, B = Z, C = 0.
- **$Y = Z^{-1}$**, so $Y_{11} = Z_{22}/\Delta Z$, $Y_{12} = -Z_{12}/\Delta Z$, and so on. Similarly $g = h^{-1}$.
- **Z in terms of ABCD**: $Z_{11} = A/C$, $Z_{12} = \Delta T/C$, $Z_{21} = 1/C$, $Z_{22} = D/C$, where $\Delta T = AD - BC$.
- **h in terms of Z**: $h_{11} = \Delta Z/Z_{22}$, $h_{12} = Z_{12}/Z_{22}$, $h_{21} = -Z_{21}/Z_{22}$, $h_{22} = 1/Z_{22}$.
- **T-network** (series arms Z_a, Z_b, shunt Z_c): **$Z_{11} = Z_a + Z_c$**, $Z_{22} = Z_b + Z_c$, **$Z_{12} = Z_{21} = Z_c$**.
- **Π-network** (shunt Y_a, Y_b, series Y_c): $Y_{11} = Y_a + Y_c$, $Y_{22} = Y_b + Y_c$, **$Y_{12} = Y_{21} = -Y_c$**.
- ABCD of a **series impedance Z** = [1 Z; 0 1]. ABCD of a **shunt admittance Y** = [1 0; Y 1]. ABCD of an **ideal transformer** n:1 = [n 0; 0 1/n].
- A **series element alone has no Z-parameters** (Z matrix does not exist), though its Y-parameters exist. A **shunt element alone has no Y-parameters**. An **ideal transformer has neither Z nor Y** parameters, but its h and ABCD parameters exist.
- **Image impedances**: $Z_{i1} = \sqrt{(AB/CD)}$, $Z_{i2} = \sqrt{(BD/AC)}$. For a symmetric network the **characteristic impedance $Z_0 = \sqrt{(B/C)} = \sqrt{(Z_{oc}Z_{sc})}$**.
- Input impedance with load Z_L: **$Z_{in} = (AZ_L + B)/(CZ_L + D)$**.

Parameter	Reciprocity condition	Symmetry condition
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
h	$h_{12} = -h_{21}$	$\Delta h = h_{11}h_{22} - h_{12}h_{21} = 1$
g	$g_{12} = -g_{21}$	$\Delta g = 1$
ABCD	**AD − BC = 1**	**A = D**

- Every **passive, linear, bilateral** network (R, L, C, mutual inductance, ideal transformer) is **reciprocal**. Networks with dependent sources are generally non-reciprocal.
- Independent parameters: a general two-port has 4, a reciprocal one has **3**, and a reciprocal and symmetric one has **2**.
- Symmetry means the ports can be interchanged without changing port voltages and currents. A symmetric network is not necessarily reciprocal unless it is also passive.

Interconnection	Parameters that add / multiply
Series–series (series)	**Z** add: $[Z] = [Z_a] + [Z_b]$
Parallel–parallel (parallel)	**Y** add
Series (input)–parallel (output)	**h** add
Parallel (input)–series (output)	**g** add
Cascade (tandem)	**ABCD multiply** (in order): $[T] = [T_a][T_b]$

MEMORY TRICK

"Z-Series, Y-Parallel, h-Hybrid (Series–Parallel), ABCD-Cascade". For reciprocity, **only the hybrids carry a minus sign** ($h_{12} = -h_{21}$). For symmetry, **only the hybrids use a determinant rule** ($\Delta h = 1$, $\Delta g = 1$). Z, Y and ABCD use equal diagonal terms ($Z_{11} = Z_{22}$, $Y_{11} = Y_{22}$, A = D).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Condition for reciprocity in ABCD parameters? → **AD − BC = 1**.
- Condition for symmetry in h-parameters? → **$h_{11}h_{22} - h_{12}h_{21} = 1$**.
- Parameters used to analyse cascaded networks? → **ABCD (transmission)**.
- Unit of the B parameter? → **Ohm**. Unit of h_{22}? → **Siemens (mho)**.
- Parameters most used for BJT small-signal analysis? → **h-parameters**.
- Two-ports connected in series–parallel. Which parameters add? → **h-parameters**.

1.8 Three-Phase Circuits ★★★

- A three-phase supply has 3 EMFs of equal magnitude displaced by **120° electrical**.

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- Advantages over single-phase: **constant (non-pulsating) total instantaneous power**, rotating magnetic field (self-starting induction motors), and about **75%** of the conductor copper for the same power, voltage and losses. Machines are also smaller for the same rating.
- **Phase sequence**: the order in which the phases reach their maximum. **RYB (ABC) = positive** sequence: $V_R \angle 0°$, $V_Y \angle -120°$, $V_B \angle +120°$. RBY is negative sequence.
- Interchanging **any two** supply lines reverses the phase sequence and hence the direction of rotation of a three-phase motor.
- Phase sequence is found with a **phase sequence indicator**: the rotating (small induction-motor) type, or the static type using 2 lamps and 1 capacitor. In the static type the lamp in the phase that **follows** the capacitor phase in sequence glows brighter.
- **Balanced load**: equal impedances (magnitude and angle) in all phases. In a balanced star, **neutral current = 0**.

Quantity	Star (Y)	Delta (Δ)
Line voltage	$\mathbf{V_L = \sqrt{3}\, V_{ph}}$	$V_L = V_{ph}$
Line current	$I_L = I_{ph}$	$\mathbf{I_L = \sqrt{3}\, I_{ph}}$
Phase relation (positive sequence)	V_L **leads** its V_{ph} by **30°**	I_L **lags** its I_{ph} by **30°**
Neutral	Available (3-phase 4-wire)	Not available (3-wire)
Typical use	LV distribution (400/230 V, 4-wire)	Motors; 11 kV (HV) winding of distribution transformers (Dyn11)
Power (both)	$\mathbf{P = \sqrt{3}\, V_L I_L \cos\varphi = 3\, V_{ph} I_{ph} \cos\varphi}$	

- In $P = \sqrt{3}\, V_L I_L \cos\varphi$, **φ is the angle between phase voltage and phase current** (the load impedance angle), **not** the angle between line quantities.
- $Q = \sqrt{3}\, V_L I_L \sin\varphi$. $S = \sqrt{3}\, V_L I_L$.
- The same three impedances **connected in Δ take 3 times the power** (and 3 times the line current) of the Y connection on the same supply. Equivalent load: $\mathbf{Z_Y = Z_\Delta/3}$.
- Indian LV supply: 415 V line, 240 V phase (now standardised as **400/230 V** per IS 12360).
- Per-phase analysis: convert Δ to equivalent Y and solve one phase (line-to-neutral). Multiply the power by 3.

KEY FORMULAE – TWO-WATTMETER METHOD (BALANCED, LAGGING PF)

- $\mathbf{W_1 = V_L I_L \cos(30° - \varphi)}$. $\mathbf{W_2 = V_L I_L \cos(30° + \varphi)}$.
- Total power $\mathbf{P = W_1 + W_2}$. This is valid for any 3-wire load, balanced or unbalanced, star or delta.
- $W_1 - W_2 = V_L I_L \sin\varphi$. $\mathbf{\tan\varphi = \sqrt{3}\,(W_1 - W_2)/(W_1 + W_2)}$.
- Reactive power $\mathbf{Q = \sqrt{3}\,(W_1 - W_2)}$.
- For a leading pf the readings interchange (W_2 becomes the larger one).

Power factor (φ)	W_1	W_2	Remark
Unity (0°)	$\sqrt{3}\, V_L I_L/2$	Equal to W_1	$W_1 = W_2$
0.866 (30°)	$V_L I_L$	$V_L I_L/2$	$W_1 = 2W_2$
0.5 (60°)	$\sqrt{3}\, V_L I_L/2$	**0**	Total power read by one meter
Between 0.5 and 0	Positive	Negative	Reverse the current-coil (or pressure-coil) connections; $P = W_1 - \lvert W_2 \rvert$
Zero (90°)	$V_L I_L/2$	$-V_L I_L/2$	$W_1 = -W_2$, P = 0

- **Blondel's theorem**: an n-wire system needs **(n − 1) wattmeters** to measure total power. A 3-phase 3-wire system needs 2, and a 3-phase 4-wire system needs **3**.
- **One-wattmeter method** (balanced load only): total P = 3 × reading, with the meter connected between line and neutral (real or artificial).
- **Reactive power with one wattmeter**: current coil in line R, pressure coil across Y–B. The reading = $V_L I_L \sin\varphi$, so **Q = √3 × reading**.
- **Unbalanced 4-wire star**: each phase voltage is fixed by the neutral. $\mathbf{I_N = I_R + I_Y + I_B}$ (phasor sum) ≠ 0.
- **Unbalanced 3-wire star** (neutral absent or broken): the load star point shifts. The **neutral shift voltage** $\mathbf{V_{N'N} = (V_R Y_R + V_Y Y_Y + V_B Y_B)/(Y_R + Y_Y + Y_B)}$ (Millman). The phase with the **highest impedance gets the highest voltage**, which can damage lamps and appliances.
- The sum of the line currents of any 3-wire system is **zero** (KCL). The phasor sum of the line voltages is always zero.
- Unbalanced systems are analysed with **symmetrical components** (Fortescue, 1918): positive, negative and zero sequence. Zero-sequence current cannot flow in a 3-wire system or into a Δ-connected load's lines.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In the two-wattmeter method one meter reads zero. Power factor? → **0.5**.
- Both wattmeters read equal values. Power factor? → **Unity**.
- Readings equal in magnitude but opposite in sign. Power factor? → **Zero**.
- Star-connected 400 V system. Phase voltage? → **≈ 231 V**.
- Delta load with phase current 10 A. Line current? → **17.32 A**.
- Wattmeters needed for a 3-phase 4-wire unbalanced load? → **3**.

1.9 Magnetically Coupled Circuits ★★☆

- **Mutual inductance M**: the flux linkage in coil 2 per ampere of current in coil 1. $M = N_2\varphi_{21}/I_1 = N_1\varphi_{12}/I_2$, with $M_{12} = M_{21} = M$ (reciprocity). Unit: **henry**.
- Mutually induced EMF: $\mathbf{e_2 = \pm M\, di_1/dt}$. The coupled-coil equation is $v_1 = L_1\, di_1/dt \pm M\, di_2/dt$.
- **Coefficient of coupling** $\mathbf{k = M/\sqrt{(L_1 L_2)}}$, with $\mathbf{0 \le k \le 1}$. k = 1 means perfect (tight) coupling with no leakage flux.
- Iron-core transformers have **k ≈ 0.99+**. Air-core coils are usually loosely coupled (k < 0.5). Coils at **right angles** have k ≈ 0.
- M depends on the number of turns, the coil geometry and orientation, the separation and the core permeability.
- **Dot convention**: if currents enter (or both leave) the **dotted terminals** of both coils, the mutual voltage term has the **same sign** as the self-inductance term (+M, fluxes aid). If one current enters a dotted terminal and the other enters an undotted terminal, the term is **−M** (fluxes oppose).
- Polarity of the mutual voltage: a current entering the dotted end of one coil makes the **dotted end of the other coil positive**.
- Energy stored in coupled coils: $\mathbf{W = \tfrac{1}{2}L_1 i_1^2 + \tfrac{1}{2}L_2 i_2^2 \pm M\, i_1 i_2}$. Requiring W ≥ 0 gives $M \le \sqrt{(L_1 L_2)}$, i.e. $k \le 1$.
- **Conductively coupled (T) equivalent** of coupled coils with a common terminal: series arms **(L_1 − M)** and **(L_2 − M)**, shunt arm **M**. An arm may be negative, which is mathematically valid.
- **Reflected (coupled) impedance** into the primary = $\mathbf{\omega^2 M^2/Z_{22}}$, where Z_{22} is the total secondary loop impedance. A secondary inductive reactance reflects as a **capacitive** one (sign reversal).

KEY FORMULAE – COUPLED COILS

- Series aiding (cumulative): $\mathbf{L_{eq} = L_1 + L_2 + 2M}$.
- Series opposing (differential): $\mathbf{L_{eq} = L_1 + L_2 - 2M}$.
- Hence $\mathbf{M = (L_{aid} - L_{opp})/4}$. This is the standard experimental method of measuring M.
- Parallel aiding: $L_{eq} = (L_1 L_2 - M^2)/(L_1 + L_2 - 2M)$. Parallel opposing: $L_{eq} = (L_1 L_2 - M^2)/(L_1 + L_2 + 2M)$.
- Ideal transformer: $\mathbf{V_1/V_2 = N_1/N_2 = a}$, $\mathbf{I_1/I_2 = N_2/N_1 = 1/a}$, $V_1 I_1 = V_2 I_2$.
- Impedance referred to primary: $\mathbf{Z_{in} = a^2 Z_L = (N_1/N_2)^2 Z_L}$.
- Matching for maximum power: $\mathbf{a = N_1/N_2 = \sqrt{(R_s/R_L)}}$.

Ideal transformer assumption	Practical meaning
Coupling coefficient k = 1	No leakage flux
$L_1, L_2 \to \infty$ ($\mu \to \infty$)	Zero magnetising current
Winding resistance = 0	No copper loss
No core loss	Efficiency 100%
$L_1/L_2 = (N_1/N_2)^2$	Inductance $\propto N^2$

- The ideal transformer is a **lossless, memoryless** two-port. It passes DC in theory, but a real transformer **does not work on DC** (no dφ/dt, and the primary draws a large current).

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- An ideal transformer changes impedance level but **not the impedance angle** (pf), since a^2 is real.
- Inductance of a coil: $L = N^2/S = \mu_0\mu_r N^2 A/l$. **$L \propto N^2$**, so doubling the turns quadruples L.

MEMORY TRICK

"Dots Together = Plus": both currents enter the dots, or both leave, gives +M. **"Aid minus Oppose, divide by Four"** gives M. Impedance seen through a transformer is **"squared by the turns ratio"**: step-down to a low R_L makes it look large on the primary side.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Two coils: L_1 = 4 H, L_2 = 9 H, k = 0.5. M? → **3 H**.
- Series aiding inductance 1.5 H, series opposing 0.5 H. M? → **0.25 H**.
- 8 Ω speaker on a 10:1 ideal transformer. Impedance seen at the primary? → **800 Ω**.
- Maximum possible value of the coefficient of coupling? → **1**.
- Coupling coefficient of two coils placed at right angles? → **≈ 0**.
- Turns ratio needed to match a 1600 Ω source to a 16 Ω load? → **10:1**.

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2. Electromagnetic Fields

NOTE

EMF gives 2–4 questions in most EE PSU papers. Most asked: E/H formulae for standard charge/current distributions, capacitance & inductance of standard geometries, Maxwell's equations (which law ↔ which equation), boundary conditions, skin depth, 377 Ω and magnetic-circuit numericals (reluctance, air gap, hysteresis/eddy loss scaling). Memorise the formulae tables – most questions are one-step substitutions.

2.1 Vector Analysis & Coordinate Systems ★★☆

- **Dot product** $A\cdot B = |A||B|\cos\theta = A_xB_x + A_yB_y + A_zB_z$ → a **scalar**; commutative. $A\cdot B = 0 \Rightarrow$ vectors are **perpendicular**.
- **Cross product** $A\times B = |A||B|\sin\theta\, a_n$ → a **vector** normal to the plane of A and B; **anti-commutative** ($A\times B = -B\times A$). $A\times B = 0 \Rightarrow$ vectors are **parallel**. $|A\times B|$ = area of parallelogram.
- Unit-vector rules: $a_x\times a_y = a_z$, $a_y\times a_z = a_x$, $a_z\times a_x = a_y$ (cyclic). Cylindrical: $a_\rho\times a_\varphi = a_z$. Spherical: $a_r\times a_\theta = a_\varphi$.
- **Scalar triple product** $A\cdot(B\times C)$ = **volume of parallelepiped**; = 0 ⇒ the three vectors are **coplanar**.
- **Vector triple product** $A\times(B\times C) = B(A\cdot C) - C(A\cdot B)$ (**"BAC – CAB"**).
- **Gradient** ∇V – vector; gives **direction and magnitude of maximum rate of increase** of a scalar; always **normal to the level (equipotential) surface**. Directional derivative = $\nabla V\cdot a_l$.
- **Divergence** ∇·A – scalar; net outward flux per unit volume (source/sink strength). Positive = source, negative = sink.
- **Curl** ∇×A – vector; circulation per unit area (rotation of field); direction by right-hand rule.
- **Laplacian** $\nabla^2V = \nabla\cdot\nabla V$ – scalar operator (of a scalar gives scalar; vector Laplacian of a vector gives vector).
- **Solenoidal** (divergence-less) field: ∇·A = 0 → e.g. **B**, J in steady state. Can be written A = ∇×F.
- **Irrotational** (conservative) field: ∇×A = 0 → e.g. **electrostatic E**. Can be written A = −∇V; $\oint A\cdot dl = 0$.
- Identities: **$\nabla\cdot(\nabla\times A) = 0$** (divergence of a curl is always zero); **$\nabla\times(\nabla V) = 0$** (curl of a gradient is always zero); $\nabla\times\nabla\times A = \nabla(\nabla\cdot A) - \nabla^2 A$.
- **Helmholtz theorem**: a vector field is uniquely determined if its **divergence and curl** are specified (plus boundary conditions).

KEY FORMULAE – INTEGRAL THEOREMS

- **Divergence (Gauss–Ostrogradsky) theorem**: $\oint_S A\cdot dS = \int_V (\nabla\cdot A)\, dv$ → converts **closed surface ↔ volume** integral.
- **Stokes' theorem**: $\oint_L A\cdot dl = \int_S (\nabla\times A)\cdot dS$ → converts **closed line ↔ open surface** integral.
- Cartesian: $\nabla V = \partial V/\partial x\, a_x + \partial V/\partial y\, a_y + \partial V/\partial z\, a_z$; $\nabla^2V = \partial^2V/\partial x^2 + \partial^2V/\partial y^2 + \partial^2V/\partial z^2$.
- Cylindrical divergence: $\nabla\cdot A = (1/\rho)\partial(\rho A_\rho)/\partial\rho + (1/\rho)\partial A_\varphi/\partial\varphi + \partial A_z/\partial z$.
- Spherical divergence: $\nabla\cdot A = (1/r^2)\partial(r^2A_r)/\partial r + (1/r\sin\theta)\partial(\sin\theta\, A_\theta)/\partial\theta + (1/r\sin\theta)\partial A_\varphi/\partial\varphi$.

Quantity	Cartesian (x, y, z)	Cylindrical (ρ, φ, z)	Spherical (r, θ, φ)
Differential length dl	$dx\,a_x + dy\,a_y + dz\,a_z$	$d\rho\,a_\rho + \rho\, d\varphi\, a_\varphi + dz\,a_z$	$dr\,a_r + r\, d\theta\, a_\theta + r\sin\theta\, d\varphi\, a_\varphi$
Differential areas	dy dz, dx dz, dx dy	ρ dφ dz (a_ρ), dρ dz (a_φ), ρ dρ dφ (a_z)	$r^2\sin\theta\, d\theta\, d\varphi$ (a_r), r sin θ dr dφ (a_θ), r dr dθ (a_φ)
Differential volume dv	dx dy dz	**ρ dρ dφ dz**	**$r^2\sin\theta\, dr\, d\theta\, d\varphi$**
Metric (scale) factors	1, 1, 1	1, ρ, 1	1, r, r sin θ
Ranges	−∞ to ∞	ρ ≥ 0; 0 ≤ φ < 2π	r ≥ 0; **0 ≤ θ ≤ π**; 0 ≤ φ < 2π
Conversion	–	x = ρ cos φ, y = ρ sin φ	x = r sin θ cos φ, y = r sin θ sin φ, z = r cos θ

- In cylindrical/spherical systems, a_ρ, a_φ, a_r, a_θ are **not constant** – they change direction with position (only a_z is fixed).
- Surface area of sphere from $\int\int r^2\sin\theta\, d\theta\, d\varphi = \mathbf{4\pi r^2}$; volume $\int r^2\, dr... = \mathbf{4\pi r^3/3}$.

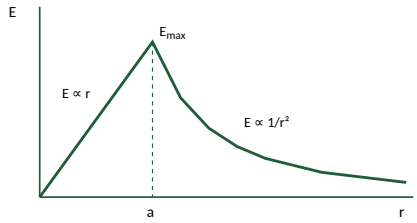
PREVIOUSLY ASKED / LIKELY QUESTIONS

- Divergence of curl of any vector is? → **Zero**.
- A field with ∇·A = 0 is called? → **Solenoidal**; with ∇×A = 0 → **Irrotational / conservative**.
- Stokes' theorem relates line integral to? → **Surface integral of curl**.
- Differential volume in spherical coordinates? → **$r^2\sin\theta\, dr\, d\theta\, d\varphi$**.
- Gradient of a scalar field is directed? → **Normal to the level surface, towards maximum increase**.

2.2 Coulomb's Law, E and D ★★★

- **Coulomb's law**: $F = Q_1Q_2/(4\pi\varepsilon_0\varepsilon_r R^2)\, a_R$ – along the line joining charges; like charges repel. Inverse-square law; obeys Newton's third law.
- **$\varepsilon_0 = 8.854\times10^{-12}$ F/m** ≈ $10^{-9}/36\pi$ F/m; **$k = 1/4\pi\varepsilon_0 \approx 9\times10^9$ N·m²/C²**.
- Electron charge e = **1.602×10^{-19} C**; electron mass 9.11×10^{-31} kg. 1 C ≈ 6.24×10^{18} electrons.
- In a medium of relative permittivity ε_r, force reduces by factor **ε_r** (water $\varepsilon_r \approx 80$ → force falls ~80 times).
- **Electric field intensity** E = F/Q (force per unit positive test charge); unit **V/m = N/C**. Superposition holds (linear medium).
- **Electric flux density** $D = \varepsilon E = \varepsilon_0\varepsilon_r E$; unit **C/m²**; D is **independent of medium** (for a given free charge), E depends on ε.
- **Electric flux** $\Psi = \int D\cdot dS$; unit **coulomb**. Total flux leaving a charge Q = **Q** (in SI, one coulomb of charge gives one coulomb of flux). Flux of E through a closed surface = Q/ε_0.
- Charge distributions: line ρ_L (C/m), surface ρ_s (C/m²), volume ρ_v (C/m³).
- Electric field lines start on +ve and end on −ve charges; never intersect; leave a conductor surface **normally**.

Charge distribution	Electric field E	Variation
Point charge Q	**$Q/(4\pi\varepsilon_0 r^2)$** a_r	∝ $1/r^2$
Infinite line charge ρ_L	**$\rho_L/(2\pi\varepsilon_0\rho)$** a_ρ	∝ $1/\rho$
Infinite sheet ρ_s	**$\rho_s/(2\varepsilon_0)$** a_n	**Independent of distance**
Two parallel sheets $+\rho_s$, $-\rho_s$ (capacitor)	**ρ_s/ε_0** between; **0** outside	Uniform
Ring (radius a, charge Q), on axis at z	$Qz/[4\pi\varepsilon_0(a^2+z^2)^{3/2}]$	**0 at centre**; maximum at **z = a/√2**
Disc (radius a, ρ_s), on axis	$(\rho_s/2\varepsilon_0)[1 - z/\sqrt{(z^2+a^2)}]$	→ $\rho_s/2\varepsilon_0$ as a → ∞ (sheet)
Spherical shell (radius a, Q)	Outside: $Q/(4\pi\varepsilon_0 r^2)$; **inside: 0**	Discontinuous at r = a
Uniformly charged solid sphere (a, Q)	Outside: $Q/(4\pi\varepsilon_0 r^2)$; inside: **$Qr/(4\pi\varepsilon_0 a^3) = \rho_v r/3\varepsilon_0$**	∝ r inside; max at surface
Conducting sphere (charged)	Same as shell (charge on surface)	0 inside
Infinite cylinder of uniform ρ_v (radius a)	Inside $\rho_v\rho/2\varepsilon_0$; outside $\rho_v a^2/(2\varepsilon_0\rho)$	∝ ρ inside, $1/\rho$ outside



E versus r for a uniformly charged solid sphere of radius a (for a shell or conductor, E = 0 for r < a and the same $1/r^2$ curve outside).

MEMORY TRICK

"Point 4π, Line 2π, Sheet 2": point $Q/4\pi\varepsilon_0 r^2$, line $\rho_L/2\pi\varepsilon_0 r$, sheet $\rho_s/2\varepsilon_0$. Power of r drops by one each time: $1/r^2 \to 1/r \to r^0$.

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- E due to an infinite sheet of charge varies with distance as? → **Independent of distance**.
- Field inside a uniformly charged spherical shell? → **Zero**.
- Value of ε_0? → **8.854×10^{-12} F/m**.
- Unit of electric flux density D? → **C/m²**.
- On the axis of a charged ring, E is maximum at? → **$z = a/\sqrt{2}$** (zero at centre).
- Two point charges in air experience force F; placed in a medium of $\varepsilon_r = 4$, force becomes? → **F/4**.

2.3 Gauss's Law, Potential & Energy ★★★

- **Gauss's law**: total electric flux through any closed surface = **charge enclosed**: $\oint D \cdot dS = Q_{enc}$. Point (differential) form: **$\nabla \cdot D = \rho_v$** (Maxwell's 1st equation in electrostatics).
- Gauss's law is useful only when a **symmetric Gaussian surface** exists (spherical, cylindrical, planar symmetry). It is valid always; but E can be extracted easily only for symmetric cases.
- Charges outside the closed surface contribute **zero net flux**.
- **Potential** V = work done per unit positive charge in bringing it from infinity: $V_{AB} = -\int_B^A E \cdot dl$; unit **volt = J/C**.
- Point charge: **$V = Q/(4\pi\varepsilon_0 r)$** ($\propto 1/r$). Line charge: $V_{AB} = (\rho_L/2\pi\varepsilon_0)\ln(\rho_B/\rho_A)$ – absolute potential of infinite line charge cannot be referenced to infinity.
- Solid sphere (uniform ρ_v): potential at centre = **$3Q/(8\pi\varepsilon_0 a)$** = 1.5 × surface potential. Conducting sphere: V inside = surface value **$Q/(4\pi\varepsilon_0 a)$** (constant).
- **$E = -\nabla V$**: E points in direction of **decreasing potential**. Potential is a scalar → easier to compute than E.
- **Equipotential surface**: all points at same V; E is **perpendicular** to it; **no work** is done moving a charge on it. For a point charge: concentric spheres. Two equipotentials never intersect.
- Electrostatic field is **conservative**: $\oint E \cdot dl = 0 \Leftrightarrow$ **$\nabla \times E = 0$** (Kirchhoff's voltage law in field form). Work done is path-independent.
- **Electric dipole** (charges ±Q separated by d): dipole moment **p = Qd** (C·m), directed from **−Q to +Q**.
- Dipole potential $V = p\cos\theta/(4\pi\varepsilon_0 r^2) \rightarrow$ **$\propto 1/r^2$**; dipole field $E = p/(4\pi\varepsilon_0 r^3)(2\cos\theta\, a_r + \sin\theta\, a_\theta) \rightarrow$ **$\propto 1/r^3$**. (Quadrupole: $V \propto 1/r^3$.)
- Torque on dipole in uniform E: **T = p × E**; net force = 0 in uniform field. PE = −p·E.
- **Electron-volt**: 1 eV = **1.602×10^{-19} J**.

KEY FORMULAE – ENERGY, POISSON, LAPLACE

- Energy in a system of charges: $W = \frac{1}{2}\Sigma Q_k V_k$; continuous: $W = \frac{1}{2}\int \rho_v V\, dv = \frac{1}{2}\int D \cdot E\, dv$.
- Energy density: **$w_e = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}D \cdot E = D^2/2\varepsilon$** (J/m³).
- **Poisson's equation**: $\nabla^2 V = -\rho_v/\varepsilon$ (charged region).
- **Laplace's equation**: $\nabla^2 V = 0$ (charge-free region). Solutions are "harmonic functions".
- Uniqueness theorem: a solution of Laplace/Poisson satisfying all boundary conditions is the **only** solution.
- Work to move Q from B to A: $W = Q \cdot V_{AB}$.

- Laplace's equation applies to: capacitor dielectric (between plates), charge-free space, potential distribution in insulators. Poisson's: **p-n junction depletion region**, vacuum diode space-charge region.
- **Method of images**: a conducting plane is replaced by an **image charge of opposite sign** at the mirror position; valid only in the region **outside** the conductor (the real region).
- Point charge Q at height h above grounded plane → image −Q at depth h; force of attraction = **$Q^2/(4\pi\varepsilon_0(2h)^2) = Q^2/(16\pi\varepsilon_0 h^2)$**.
- Line charge above ground → image line charge $-\rho_L$ (used for capacitance of overhead line to earth, $2\pi\varepsilon/\ln(2h/a)$).
- Two conducting planes at angle θ: number of images **$N = (360°/\theta) - 1$** (θ = 90° → 3 images; 60° → 5).
- Charge near grounded conducting sphere (radius a, charge at distance d): image $Q' = -Qa/d$ at a^2/d from centre.

Quantity	Point charge	Dipole	Line charge
Potential V	$\propto 1/r$	**$\propto 1/r^2$**	$\propto \ln(1/\rho)$
Field E	$\propto 1/r^2$	**$\propto 1/r^3$**	$\propto 1/\rho$
Equipotentials	Spheres	Non-spherical	Coaxial cylinders

MEMORY TRICK

Poisson has Charge, Laplace has nothing ($\nabla^2 V = -\rho/\varepsilon$ vs 0). Each extra "pole" adds one power of r: monopole $V \propto 1/r$, dipole $1/r^2$, quadrupole $1/r^3$ (E always one power higher).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Point form of Gauss's law? → **$\nabla \cdot D = \rho_v$**.
- Potential due to an electric dipole varies as? → **$1/r^2$**; field → **$1/r^3$**.
- Laplace's equation applies to? → **Charge free region**.
- Energy density in electrostatic field? → **$\frac{1}{2}\varepsilon E^2$**.
- Number of images for a charge between two conducting planes at 60°? → **5**.
- E and equipotential surface are at angle? → **90°**.

2.4 Dielectrics, Conductors & Boundary Conditions ★★★

- **Polarization** P = dipole moment per unit volume (C/m²). Linear isotropic dielectric: **$P = \chi_e \varepsilon_0 E$**.
- **$D = \varepsilon_0 E + P = \varepsilon_0(1 + \chi_e)E = \varepsilon_0 \varepsilon_r E \Rightarrow \varepsilon_r = 1 + \chi_e$**. Vacuum: $\chi_e = 0$, $\varepsilon_r = 1$.
- Bound charges: surface $\rho_{ps} = P \cdot a_n$; volume $\rho_{pv} = -\nabla \cdot P$.
- Types of polarization: electronic, ionic, orientational (dipolar), interfacial (space-charge). Electronic is fastest (up to optical frequencies); only electronic exists in **monatomic gases**.
- **Dielectric strength** = max E a dielectric can withstand without breakdown. **Air ≈ 3 kV/mm (30 kV/cm peak, 21.2 kV/cm rms)** at NTP.
- Linear (ε independent of E), homogeneous (ε independent of position), isotropic (ε independent of direction). Crystals are anisotropic (ε is a tensor).
- **Conductors in electrostatic equilibrium**: **E = 0 inside**, $\rho_v = 0$ inside, charge resides on the **outer surface**, conductor is an **equipotential body**, E at surface is **normal** and = ρ_s/ε. Charge density is highest at **sharp points** (corona, lightning arrester points).
- **Current density** J = **σE** (point form of Ohm's law), unit A/m². Convection current $J = \rho_v u$ (no Ohm's law, e.g. electron beam). Drift velocity $u = \mu_e E$.
- **Continuity equation** (conservation of charge): **$\nabla \cdot J = -\partial\rho_v/\partial t$**. Steady current: $\nabla \cdot J = 0$ (KCL in field form).
- **Relaxation time** $\tau = \varepsilon/\sigma$ – time for charge placed inside a conductor to decay to 1/e (36.8%) of initial value: $\rho_v = \rho_0 e^{-t/\tau}$. Copper: **≈ 1.5×10^{-19} s**; fused quartz ≈ 51 days. Good conductor → very small τ.
- Resistance R = l/(σA); conductivity of copper **5.8×10^7 S/m**, aluminium 3.5–3.8 × 10^7 S/m, silver 6.1–6.3 × 10^7 S/m (highest).

Material	Typical ε_r	Material	Typical ε_r
Air	**1.0006**	Mica	5–7
Paper (dry)	2–4	Glass	4–10
Transformer oil	2.2–2.5	Porcelain	5–7
Polyethylene / XLPE	2.3	Distilled water	**≈ 80**
PVC	3–6	Barium titanate	1000–5000+
Rubber	2.5–3	SF_6 (dielectric strength ≈ 2.5 × air)	≈ 1.002

Interface	Tangential component	Normal component
Dielectric–dielectric (electric)	**$E_{t1} = E_{t2}$** (continuous); $D_{t1}/\varepsilon_1 = D_{t2}/\varepsilon_2$	**$D_{n1} - D_{n2} = \rho_s$**; if no free surface charge $D_{n1} = D_{n2} \rightarrow \varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$
Conductor–dielectric	**$E_t = 0$**	$D_n = \rho_s$ ($E_n = \rho_s/\varepsilon$)
Magnetic (two media)	**$H_{t1} - H_{t2} = K$** (surface current, A/m); if K = 0, H_t continuous	**$B_{n1} = B_{n2}$** (always continuous); $\mu_1 H_{n1} = \mu_2 H_{n2}$
Steady current (two conductors)	$J_{t1}/\sigma_1 = J_{t2}/\sigma_2$	**$J_{n1} = J_{n2}$**

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- **Law of refraction** of field lines (no surface charge): **$\tan\theta_1/\tan\theta_2 = \varepsilon_{r1}/\varepsilon_{r2}$** ($\theta$ from the normal); magnetic: $\tan\theta_1/\tan\theta_2 = \mu_{r1}/\mu_{r2}$.
- Flux lines bend **away from normal** on entering a medium of higher ε (or μ). Field lines leaving iron into air are nearly **normal** to the iron surface.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- At a dielectric boundary which component of E is continuous? → **Tangential**.
- Which component of B is always continuous? → **Normal**.
- Relaxation time of a medium? → **ε/σ**.
- Relation between ε_r and χ_e? → **$\varepsilon_r = 1 + \chi_e$**.
- Dielectric strength of air at NTP? → **≈ 3 kV/mm (30 kV/cm peak)**.
- Equation of continuity expresses? → **Conservation of charge**.

2.5 Capacitance of Simple Configurations ★★★

- **Capacitance** C = Q/V; unit farad (F = C/V). Depends only on **geometry and ε**, not on Q or V.
- General: $C = \varepsilon\oint E\cdot dS / \int E\cdot dl$. Product **$RC = \varepsilon/\sigma$** for any geometry filled with a homogeneous medium (relaxation time).
- Capacitance of an isolated sphere of radius equal to the **Earth (6400 km) ≈ 711 μF**.

Configuration	Capacitance
Parallel plate (area A, separation d)	**$C = \varepsilon A/d = \varepsilon_0\varepsilon_r A/d$**
Coaxial cylinder (length L, inner a, outer b)	**$C = 2\pi\varepsilon L/\ln(b/a)$**
Concentric spheres (inner a, outer b)	**$C = 4\pi\varepsilon\, ab/(b-a)$**
Isolated sphere (radius a)	**$C = 4\pi\varepsilon a$** ($b \to \infty$)
Two parallel wires (radius a, spacing d ≫ a), per metre	**$C = \pi\varepsilon/\ln(d/a)$** (exact: $\pi\varepsilon/\cosh^{-1}(d/2a)$)
Single wire at height h above ground, per metre	$C = 2\pi\varepsilon/\ln(2h/a)$
Single-phase line to neutral, per metre	$C_n = 2\pi\varepsilon/\ln(d/a) = 2 \times$ line-to-line value
n parallel plates (alternate connected)	$C = (n-1)\varepsilon A/d$

Arrangement	Equivalent	Formula
Dielectrics in layers **parallel to plates** (stacked between plates, thickness t_1, t_2...)	**Series**	$C = \varepsilon_0 A/(t_1/\varepsilon_{r1} + t_2/\varepsilon_{r2} + \ldots)$
Dielectrics side-by-side (each spanning full gap, areas A_1, A_2)	**Parallel**	$C = (\varepsilon_0/d)(\varepsilon_{r1}A_1 + \varepsilon_{r2}A_2)$
Slab of thickness t (ε_r) in air gap d	Series air + slab	**$C = \varepsilon_0 A/[d - t + t/\varepsilon_r]$**
Metal plate of thickness t inserted	$\varepsilon_r \to \infty$	$C = \varepsilon_0 A/(d - t)$

- In series layers, D is the same in each layer; **E is inversely proportional to ε_r** → the layer with **lower ε_r (e.g. air) is stressed more** – reason for air voids causing breakdown/partial discharge in cables. Principle of **grading of cables** (capacitance grading: higher ε near the core).
- Coaxial cable stress $g = V/[r\ln(R/r)]$; **maximum at the conductor surface**; minimum at sheath. For minimum stress at core, **R/r = e = 2.718**.
- Combination: capacitors in series $1/C = \Sigma 1/C_i$ (same Q); parallel $C = \Sigma C_i$ (same V) – opposite of resistors.

KEY FORMULAE – ENERGY & FORCE

- Stored energy **$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = Q^2/2C$**.
- Force between plates of a parallel-plate capacitor: **$F = Q^2/(2\varepsilon A) = \tfrac{1}{2}\varepsilon E^2 A$** (attractive).
- Battery connected (V constant) and d increased → C ↓, Q ↓, W ↓. Battery disconnected (Q constant) and d increased → C ↓, V ↑, **W ↑** (work done against attraction).
- Dielectric inserted with battery connected → Q and W increase ε_r times; isolated → V and W decrease ε_r times.

MEMORY TRICK

Cylinder → **ln** appears ($2\pi\varepsilon L/\ln(b/a)$); sphere → **product over difference** ($4\pi\varepsilon\, ab/(b-a)$); flat plates → **area over gap**. Two-wire line has **π** (half of 2π) because the two conductors act like two capacitances in series.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Capacitance of an isolated sphere of radius a? → **$4\pi\varepsilon_0 a$**.
- If plate separation is doubled and area halved, capacitance becomes? → **One-fourth**.
- Capacitance per metre of a coaxial cable? → **$2\pi\varepsilon/\ln(b/a)$**.
- In a cable, dielectric stress is maximum at? → **Conductor (core) surface**.
- Energy stored in a 10 μF capacitor at 100 V? → **0.05 J**.
- Two dielectrics stacked between plates behave as capacitors in? → **Series**.

2.6 Magnetostatics – Biot–Savart & Ampere ★★★

- Steady currents (DC) produce static magnetic fields; static charges produce static E. Moving charge with constant velocity produces both E and H.
- **Biot–Savart law**: $dH = I\,dl \times a_R/(4\pi R^2)$ → $dH \propto I\,dl\sin\theta/R^2$. Analogous to Coulomb's law. Direction: right-hand rule.
- **Magnetic field intensity H**: unit **A/m** (A-turn/m). **Magnetic flux density B = μH**: unit **tesla (T) = Wb/m²**; 1 T = **10^4 gauss**.
- **$\mu_0 = 4\pi \times 10^{-7}$ H/m** (≈ 1.2566×10^{-6}). $\mu = \mu_0\mu_r$.
- **Magnetic flux** $\Phi = \int B\cdot dS$; unit **weber (Wb)** = V·s. 1 Wb = 10^8 maxwell (lines).
- **Gauss's law for magnetism**: $\oint B\cdot dS = 0 \Rightarrow$ **$\nabla\cdot B = 0$** – no isolated magnetic poles (monopoles); flux lines are **closed loops**; B is solenoidal.
- **Ampere's circuital law**: $\oint H\cdot dl = I_{enc}$; point form **$\nabla\times H = J$** (steady currents). Consequence: $\nabla\cdot J = 0$ (only valid for steady currents → Maxwell added displacement current).
- Magnetic field is **non-conservative** ($\nabla\times H \neq 0$ where J exists).
- **Magnetic vector potential A**: **$B = \nabla\times A$**; unit Wb/m. $A = (\mu/4\pi)\int I\,dl/R$ (direction of current). $\Phi = \oint A\cdot dl$. Vector Poisson: **$\nabla^2 A = -\mu J$** (Coulomb gauge $\nabla\cdot A = 0$).
- Magnetic scalar potential V_m ($H = -\nabla V_m$) is defined only where **J = 0**.

Current configuration	Magnetic field H
Infinite straight wire, distance r	**$H = I/(2\pi r)$** (circles around wire, $\propto 1/r$)
Finite wire, perpendicular distance r (angles θ_1, θ_2 subtended at point from the perpendicular)	$H = (I/4\pi r)(\sin\theta_1 + \sin\theta_2)$
Semi-infinite wire, point on perpendicular at end	**$H = I/(4\pi r)$**
Circular loop, at centre (radius a)	**$H = I/(2a)$**; N turns: NI/2a
Circular loop, on axis at distance z	**$H = Ia^2/[2(a^2 + z^2)^{3/2}]$**
Square loop side a, at centre	**$H = 2\sqrt{2}\, I/(\pi a)$**
Long solenoid (n turns/m)	Inside **$H = nI = NI/l$** (uniform); at ends **nI/2**; outside ≈ 0
Toroid (N turns, mean radius r)	Inside **$H = NI/(2\pi r)$**; outside **0**
Infinite current sheet K (A/m)	**$H = K/2$** (independent of distance)
Coaxial cable (inner a, outer b)	$r < a$: $Ir/(2\pi a^2)$; $a < r < b$: **$I/(2\pi r)$**; outside: **0**
Inside a solid wire (radius a, uniform J)	$H = Ir/(2\pi a^2)$ – zero at axis, max at surface

KEY FORMULAE – FORCES & TORQUE

- **Lorentz force**: **$F = q(E + v\times B)$**. Magnetic part $q\,v\times B$ is $\perp v$ → **does no work**, changes only direction (speed and KE unchanged).
- Radius of circular motion $r = mv/(qB)$; cyclotron frequency $f = qB/(2\pi m)$ (independent of speed).
- Force on conductor: **$F = I\,L\times B$** → $F = BIL\sin\theta$ (Fleming's left-hand rule – motor).
- Force per metre between parallel conductors: **$F/l = \mu_0 I_1 I_2/(2\pi d) = 2\times 10^{-7} I_1 I_2/d$** N/m. **Same direction → attract**; opposite → repel.
- Magnetic dipole moment **$m = NIA$** ($A\cdot m^2$), direction normal to loop (right-hand rule).
- Torque on loop: **$T = m\times B$** → $T = NBIA\sin\theta$; maximum when plane of coil is **parallel** to B ($m \perp B$).
- Far field of a small current loop (magnetic dipole) $\propto$ **$1/r^3$**, like the electric dipole.

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CODE UPDATE

Since the **SI redefinition of 20 May 2019**, the ampere is defined by fixing e = 1.602176634 × 10^{-19} C exactly. μ_0 is now an experimentally determined value (≈ 4π × 10^{-7} H/m, within about 1 part in 10^9). The older definition (2 × 10^{-7} N/m between two conductors 1 m apart carrying 1 A) is still asked in MCQs as the "definition of ampere".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- H at the centre of a circular loop of radius a? → **I/2a**.
- Field outside an ideal toroid? → **Zero**.
- Two parallel wires carrying currents in the same direction? → **Attract**.
- ∇·B = 0 implies? → **Magnetic monopoles do not exist**.
- Point form of Ampere's law? → **∇×H = J**.
- Work done by magnetic force on a moving charge? → **Zero**.

2.7 Faraday's Law, Maxwell's Equations & Waves ★★★

- **Faraday's law**: induced emf = **−N dΦ/dt** = −dλ/dt (λ = flux linkage). **Lenz's law** gives the minus sign – induced current opposes the **cause** (change of flux); consequence of **conservation of energy**.
- **Transformer (statically induced) emf**: stationary loop, time-varying B: emf = $-\int(\partial B/\partial t)\cdot dS$. Example: transformer.
- **Motional (dynamically induced) emf**: moving conductor in static B: emf = $\oint(v \times B)\cdot dl$ = **Blv** (Fleming's right-hand rule – generator).
- General: both present → emf = $-\int\partial B/\partial t\cdot dS + \oint(v\times B)\cdot dl$ (e.g. AC generator / Faraday disc).
- **Displacement current** (Maxwell): $\mathbf{J_d = \partial D/\partial t}$; I_d = C dV/dt. Makes Ampere's law consistent with continuity equation; current "through" a capacitor. Exists even in vacuum.
- Ratio conduction to displacement current density = **σ/ωε** (= loss tangent).

Law	Point (differential) form	Integral form	Meaning
Gauss's law (electric)	$\nabla\cdot D = \rho_v$	$\oint D\cdot dS = Q_{enc}$	Charges are sources of E/D
Gauss's law for magnetism	$\nabla\cdot B = 0$	$\oint B\cdot dS = 0$	No monopoles; flux lines closed
Faraday's law	$\nabla\times E = -\partial B/\partial t$	$\oint E\cdot dl = -(d/dt)\int B\cdot dS$	Changing B induces E (emf)
Ampere–Maxwell law	$\nabla\times H = J + \partial D/\partial t$	$\oint H\cdot dl = I + \int(\partial D/\partial t)\cdot dS$	Current & changing D produce H

- For **static** fields: ∇×E = 0 and ∇×H = J. Divergence equations are **not independent** – they follow from the curl equations + continuity equation.
- Phasor (time-harmonic $e^{j\omega t}$) forms: ∇×E = −jωμH; ∇×H = (σ + jωε)E.
- Constitutive relations: D = εE, B = μH, J = σE.
- Wave equation (source-free, lossless): $\nabla^2 E = \mu\varepsilon\,\partial^2E/\partial t^2$. **TEM wave**: E ⊥ H ⊥ direction of propagation; E × H gives direction.
- Velocity **v = 1/√(με)**; free space **c = 1/√($\mu_0\varepsilon_0$) = 3 × 10^8 m/s**. In a dielectric v = c/√($\mu_r\varepsilon_r$); refractive index n = √($\mu_r\varepsilon_r$).
- **Intrinsic impedance** η = E/H = √(μ/ε); free space **η_0 = 120π ≈ 377 Ω**. Dielectric: η = 120π√(μ_r/ε_r) (e.g. ε_r = 4 → 60π ≈ 188.5 Ω).
- Phase constant β = ω√(με) = 2π/λ; λ = v/f.
- **Propagation constant** γ = α + jβ = √[jωμ(σ + jωε)]; α = attenuation (Np/m), 1 Np = **8.686 dB**.
- **Loss tangent** tan θ = **σ/(ωε)** → good dielectric if σ/ωε ≪ 1; good conductor if σ/ωε ≫ 1. A material's class depends on **frequency** (e.g. sea water/earth is a conductor at low f, dielectric at very high f).
- **Skin depth** δ = distance at which amplitude falls to **1/e (36.8%)** = **1/√(πfμσ)** = 1/α. Copper: **δ ≈ 66/√f mm** → ≈ **9.3 mm at 50 Hz**, ≈ 66 μm at 1 MHz. δ ∝ 1/√f.
- **Poynting vector** P = **E × H** (W/m²) – instantaneous power flow per unit area; average power density P_{avg} = ½Re(E × H*) = **E_0^2/(2η)**. Poynting theorem = energy conservation.

Parameter	Lossless dielectric (σ = 0)	Lossy dielectric	Good conductor (σ ≫ ωε)
α	**0**	≈ (σ/2)√(μ/ε) (low loss)	**√(πfμσ)**
β	ω√(με)	≈ ω√(με)	**√(πfμσ) = α**
η	√(μ/ε), real	complex, angle < 45°	(1 + j)/(σδ) = √(ωμ/σ)∠**45°**
E–H phase	**In phase**	H lags E by θ_η	**H lags E by 45°**
Velocity	1/√(με)	< 1/√(με)	ωδ (very small)

MEMORY TRICK

"**Gauss-Gauss-Faraday-Ampere**" = div D, div B, curl E, curl H (order of Maxwell's equations). Numbers to remember together: **3 × 10^8 m/s, 377 Ω (120π), 66/√f mm** for copper skin depth.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- ∇×H = J + ∂D/∂t is derived from? → **Ampere's law (with Maxwell's displacement current)**.
- Intrinsic impedance of free space? → **120π ≈ 377 Ω**.
- Skin depth varies with frequency as? → **1/√f**.
- Poynting vector represents? → **Power flow per unit area (W/m²)**.
- Phase difference between E and H in a good conductor? → **45°**.
- Lenz's law is a consequence of? → **Conservation of energy**.

2.8 Magnetic Circuits & Inductance ★★★

- **MMF** F = **NI** (ampere-turns, AT) – driving force for flux.
- **Reluctance** S = **l/($\mu_0\mu_r$A)**; unit **AT/Wb (= H^{-1})**. Flux Φ = MMF/S ("Ohm's law of magnetic circuit").
- **Permeance** = 1/S; unit **Wb/AT = henry**. Reluctivity = 1/μ.
- **Series magnetic circuit**: $S_{total} = \Sigma S_i$; total AT = Σ $H_i l_i$. **Parallel**: same as parallel resistances; flux divides in inverse ratio of reluctances.
- **Air gap**: although very short, it usually has reluctance **much greater than the iron path** (μ_r of iron ≈ 1000s) → most MMF is consumed in the gap. AT for gap = B l_g/μ_0 ≈ **8 × 10^5 B l_g**. Gaps are used in chokes/reactors to make inductance **linear**.
- **Fringing**: spreading of flux at the air gap → **increases effective gap area**, reduces B in gap; more for longer gaps. Usually ignored or gap area increased by gap length on each side.
- **Leakage flux**: flux not following the intended path. **Leakage coefficient (Hopkinson's)** λ = total flux/useful flux, typically **1.1–1.25** (always > 1). Leakage causes leakage reactance in transformers/machines.

Electric circuit	Magnetic circuit
EMF (V)	**MMF (AT)**
Current I (A)	**Flux Φ (Wb)**
Resistance R = ρl/A = l/σA	**Reluctance S = l/μA**
Conductance G	**Permeance**
Conductivity σ	**Permeability μ**
Resistivity ρ	Reluctivity
Current density J	Flux density B
Electric field E (V/m)	Magnetising force H (AT/m)
Energy lost continuously (I^2R) while current flows	Energy needed only to **create** flux, not to maintain it
Good insulators exist (current confined)	**No magnetic insulator** → leakage flux unavoidable
R independent of current (linear)	S depends on flux (μ varies; saturation – non-linear)

- **B–H (magnetisation) curve**: initial linear region → knee → **saturation**. μ_r = B/(μ_0H) is not constant – rises to a maximum then falls.
- **Retentivity / remanence (B_r)**: B left when H = 0. **Coercivity / coercive force (H_c)**: reverse H needed to reduce B to zero.
- **Hysteresis loss ∝ area of B–H loop** (energy per cycle per unit volume). **Steinmetz formula: P_h = η $B_m^{1.6}$ f V** watts (η = Steinmetz coefficient; exponent 1.6 empirical, 1.5–2.5 in practice).

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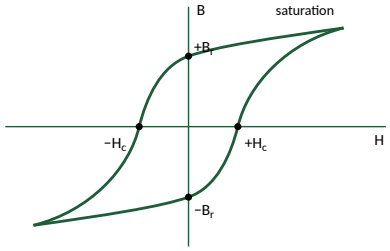
- **Eddy-current loss: $P_e = K_e B_m^2 f^2 t^2 V$**; per unit volume $P_e = \pi^2 B_m^2 f^2 t^2/(6\rho)$. Reduced by **laminations** (thin t: 0.35–0.5 mm for 50 Hz) and **high resistivity** (silicon steel 3–4% Si, ferrites).
- At constant V/f (constant B_m): $P_h \propto f$, $P_e \propto f^2$. At constant f: $P_h \propto B_m^{1.6}$, $P_e \propto B_m^2 \propto V^2$. Eddy loss in a transformer is **independent of frequency** if V is constant (since $B_m \propto V/f$).
- **Soft magnetic materials** (narrow loop, low H_c, high μ): CRGO silicon steel, permalloy, soft ferrites → transformers, motors, relays. **Hard** (wide loop, high B_r & H_c): Alnico, NdFeB (strongest), SmCo, barium/strontium ferrite → permanent magnets.



Hysteresis (B–H) loop: B_r = remanence, H_c = coercivity; loop area = hysteresis energy loss per cycle per m^3.

Type	χ_m / μ_r	Examples	Key feature
Diamagnetic	χ small **negative** (~-10^{-5}); μ_r slightly < 1	Bismuth (strongest), copper, silver, gold, water, Zn, superconductors (χ = −1, Meissner effect)	Weakly repelled; independent of temperature
Paramagnetic	χ small **positive** (10^{-5}–10^{-3}); μ_r slightly > 1	Aluminium, platinum, tungsten, oxygen, Mg	Weakly attracted; **Curie law χ = C/T**
Ferromagnetic	χ large positive; μ_r 10^2–10^5+	**Fe, Co, Ni, Gd**	Domains, hysteresis, saturation; paramagnetic above **Curie temperature** (Fe 770 °C, Ni ≈ 358 °C, Co ≈ 1120 °C); χ = C/(T − θ) (Curie–Weiss)
Antiferromagnetic	Small positive	MnO, NiO, Cr	Antiparallel equal moments; Néel temperature
Ferrimagnetic (ferrites)	Large	Fe_3O_4, MnZn/NiZn ferrites	Unequal antiparallel moments; **very high resistivity → negligible eddy loss**; HF cores

- **Self inductance** L = NΦ/I = λ/I (henry, H = Wb/A); also L = N^2/S = N^2 × permeance. L ∝ **N^2**.
- **Mutual inductance** M = $N_2\Phi_{12}/I_1$; **$M = k\sqrt{(L_1 L_2)}$**, coupling coefficient $0 \le k \le 1$ (k = 1 ideal, no leakage). $M_{12} = M_{21}$ (reciprocity). $M = N_1 N_2/S$ for a common core.
- Series aiding **$L_1 + L_2 + 2M$**; opposing **$L_1 + L_2 - 2M$** → $M = (L_{aid} - L_{opp})/4$. Parallel aiding: $(L_1L_2 - M^2)/(L_1 + L_2 - 2M)$; opposing: $(L_1L_2 - M^2)/(L_1 + L_2 + 2M)$.
- Internal inductance of a straight conductor (uniform current) = **μ/8π H/m** (= 0.5×10^{-7} H/m for $\mu_r = 1$), independent of radius.

Configuration	Self inductance
Long solenoid (N turns, area A, length l)	**$L = \mu N^2 A/l$**
Toroid, thin (mean radius r, cross-section A)	**$L = \mu N^2 A/(2\pi r)$**
Toroid, rectangular section (height h, radii a, b)	$L = (\mu N^2 h/2\pi) \ln(b/a)$
Coaxial cable, per metre (external)	**$L = (\mu/2\pi) \ln(b/a)$**; with inner-conductor internal: $(\mu/2\pi)[¼ + \ln(b/a)]$
Two-wire line, per metre (spacing d, radius a)	External **$(\mu_0/\pi) \ln(d/a)$**; total **$(\mu_0/\pi)[¼ + \ln(d/a)] = 4 \times 10^{-7} \ln(d/r')$**, r′ = 0.7788a (GMR)
Per conductor of 1-φ / 3-φ symmetric line	$2 \times 10^{-7} \ln(d/r')$ H/m

KEY FORMULAE – MAGNETIC ENERGY

- Energy stored in inductor **$W = ½LI^2$**; coupled coils $W = ½L_1 I_1^2 + ½L_2 I_2^2 \pm MI_1 I_2$.
- Energy density **$w_m = B^2/(2\mu) = ½BH = ½\mu H^2$** (J/$m^3$).
- Lifting (tractive) force of magnet: **$F = B^2A/(2\mu_0)$** N (A = pole-face area; per face).
- Magnetic circuit: Φ = NI/S; H = NI/l; B = $\mu_0\mu_r$NI/l.
- $L = \mu N^2 A/l$ → doubling N quadruples L; inserting core of μ_r multiplies L by μ_r.

MEMORY TRICK

"**Hysteresis 1.6 & f, Eddy all squared**": $P_h \propto B^{1.6} f$, $P_e \propto B^2 f^2 t^2$. Magnetic material order by χ: **Dia (−) < Para (+ small) < Ferro (+ huge)**; ferro elements "**Fe Co Ni Gd**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Unit of reluctance? → **AT/Wb (H^{-1})**.
- Hysteresis loss is proportional to? → **Area of the B–H loop**; Steinmetz exponent → **1.6**.
- Eddy-current loss varies with lamination thickness as? → **t^2**.
- If the number of turns of a coil is doubled, inductance becomes? → **4 times**.
- Coefficient of coupling when M = 0.5 H, $L_1 = L_2 = 1$ H? → **0.5**.
- Ferrites are used at high frequency because of? → **High resistivity (low eddy loss)**.

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3. Signals & Systems

NOTE

Signals & Systems gives 3–5 quick MCQs in GATE-EE style papers. Usual targets: **fundamental period**, **energy/power** classification, **system property** checks (linear/TI/causal/stable), **convolution length**, Fourier series **symmetry**, FT pairs, **Nyquist rate**, **final value theorem** and **ROC** of Laplace/Z-transforms. Most questions take under a minute if the standard results below are memorised.

3.1 Classification & Standard Signals ★★★

- **Continuous-time (CT)** signal x(t): defined for every t. **Discrete-time (DT)** signal x[n]: defined only at integer n. A DT signal with quantised amplitude = **digital** signal.
- **Periodic**: x(t + T) = x(t) for all t; smallest T > 0 = **fundamental period T_0**; $\omega_0 = 2\pi/T_0$. A constant (DC) is periodic but has **no fundamental period** (undefined).
- **Sum of two periodic CT signals** is periodic only if **T_1/T_2 is rational** (= p/q); then T_0 = **LCM(T_1, T_2)** = $qT_1 = pT_2$. Equivalently ω_0 = **HCF** of the frequencies.
- Example: cos 2t + sin 3t → $T_1 = \pi$, $T_2 = 2\pi/3$, ratio 3/2 → T_0 = **2π**. cos t + sin √2 t → ratio irrational → **aperiodic**.
- **DT sinusoid** cos(ω_0n) is periodic only if **$\omega_0/2\pi$ = m/N is rational**; fundamental period N = 2πm/ω_0 (smallest integer, m and N coprime). cos(n/4) is **not periodic** ($\omega_0/2\pi = 1/(8\pi)$, irrational) although cos(t/4) is.
- DT frequency is unique only over a **2π range**: cos(ω_0n) = cos((ω_0 + 2πk)n). Highest oscillation rate at **ω = π** ($(-1)^n$); lowest at ω = 0 or 2π.
- |sin t|, $\sin^2 t$, $\cos^2 t$ have period **π** (half of sin t).
- **Even**: x(−t) = x(t) (cos t, t^2, |t|); **Odd**: x(−t) = −x(t) (sin t, t, sgn t). Odd signal: x(0) = 0 and $\int_{-a}^{a} x\,dt = 0$.
- Every signal = even part + odd part: $x_e(t)$ = **[x(t) + x(−t)]/2**, $x_o(t)$ = **[x(t) − x(−t)]/2**. For u(t): x_e = 1/2, x_o = ½ sgn(t).
- Even × even = even; odd × odd = **even**; even × odd = **odd**. Energy of x = energy of x_e + energy of x_o.
- **Energy signal**: 0 < E < ∞, average power P = **0**. **Power signal**: 0 < P < ∞, E = **∞**. A signal cannot be both; some are **neither** (ramp t·u(t), e^{at} for all t, t u(t)).
- All bounded **finite-duration** signals are energy signals; all **periodic** (non-zero) signals are power signals.
- **Deterministic**: fully described by a formula/table. **Random**: described statistically (noise, speech). **Causal** signal: x(t) = 0 for t < 0; **anti-causal**: zero for t > 0.

Signal	Energy E	Power P	Type
$e^{-at}u(t)$, a > 0	**1/(2a)**	0	Energy
$e^{-a\|t\|}$, a > 0	**1/a**	0	Energy
rect pulse, amplitude A, width τ	**$A^2\tau$**	0	Energy
A cos(ω_0t + φ)	∞	**$A^2/2$**	Power
A $e^{j\omega_0 t}$	∞	**A^2**	Power
u(t) (unit step)	∞	**1/2**	Power
DC level A	∞	**A^2**	Power
u[n] (DT step)	∞	**1/2**	Power
$(0.5)^n u[n]$	**4/3** (=1/(1−0.25))	0	Energy
r(t) = t u(t), e^t	∞	∞	**Neither**

KEY FORMULAE

- $E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$; $P = \lim_{T\to\infty} (1/2T) \int_{-T}^{T} |x(t)|^2\,dt$
- DT: $E = \Sigma |x[n]|^2$; $P = \lim_{N\to\infty} [1/(2N+1)] \Sigma_{-N}^{N} |x[n]|^2$
- Periodic: $P = (1/T_0) \int_{T_0} |x(t)|^2\,dt$; $x_{rms} = \sqrt{P}$
- Scaling/shifting: $E\{A\,x(at - b)\} = A^2E/|a|$ (shift does not change E or P)

Standard signal	Definition	Key facts
Unit step u(t)	1 for t > 0, 0 for t < 0	u(0) undefined (or ½); du/dt = **δ(t)**
Unit impulse δ(t)	0 for t ≠ 0, ∫δ dt = 1	Even; area 1; δ(t) = du/dt
Unit ramp r(t)	t u(t)	dr/dt = **u(t)**; r = ∫u
Parabolic	$(t^2/2)$ u(t)	derivative = r(t)
Signum sgn(t)	+1 (t > 0), −1 (t < 0)	sgn(t) = **2u(t) − 1**; odd
rect(t/τ)	1 for |t| < τ/2	width τ, area τ; = u(t + τ/2) − u(t − τ/2)
Triangular Λ(t/τ)	1 − |t|/τ for |t| < τ	base 2τ; = rect * rect (scaled)
sinc	sinc(x) = **sin(πx)/(πx)** (normalised); Sa(x) = sin x/x	sinc(0) = 1; zeros at x = ±1, ±2…; ∫sinc dx = 1; even
Real exponential	Ae^{at}	a < 0 decaying; a > 0 growing; time constant 1/|a|
Complex exponential	e^{st}, s = σ + jω	Eigenfunction of LTI systems
Unit sample δ[n]	1 at n = 0, else 0	δ[n] = u[n] − u[n−1]; u[n] = $\Sigma_{k\le n}$ δ[k]

KEY FORMULAE – IMPULSE PROPERTIES

- Sifting: $\int x(t)\,\delta(t - t_0)\,dt = x(t_0)$ (t_0 inside limits, else 0)
- Product: $x(t)\,\delta(t - t_0) = x(t_0)\,\delta(t - t_0)$
- Scaling: $\delta(at) = \delta(t)/|a|$; $\delta(at - b) = (1/|a|)\,\delta(t - b/a)$; $\delta(-t) = \delta(t)$
- Doublet: $\int x(t)\,\delta'(t - t_0)\,dt = -x'(t_0)$; δ′(t) is odd
- $x(t) * \delta(t - t_0) = x(t - t_0)$; DT: $\Sigma\, x[n]\,\delta[n - k] = x[k]$

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Fundamental period of cos(3πn/8)? → $\omega_0/2\pi = 3/16$ → **N = 16**.
- Is x(t) = cos 4t + sin 6t periodic? → Yes, **T_0 = π**.
- $\int_{-\infty}^{\infty} e^{-t}\,\delta(2t - 2)\,dt$? → $(1/2)e^{-1}$ = **0.184**.
- Unit step u(t) is → **power signal**, P = 1/2.
- Energy of $e^{-2t}u(t)$? → **1/4**.
- Odd part of u(t)? → **½ sgn(t)**.

3.2 Operations on Signals ★★★

- **Time shifting**: x(t − t_0) → **delay** (shift right) if $t_0 > 0$; x(t + t_0) → **advance** (shift left).
- **Time scaling**: x(at) → **compression** by a if |a| > 1; **expansion** if |a| < 1. A point at t = t_1 in x(t) moves to t_1/a.
- **Time reversal (folding)**: x(−t) = mirror image about t = 0. x(−t + t_0) = x(−(t − t_0)): fold, then shift right by t_0.
- **Amplitude scaling**: A x(t) (|A| > 1 amplification, −x(t) = amplitude reversal). **Amplitude shift**: x(t) + B.
- **Order for x(at − b)**: Method 1 – **shift first** by b (x(t − b)), **then scale** by a (replace t by at). Method 2 – scale by a first, then shift by **b/a** since x(at − b) = x[a(t − b/a)]. Scale-then-shift-by-b is the common **wrong** answer.
- Quick check: the point of x at argument t_1 appears at t = **(t_1 + b)/a** in x(at − b).
- Example: x(t) non-zero over 0 ≤ t ≤ 4; x(2t − 2) non-zero over **1 ≤ t ≤ 3**; x(−t/2 + 1) over **−6 ≤ t ≤ 2**.
- **Time shift and reversal do not alter energy**; time scaling x(at) changes energy to E/|a|.
- Even/odd of shifted signal: x(t − t_0) loses its even/odd symmetry about origin in general.

DT operation	Form	Effect
Shift	x[n − k]	Delay by k samples (k integer only)
Reversal	x[−n]	Fold about n = 0
Decimation (down-sampling) by M	x[Mn]	Keeps every M-th sample; **information lost**; spectrum stretches → may alias; needs anti-aliasing filter
Interpolation (up-sampling) by L	x[n/L] for n multiple of L, else 0 (or interpolated)	Inserts **L − 1 zeros**; spectrum compresses, images appear → needs anti-imaging LPF
Sample-rate change by L/M	Up by L, then down by M	Interpolate **before** decimating to avoid loss

- x[2n] of {x[0], x[1], x[2], x[3], x[4]} = {x[0], x[2], x[4]} – odd samples **dropped**; hence x[n] → x[2n] is **not invertible**.

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- DT time scaling is defined only for integer results of n/a; x[n/2] is defined at even n.
- Length after decimation by M ≈ N/M; after zero-insertion up-sampling by L ≈ NL.

MEMORY TRICK

"**S**hift **B**efore **S**cale" (SBS) – in x(at – b) first shift by b, then scale by a. If you scale first, shift only by **b/a**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- x(t) = 0 outside 2 < t < 6; where is x(3t) non-zero? → **2/3 < t < 2**.
- x(–2t + 4) is obtained from x(t) by? → shift left 4, compress by 2, fold (or fold, compress by 2, shift **right by 2**).
- Up-sampling by L inserts how many zeros between samples? → **L – 1**.
- Which DT operation can lose information? → **Decimation** (down-sampling).
- If E{x(t)} = 10 J, energy of x(2t – 5)? → **5 J**.

3.3 System Properties & LTI Systems ★★★

- **Linearity** = additivity + homogeneity (**superposition**): a x_1 + b x_2 → a y_1 + b y_2. Test: zero input must give **zero output** – so y = 2x + 3 is **non-linear** (incrementally linear).
- **Time invariance**: x(t – t_0) → y(t – t_0). Red flags for **time-variance**: t multiplying x or appearing outside x (t x(t), x(t)cos ω_ct), scaled/reversed argument (x(2t), x(–t), x(sin t)).
- **Causality**: output at t depends only on present and **past** inputs. Any x(t + positive), x(–t), x(2t) → **non-causal**. All real-time physical systems are causal.
- **Memoryless (static)**: output depends on **present input only** (resistor, y = x^2). Dynamic: involves past/future (capacitor, inductor, delay, integrator). Memoryless ⇒ causal (not conversely).
- **BIBO stability**: every bounded input gives a bounded output. For LTI: $\int|h(t)|\,dt < \infty$ (DT: $\Sigma|h[n]| < \infty$) – **absolutely integrable/summable**.
- **Invertibility**: distinct inputs → distinct outputs (one-to-one). y = x^2, y = |x|, y = 0, x[2n] are **not invertible**. Inverse of delay T is advance T; inverse of integrator is differentiator. Inverse system: h(t) * h_{inv}(t) = δ(t).

System	Linear	TI	Causal	Memoryless	Stable
y = x^2(t)	No	Yes	Yes	Yes	Yes
y = t x(t)	Yes	**No**	Yes	Yes	**No**
y = x(2t)	Yes	No	**No**	No	Yes
y = x(t/2)	Yes	No	No (t < 0)	No	Yes
y = x(–t)	Yes	No	No	No	Yes
y = x(t – 2)	Yes	Yes	Yes	No	Yes
y = x(t + 2)	Yes	Yes	**No**	No	Yes
y = 2x(t) + 3	**No**	Yes	Yes	Yes	Yes
y = $\int_{-\infty}^{t} x(\tau)d\tau$	Yes	Yes	Yes	No	**No** (u(t) → ramp)
y = dx/dt	Yes	Yes	Yes	No	No
y = x(t) cos ω_ct (modulator)	Yes	**No**	Yes	Yes	Yes
y = $e^{x(t)}$	No	Yes	Yes	Yes	Yes
y = x(sin t)	Yes	No	No	No	Yes
y[n] = n x[n]	Yes	No	Yes	Yes	No
y[n] = x[n] + x[n – 1]	Yes	Yes	Yes	No	Yes
y[n] = $\Sigma_{k=-\infty}^{n}$ x[k] (accumulator)	Yes	Yes	Yes	No	**No**
y[n] = x[n]·x[n – 1]	No	Yes	Yes	No	Yes
y = Even{x(t)}	Yes	No	No	No	Yes

LTI systems and convolution

- An LTI system is **completely characterised by its impulse response h(t)**; y(t) = x(t) * h(t) = $\int$ x(τ) h(t – τ) dτ. DT: y[n] = Σ x[k] h[n – k].
- Convolution properties: **commutative** (x*h = h*x), **associative** (cascade: h_1*h_2), **distributive** (parallel: h_1 + h_2), identity **x * δ = x**, shift x(t)*δ(t – t_0) = x(t – t_0).
- Order of LTI systems in cascade **can be interchanged** (commutativity) – not true for non-linear/time-variant systems.
- **Graphical convolution** steps: (1) replace t by τ; (2) **fold** h(τ) → h(–τ); (3) **shift** by t → h(t – τ); (4) **multiply** by x(τ); (5) **integrate** (area) – repeat for each t.
- CT duration: if x lies in [a_1, b_1] and h in [a_2, b_2], y lies in **[a_1 + a_2, b_1 + b_2]**; width = sum of widths.
- DT length: N_1 and N_2 samples → **N_1 + N_2 – 1** samples; first index = sum of first indices.
- Area property: area of y = (area of x) × (area of h); DT: Σy = (Σx)(Σh).
- Example: {1, 2, 1} * {1, 1} = **{1, 3, 3, 1}** (length 3 + 2 – 1 = 4; sum 4 × 2 = 8 ✓).
- rect * rect (equal widths T) = **triangle** of base 2T; unequal widths → **trapezoid**. u(t) * u(t) = **r(t) = t u(t)**.
- e^{-at}u(t) * e^{-bt}u(t) = [(e^{-at} – e^{-bt})/(b – a)] u(t), a ≠ b; if a = b → **t e^{-at}u(t)**.
- **Eigenfunction**: e^{st} → H(s) e^{st}; $e^{j\omega t}$ → H(jω)$e^{j\omega t}$; sinusoid in → sinusoid out of **same frequency**, changed amplitude and phase only.
- Total response of LTI = **zero-input** (due to initial conditions) + **zero-state** (due to input) response; LTI convolution gives the zero-state part. (Also = natural + forced, but zero-state ≠ forced: it contains natural-mode terms too.)

LTI property	Condition on h(t)	Condition on h[n]
Causal	**h(t) = 0 for t < 0**	h[n] = 0 for n < 0
BIBO stable	$\int\|h(t)\|dt < \infty$	$\Sigma\|h[n]\| < \infty$
Memoryless	h(t) = **K δ(t)**	h[n] = K δ[n]
Invertible	$\exists$ h_i: h * h_i = δ(t)	h * h_i = δ[n]
FIR	–	h[n] of finite length → **always stable**

KEY FORMULAE

- Step response s(t) = $\int_{-\infty}^{t}$ h(τ) dτ = h(t) * u(t) ; h(t) = **ds/dt**
- DT: s[n] = $\Sigma_{k=-\infty}^{n}$ h[k] ; h[n] = **s[n] – s[n – 1]**
- Ramp response = $\int$ s(t) dt ; impulse response = d^2(ramp response)/dt^2
- Cascade: h = h_1 * h_2 (H = H_1H_2) ; Parallel: h = h_1 + h_2

PREVIOUSLY ASKED / LIKELY QUESTIONS

- y(t) = x(t) + 5 is → **non-linear**, time-invariant.
- h(t) = $e^{-2|t|}$: causal? stable? → **Non-causal, stable**.
- h[n] = 2^nu[n] → causal, **unstable**.
- Lengths 5 and 7 convolved → **11** samples.
- Step response 1 – e^{-t}; impulse response? → **e^{-t}u(t)**.
- An LTI system is memoryless only if h(t) = → **Kδ(t)**.

3.4 Fourier Series ★★★

- Applicable to **periodic** signals; gives a **discrete (line) spectrum** at multiples of $\omega_0 = 2\pi/T_0$. n = 1 → fundamental; n = 2 → second harmonic…
- **Dirichlet conditions** (sufficient, not necessary): over one period x(t) must be (1) **absolutely integrable**, (2) have a **finite number of maxima and minima**, (3) have a **finite number of finite discontinuities**.
- At a discontinuity the FS converges to the **average** of left and right limits (mid-point).
- **Gibbs phenomenon**: near a jump, the truncated series overshoots by **≈ 9% of the jump** (square wave ±A peaks at ≈ 1.18A); the overshoot **does not decrease** as more terms are added – it only narrows. Occurs only at **discontinuities**.
- For real x(t): $c_{-n} = c_n^*$ → **$|c_n|$ even**, **$\angle c_n$ odd** (magnitude spectrum even, phase spectrum odd).
- Time shift x(t – t_0) → $c_n e^{-jn\omega_0 t_0}$ (magnitudes unchanged). Differentiation → $jn\omega_0 c_n$. Time scaling x(at) → same c_n, fundamental becomes $a\omega_0$.
- **Coefficient decay**: discontinuous signal (square, sawtooth) → **1/n**; continuous but derivative discontinuous (triangular, full-wave rectified) → **$1/n^2$**; impulse train → **constant** (c_n = 1/T). Smoother signal → faster decay → less bandwidth.

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KEY FORMULAE

- Trigonometric: $x(t) = a_0 + \Sigma_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$
- $a_0 = (1/T)\int_T x\,dt$ (**average/DC**) ; $a_n = (2/T)\int_T x \cos n\omega_0 t\,dt$; $b_n = (2/T)\int_T x \sin n\omega_0 t\,dt$
- Polar: $x(t) = C_0 + \Sigma C_n \cos(n\omega_0 t + \theta_n)$; $C_n = \sqrt{(a_n^2 + b_n^2)}$, $\theta_n = -\tan^{-1}(b_n/a_n)$
- Exponential: $x(t) = \Sigma_{-\infty}^{\infty} c_n e^{jn\omega_0 t}$; $c_n = (1/T)\int_T x(t) e^{-jn\omega_0 t}\,dt$
- Relations: $c_0 = a_0$; $c_n = (a_n - jb_n)/2$; $|c_n| = C_n/2$
- **Parseval**: $P = (1/T)\int|x|^2 dt = \Sigma|c_n|^2 = a_0^2 + ½\,\Sigma(a_n^2 + b_n^2)$
- RMS $= \sqrt{[a_0^2 + \Sigma\,(C_n/\sqrt{2})^2]}$

Symmetry	Condition	Coefficients present
Even	$x(-t) = x(t)$	a_0, a_n only – $\mathbf{b_n = 0}$ (cosine terms); c_n real
Odd	$x(-t) = -x(t)$	$\mathbf{a_0 = a_n = 0}$; sine terms only; c_n imaginary
Half-wave (rotational)	$x(t \pm T/2) = -x(t)$	**Odd harmonics only**; $a_0 = 0$
Even + half-wave (quarter-wave even)	both	Odd-harmonic **cosines** only
Odd + half-wave (quarter-wave odd)	both	Odd-harmonic **sines** only
Hidden symmetry	$x(t) - a_0$ is even/odd	Apply rule after removing DC

Waveform (peak A, period T)	Fourier series	Harmonics / decay
Square wave ±A, odd	**(4A/π)**[sin $\omega_0 t$ + (1/3)sin $3\omega_0 t$ + (1/5)sin $5\omega_0 t$ + …]	Odd only, **1/n**
Square wave ±A, even	(4A/π)[cos $\omega_0 t$ − (1/3)cos $3\omega_0 t$ + (1/5)cos $5\omega_0 t$ − …]	Odd only, 1/n
Triangular ±A, even	**(8A/π²)**[cos $\omega_0 t$ + (1/9)cos $3\omega_0 t$ + (1/25)cos $5\omega_0 t$ + …]	Odd only, **1/n²**
Sawtooth −A to +A, odd	**(2A/π)**[sin $\omega_0 t$ − (1/2)sin $2\omega_0 t$ + (1/3)sin $3\omega_0 t$ − …]	**All** harmonics, 1/n
Half-wave rectified sine (peak A)	A/π + (A/2)sin $\omega_0 t$ − (2A/π)Σ cos $2k\omega_0 t/(4k^2 - 1)$	DC, fundamental, even harmonics
Full-wave rectified sine (peak A)	2A/π − (4A/π)Σ cos $2k\omega_0 t/(4k^2 - 1)$	DC + even harmonics of input (1/n²)
Impulse train $\delta_T(t)$	(1/T) Σ $e^{jn\omega_0 t}$	All, **equal** (c_n = 1/T)

- Square wave ±A: total power A^2; fundamental power = $(4A/\pi)^2/2 = 8A^2/\pi^2 \approx$ **$0.81A^2$ (81%)**.
- Rectangular pulse train (duty d = τ/T, amplitude A): c_n = **A d sinc(nd)**; DC = Ad; harmonics at multiples of 1/d are **missing** (e.g. d = 1/3 → every 3rd harmonic absent; d = ½ → even harmonics absent).

MEMORY TRICK

"**E**ven = **C**osine (E–C), **O**dd = **S**ine (O–S), **H**alf-wave = **O**dd harmonics." Decay: **Jump → 1/n, Kink → $1/n^2$, Spike → constant**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- FS of a signal with half-wave symmetry contains → **only odd harmonics**.
- Gibbs overshoot ≈ → **9% of the discontinuity**, independent of number of terms.
- FS coefficients of a triangular wave decay as → **$1/n^2$**.
- $x(t) = 3 + 4\cos\omega_0 t + 2\sin 2\omega_0 t$; average power? → 9 + 8 + 2 = **19 W**.
- An even periodic signal's FS has → **DC and cosine terms only**.
- Amplitude of fundamental of square wave of peak A → **4A/π ≈ 1.27A**.

3.5 Fourier Transform ★★★

- FT extends FS to **aperiodic** signals ($T_0 \to \infty$): spectrum becomes **continuous**.
- Existence (Dirichlet): **absolutely integrable** $\int|x|dt < \infty$ (sufficient). Energy signals always have FT; power signals (sin, u(t), constant) have FT **only with impulses**. $e^{at}u(t)$, a > 0, and ramp have **no FT**.
- $X(0) = \int x(t)dt$ (area); $x(0) = (1/2\pi)\int X(\omega)d\omega$.
- Real & even x → X **real & even**; real & odd x → X **purely imaginary & odd**; real x → $X(-\omega) = X^*(\omega)$ (conjugate symmetry).
- **Time–bandwidth**: compression in time ⇒ expansion in frequency (scaling property); duration × bandwidth = constant. A time-limited signal cannot be band-limited.

KEY FORMULAE

- $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt$; $x(t) = (1/2\pi)\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\,d\omega$
- In f: $X(f) = \int x(t)e^{-j2\pi ft}dt$; $x(t) = \int X(f)e^{j2\pi ft}df$ (no 2π factor)
- **Parseval / Rayleigh energy theorem**: $E = \int|x(t)|^2dt = (1/2\pi)\int|X(\omega)|^2d\omega = \int|X(f)|^2df$
- $|X(f)|^2$ = **energy spectral density**; ESD of output = $|H(f)|^2$ × ESD of input

Property	Time domain	Frequency domain
Linearity	$a\,x_1 + b\,x_2$	$a\,X_1 + b\,X_2$
Time shift	$x(t - t_0)$	$\mathbf{e^{-j\omega t_0}\,X(\omega)}$ (magnitude unchanged, linear phase)
Frequency shift	$e^{j\omega_0 t}\,x(t)$	$X(\omega - \omega_0)$
Time scaling	x(at)	**(1/|a|) X(ω/a)**
Time reversal	x(−t)	X(−ω)
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	X(t)	**$2\pi\,x(-\omega)$**
Differentiation (time)	dx/dt	**jω X(ω)**
Integration	$\int_{-\infty}^{t} x\,d\tau$	$X(\omega)/(j\omega) + \pi X(0)\,\delta(\omega)$
Differentiation (freq)	t x(t)	j dX/dω
Convolution	x(t) * h(t)	**X(ω) H(ω)**
Multiplication	x(t) h(t)	$(1/2\pi)\,X(\omega) * H(\omega)$
Modulation	$x(t)\cos\omega_0 t$	**$½[X(\omega - \omega_0) + X(\omega + \omega_0)]$**

x(t)	X(ω)
δ(t)	**1** (flat spectrum)
1 (DC)	**$2\pi\,\delta(\omega)$**
$\delta(t - t_0)$	$e^{-j\omega t_0}$
$e^{j\omega_0 t}$	$2\pi\,\delta(\omega - \omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin\omega_0 t$	$(\pi/j)[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
u(t)	**$\pi\,\delta(\omega) + 1/(j\omega)$**
sgn(t)	**$2/(j\omega)$**
$e^{-at}u(t)$, a > 0	**$1/(a + j\omega)$**
$t\,e^{-at}u(t)$	$1/(a + j\omega)^2$
$e^{-a\|t\|}$	**$2a/(a^2 + \omega^2)$**
rect(t/τ) (width τ, height 1)	**$\tau\,\mathrm{sinc}(\omega\tau/2\pi) = \tau\,\mathrm{Sa}(\omega\tau/2)$**; first zero at $\omega = 2\pi/\tau$ (f = 1/τ)
Λ(t/τ) (base 2τ)	$\tau\,Sa^2(\omega\tau/2)$
sin(Wt)/(πt)	rect(ω/2W): 1 for |ω| < W (ideal LPF)
$e^{-\pi t^2}$ (Gaussian)	$e^{-\pi f^2}$ (Gaussian – **self-reciprocal**)
Impulse train Σδ(t − nT)	$(2\pi/T)\,\Sigma\,\delta(\omega - 2\pi k/T)$ (impulse train)

DTFT, DFT and FFT

- **DTFT**: $X(e^{j\omega}) = \Sigma_n x[n]\, e^{-j\omega n}$; continuous in ω and **periodic with period 2π**. Inverse: $x[n] = (1/2\pi)\int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\,d\omega$. Exists if $\Sigma|x[n]| < \infty$.
- Sampling in one domain ⇒ **periodicity** in the other; periodic in one ⇒ discrete in the other.
- **DFT** (N-point): $X[k] = \Sigma_{n=0}^{N-1} x[n]\, e^{-j2\pi kn/N}$, k = 0…N−1 = samples of DTFT at $\omega = 2\pi k/N$. Twiddle factor $W_N = e^{-j2\pi/N}$.
- DFT product ↔ **circular** convolution. Linear convolution via DFT needs $N \geq \mathbf{N_1 + N_2 - 1}$ (zero-padding).
- Frequency resolution $\Delta f = \mathbf{f_s/N}$; X[0] = Σx[n] (DC); for real x, $X[N - k] = X^*[k]$.
- Direct DFT: **N^2** complex multiplications, N(N − 1) additions. Radix-2 **FFT**: **$(N/2)\log_2 N$** multiplications, **$N \log_2 N$** additions → order **N log N**. N must be a

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power of 2; $\log_2 N$ stages; N/2 butterflies per stage.

- N = 1024: DFT ≈ 1.05×10^6 multiplications vs FFT 5120 → ≈ **200 times** faster. DIT (decimation-in-time): input in **bit-reversed** order; DIF: output bit-reversed.
- Applications of FT: **filtering** (Y = HX), **bandwidth** estimation, modulation analysis, spectrum analysis, system frequency response H(jω).

Signal in time	Transform	Spectrum
CT periodic	CTFS	Discrete, aperiodic
CT aperiodic	CTFT	Continuous, aperiodic
DT aperiodic	DTFT	Continuous, **periodic (2π)**
DT periodic / finite	DTFS / DFT	Discrete, periodic (N)

MEMORY TRICK

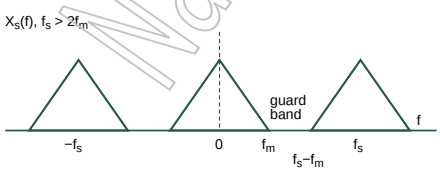
"**Narrow in time, wide in frequency**": δ(t) (zero width) → flat spectrum; DC (infinite width) → single impulse at ω = 0. Rect ↔ sinc and back again by **duality**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- FT of $e^{-3t}u(t)$? → **$1/(3 + j\omega)$**.
- FT of x(t − 2) compared with X(ω)? → magnitude same, phase −2ω added ($e^{-j2\omega}X(\omega)$).
- FT of a real and even signal is → **real and even**.
- DTFT is periodic with period → **2π**.
- Number of complex multiplications in 8-point radix-2 FFT → (8/2)×3 = **12**.
- Energy of x(t) = sin(Wt)/(πt) via Parseval → **W/π**.

3.6 Sampling Theorem ★★★

- **Sampling theorem** (Nyquist–Shannon): a band-limited signal with highest frequency f_m is completely represented by, and recoverable from, samples taken at $f_s \geq 2f_m$.
- **Nyquist rate** = $2f_m$ (minimum sampling rate); **Nyquist interval** = $1/(2f_m)$ (maximum sampling interval). Nyquist frequency = $f_s/2$ (folding frequency).
- Spectrum of sampled signal = X(f) repeated at every multiple of f_s (scaled by $1/T_s$ for ideal sampling).
- **Aliasing**: $f_s < 2f_m$ → spectral copies overlap; a high frequency appears as a low one. Alias of f: $|f - k f_s|$ (nearest integer k) folded into 0…$f_s/2$. Example: 7 kHz tone sampled at 10 kHz appears at **3 kHz**.
- **Anti-aliasing filter**: analog **LPF before the sampler**, cut-off ≤ $f_s/2$. Aliasing cannot be removed after sampling.
- Practical: sample above Nyquist rate to allow a **guard band** $f_s > 2f_m$ (realisable filters). Telephone speech: band limited to 3.4 kHz, sampled at **8 kHz**; audio CD 44.1 kHz (20 kHz audio).
- Sinusoid sampled exactly at $2f_m$ may give all zero samples (sin) – hence strict inequality $f_s > 2f_m$ for pure sinusoids.
- **Reconstruction**: ideal LPF of cut-off f_c with $f_m \leq f_c \leq f_s - f_m$, gain T_s; interpolation by sinc functions (ideal), zero-order hold in practice (staircase; ZOH response ∝ sinc).
- **Bandpass sampling**: signal occupying f_L…f_H, bandwidth $B = f_H - f_L$. Minimum $f_s = 2f_H/k$, k = largest integer ≤ f_H/B. If f_H is an integer multiple of B, $f_s(\min) = 2B$. Always $2B \leq f_s(\min) \leq 4B$.



Spectrum of an ideally sampled signal: replicas at multiples of f_s. If $f_s < 2f_m$ the triangles overlap (aliasing).

Sampling type	Pulse	Spectrum / effect
Ideal (impulse)	Impulse train	Exact replicas; theoretical only
Natural	Pulses whose tops **follow** the signal (gating/chopping)	Replicas weighted by sinc of pulse width; **no distortion** of baseband; recoverable by LPF
Flat-top (sample-and-hold, PAM)	Pulse tops **flat** at sample value	Spectrum multiplied by τ sinc(fτ) → high-frequency attenuation = **aperture effect**; corrected by **equaliser** 1/sinc(fτ); negligible if $\tau \ll T_s$

Signal	Highest frequency	Nyquist rate
$x_1(t) + x_2(t)$	**max**(f_1, f_2)	2 max(f_1, f_2)
$x_1(t) \cdot x_2(t)$	**$f_1 + f_2$**	2($f_1 + f_2$)
$x^2(t)$	$2f_m$	**$4f_m$**
$x^n(t)$	n f_m	2n f_m
$x_1(t) * x_2(t)$	**min**(f_1, f_2)	2 min(f_1, f_2)
x(at)	a f_m	2a f_m
dx/dt, x(t − t_0)	f_m (unchanged)	$2f_m$

KEY FORMULAE

- $f_s \geq 2f_m$; $T_s \leq 1/(2f_m)$; guard band = $f_s - 2f_m$
- $x_s(t) = x(t)\Sigma\delta(t - nT_s) \Rightarrow X_s(f) = f_s \Sigma X(f - n f_s)$
- Reconstruction: $x(t) = \Sigma\, x(nT_s)\, \mathrm{sinc}[(t - nT_s)/T_s]$ (for $f_s = 2f_m$ case: $\mathrm{sinc}(2f_m t - n)$)
- Flat-top aperture loss: $H(f) = \tau\, \mathrm{sinc}(f\tau) = \tau \sin(\pi f\tau)/(\pi f\tau)$

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Nyquist rate of x(t) = 3cos 50πt + 10 sin 300πt − cos 100πt? → f_{max} = 150 Hz → **300 Hz**.
- Nyquist rate of sin(200πt)/(πt)? → f_m = 100 Hz → **200 Hz**; of its square → **400 Hz**.
- Aperture effect occurs in → **flat-top sampling**.
- Anti-aliasing filter is placed → **before the sampler** (an LPF).
- Bandpass signal 20–25 kHz: minimum f_s? → $f_H/B = 5$ (integer) → **10 kHz**.
- Signal of f_m = 4 kHz sampled at 6 kHz; a 4 kHz component appears at → **2 kHz**.

3.7 Laplace Transform ★★★

- Generalises FT with s = **σ + jω**; handles unstable/growing signals. FT = Laplace on the **jω-axis** (if ROC includes it): $X(\omega) = X(s)|_{s = j\omega}$.
- **Bilateral**: $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$. **Unilateral**: $\int_{0^-}^{\infty}$ – used for causal signals and initial-condition problems.
- The same X(s) with **different ROCs** gives different signals: $1/(s + a) \leftrightarrow e^{-at}u(t)$ (Re s > −a) or $-e^{-at}u(-t)$ (Re s < −a). **ROC must always be stated** for bilateral LT.

ROC rule (Laplace)	Statement
Shape	**Vertical strips** parallel to jω-axis (depends only on σ)
Poles	ROC contains **no poles**
Finite-duration, absolutely integrable	**Entire s-plane**
Right-sided (causal)	**Right of the rightmost pole**: Re s > σ_{max}
Left-sided (anti-causal)	**Left of the leftmost pole**
Two-sided	**Strip** between two poles (or none – LT does not exist)
FT exists / stable	ROC **includes jω-axis**
Causal + stable	All poles in **LHP**, ROC right of rightmost pole

Property	x(t)	X(s)
Time shift	x(t − t_0)u(t − t_0)	e^{-st_0} X(s)
s-shift	e^{-at} x(t)	**X(s + a)**
Scaling	x(at)	(1/|a|) X(s/a)
Differentiation	dx/dt	**sX(s) − x(0⁻)**
2nd derivative	d^2x/dt^2	$s^2X(s) - s\,x(0^-) - x'(0^-)$
Integration	$\int_0^t x\,d\tau$	**X(s)/s**
Multiplication by t	t x(t)	−dX/ds

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Division by t	x(t)/t	$\int_s^\infty X(s)ds$
Convolution	x * h	X(s)H(s)
Periodic (period T)	x(t) = x(t + T), t ≥ 0	$X_1(s)/(1 - e^{-sT})$, X_1 = LT of first period

x(t) (t ≥ 0)	X(s)	ROC
δ(t)	**1**	all s
u(t)	**1/s**	Re s > 0
t u(t)	$1/s^2$	Re s > 0
$t^n u(t)$	$\mathbf{n!/s^{n+1}}$	Re s > 0
$e^{-at}u(t)$	**1/(s + a)**	Re s > −a
$t\,e^{-at}u(t)$	$1/(s + a)^2$	Re s > −a
$t^n e^{-at}u(t)$	$n!/(s + a)^{n+1}$	Re s > −a
$\sin \omega_0 t\, u(t)$	$\boldsymbol{\omega_0/(s^2 + \omega_0^2)}$	Re s > 0
$\cos \omega_0 t\, u(t)$	$\boldsymbol{s/(s^2 + \omega_0^2)}$	Re s > 0
$e^{-at}\sin \omega_0 t\, u(t)$	$\omega_0/[(s + a)^2 + \omega_0^2]$	Re s > −a
$e^{-at}\cos \omega_0 t\, u(t)$	$(s + a)/[(s + a)^2 + \omega_0^2]$	Re s > −a
sinh at u(t) ; cosh at u(t)	$a/(s^2 - a^2)$; $s/(s^2 - a^2)$	Re s > |a|
u(t) − u(t − T) (pulse)	$(1 - e^{-sT})/s$	all s

KEY FORMULAE

- **Initial value theorem**: $x(0^+) = \lim_{s\to\infty} sX(s)$ (valid if X(s) strictly proper; no impulse at t = 0)
- **Final value theorem**: $x(\infty) = \lim_{s\to 0} sX(s)$ – valid **only if all poles of sX(s) lie in the open LHP** (X(s) may have one simple pole at origin)
- Transfer function H(s) = Y(s)/X(s) = L{h(t)} with **all initial conditions zero**
- Partial fractions: simple pole p: $K = [(s - p)X(s)]_{s=p}$; repeated pole of order r: $K_{r-i} = (1/i!)\, d^i/ds^i[(s - p)^r X(s)]_{s=p}$

- FVT example: X(s) = 5/[s(s + 2)] → x(∞) = **2.5**. FVT **fails** for $\omega/(s^2 + \omega^2)$ (poles on jω-axis, sin oscillates) and 1/(s − 1) (RHP pole) – formula gives a wrong finite value.
- IVT example: X(s) = $(2s + 1)/(s^2 + 3s + 2)$ → $x(0^+)$ = **2**.
- **Poles** = roots of denominator (H → ∞); **zeros** = roots of numerator (H = 0). Order of system = **highest power of s in denominator** = number of poles. **Characteristic equation**: denominator = 0.
- **Proper** TF: deg N ≤ deg D; **strictly proper**: deg N < deg D. Improper → long division first, then partial fractions.
- Complex poles occur in **conjugate pairs** for real systems.
- Impulse response = inverse LT of H(s) = sum of terms from poles (modes): LHP real pole → decaying exponential; complex LHP → **damped oscillation**; jω-axis simple → sustained oscillation; RHP → growing.

Pole location (causal system)	h(t)	Stability
All in **open LHP**	Decays to 0	**Stable**
Simple (non-repeated) poles on jω-axis, rest LHP	Constant / sustained oscillation	**Marginally stable** (BIBO unstable)
Repeated poles on jω-axis	Grows as t, t sin ωt	Unstable
Any pole in RHP	Grows exponentially	Unstable

MEMORY TRICK

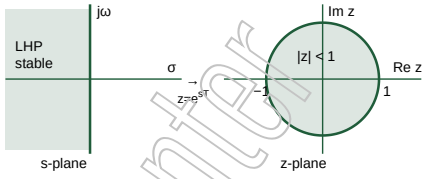
"**FVT needs a calm Left**": before applying FVT, check that sX(s) has every pole strictly in the Left half-plane. "**IVT: s → Infinity**, FVT: s → zero (Finish)".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Laplace transform of $t\,e^{-2t}u(t)$? → $\mathbf{1/(s + 2)^2}$.
- X(s) = 1/(s + 1)(s − 2), ROC −1 < Re s < 2: signal is → **two-sided**; stable (ROC includes jω-axis) but non-causal.
- Final value of x(t) for X(s) = (s + 3)/[s(s + 1)(s + 2)]? → **1.5**.
- H(s) = $1/(s^2 + 4)$: system is → **marginally stable**.
- Transfer function is defined with initial conditions → **zero**.
- Inverse LT of $1/(s^2 + 2s + 5)$? → $\mathbf{½\, e^{-t} \sin 2t\, u(t)}$.

3.8 Z-Transform ★★★

- DT counterpart of the Laplace transform: $X(z) = \boldsymbol{\sum_{n=-\infty}^{\infty} x[n]\, z^{-n}}$, $z = re^{j\omega}$. DTFT = X(z) on the **unit circle** |z| = 1 (if ROC includes it).
- **Relation with sampling**: $\mathbf{z = e^{sT}}$ → σ < 0 (LHP) maps **inside** unit circle; jω-axis → **on** unit circle; RHP → outside; s = 0 → **z = 1**; $s = j\omega_s/2$ → z = −1. Mapping is many-to-one (strips of width ω_s overlap → aliasing).
- **Bilinear transformation** $s = (2/T)(z - 1)/(z + 1)$: one-to-one, no aliasing, but **frequency warping** ($\omega_a = (2/T)\tan(\omega_d/2)$) → pre-warping needed. **Impulse invariance**: suffers aliasing, unsuitable for HPF/BSF.



$z = e^{sT}$: left half s-plane → inside unit circle; jω-axis → unit circle (stability boundary).

ROC rule (Z)	Statement
Shape	**Ring/annulus** centred at origin (depends only on |z|)
Poles	ROC contains **no poles**; bounded by poles
Finite-duration	Entire z-plane, except possibly **z = 0** (if x[n] ≠ 0 for some n > 0) and/or **z = ∞** (if x[n] ≠ 0 for some n < 0)
Right-sided / causal	$\mathbf{\|z\| > r_{max}}$ (outside outermost pole); causal includes z = ∞
Left-sided / anti-causal	$\mathbf{\|z\| < r_{min}}$ (inside innermost pole)
Two-sided	**Annulus** $r_1 < |z| < r_2$
Stable (DTFT exists)	ROC includes **unit circle**
Causal + stable	**All poles inside unit circle** (|p| < 1)

Property	x[n]	X(z)
Linearity	$a\,x_1 + b\,x_2$	$aX_1 + bX_2$ (ROC ⊇ intersection)
Time shift	x[n − k]	$\mathbf{z^{-k}\, X(z)}$
Unilateral delay	x[n − 1]	$z^{-1}X(z) + x[-1]$
Unilateral advance	x[n + 1]	zX(z) − z x[0]
Scaling in z	$\mathbf{a^n\, x[n]}$	**X(z/a)**, ROC |a|R
Time reversal	x[−n]	X(1/z), ROC 1/R
Differentiation in z	**n x[n]**	**−z dX/dz**
Convolution	x[n] * h[n]	X(z)H(z)
Accumulation	$\sum_{k\le n} x[k]$	$X(z)/(1 - z^{-1}) = zX(z)/(z - 1)$
Initial value (causal)	x[0]	$\mathbf{\lim_{z\to\infty} X(z)}$
Final value	x[∞]	$\mathbf{\lim_{z\to 1} (z - 1)X(z)}$ (poles of (z − 1)X(z) inside unit circle)

x[n]	X(z)	ROC
δ[n]	**1**	all z
δ[n − k], k > 0	z^{-k}	z ≠ 0
u[n]	$\mathbf{z/(z - 1)} = 1/(1 - z^{-1})$	|z| > 1
$a^n u[n]$	$\mathbf{z/(z - a)} = 1/(1 - az^{-1})$	|z| > |a|
$-a^n u[-n - 1]$	z/(z − a)	**|z| < |a|**
n u[n]	$z/(z - 1)^2$	|z| > 1
$n\, a^n u[n]$	$az/(z - a)^2$	|z| > |a|
$\cos(\omega_0 n)\, u[n]$	$z(z - \cos\omega_0)/(z^2 - 2z\cos\omega_0 + 1)$	|z| > 1
$\sin(\omega_0 n)\, u[n]$	$z \sin\omega_0/(z^2 - 2z\cos\omega_0 + 1)$	|z| > 1
$a^{\|n\|}$, |a| < 1	$z/(z - a) - z/(z - 1/a)$	|a| < |z| < 1/|a|

- **Inverse Z-transform methods**: (1) **partial fractions** – expand X(z)/z, then multiply by z; (2) **power series / long division** – gives samples directly

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(causal: divide in powers of z^{-1}); (3) **residue / contour integration**: $x[n] = (1/2\pi j)\oint X(z)z^{n-1}dz$; (4) convolution method.

- **Difference equation**: $\Sigma\, a_k\, y[n-k] = \Sigma\, b_k\, x[n-k] \rightarrow H(z) = (\Sigma\, b_k z^{-k})/(\Sigma\, a_k z^{-k})$. Example: $y[n] - 0.5y[n-1] = x[n] \rightarrow H(z) = 1/(1 - 0.5z^{-1}) = z/(z - 0.5)$; $h[n] =$ **$(0.5)^n u[n]$** – causal, stable (pole 0.5 inside unit circle).
- Order of a difference equation = largest delay of output (or highest power of z^{-1} in denominator).
- **Recursive** system: output depends on past outputs (feedback) → **IIR**. **Non-recursive**: output depends only on inputs → **FIR**.

Feature	FIR	IIR
Impulse response	**Finite** length	Infinite length
Structure	Non-recursive (no feedback)	**Recursive** (feedback)
Poles	All at **z = 0** (only zeros matter)	Poles anywhere
Stability	**Always stable**	Stable only if poles inside unit circle
Phase	**Exact linear phase** possible (symmetric/antisymmetric h[n])	Non-linear phase
Order for given specs	Higher (more coefficients)	**Lower** (efficient)
Design methods	Windowing (rectangular, Hanning, Hamming, Blackman, Kaiser), frequency sampling	From analog prototypes (Butterworth, Chebyshev, elliptic) via **impulse invariance / bilinear**
Quantisation sensitivity / limit cycles	Low / none	Higher / possible

KEY FORMULAE

- $X(z) = \Sigma\, x[n]z^{-n}$; $X(e^{j\omega}) = X(z)|_{z = e^{j\omega}}$
- $z = e^{sT}$; $|z| = e^{\sigma T}$; $\angle z = \omega T$
- Geometric series: $\Sigma_{n=0}^{\infty} a^n = 1/(1-a)$, $|a| < 1$; $\Sigma_{n=0}^{N-1} a^n = (1 - a^N)/(1 - a)$
- Linear-phase FIR of length N: group delay = **(N − 1)/2** samples

Laplace (s-plane)	Z (z-plane)
ROC = vertical strip	ROC = annulus
Stability boundary: **jω-axis**	Stability boundary: **unit circle**
Causal stable: poles in LHP	Causal stable: poles inside $\|z\| = 1$
Delay T: e^{-sT}	Delay 1 sample: z^{-1}
Integrator 1/s	Accumulator $1/(1 - z^{-1})$
FVT: $\lim_{s\to 0} sX(s)$	FVT: $\lim_{z\to 1} (z-1)X(z)$

MEMORY TRICK

"**Causal looks Outward, Stable holds the Circle**": causal ROC lies **outside** the outermost pole; stable ROC must **contain the unit circle**; both together ⇒ every pole inside $|z| = 1$.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Z-transform of $(0.5)^n u[n]$ and its ROC? → **$z/(z - 0.5)$, $|z| > 0.5$**.
- $H(z) = z/(z - 2)$, ROC $|z| < 2$: system is → **anti-causal and stable** (ROC contains unit circle).
- ROC of a finite-duration causal sequence? → entire z-plane **except z = 0**.
- Z-transform of $x[n - 3]$? → **$z^{-3}X(z)$**.
- FIR filters are always stable because → all poles are at **z = 0** (inside unit circle).
- s-plane jω-axis maps to which z-plane contour under $z = e^{sT}$? → **unit circle**.

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4. Electrical Machines I – Transformers & DC Machines

NOTE

Transformers and DC machines together give about **5–8 MCQs** in AE/AM papers. Most asked: EMF equations, OC/SC tests, condition and load for maximum efficiency, regulation, vector groups, open-delta, autotransformer copper saving, DC generator build-up and critical resistance, DC motor characteristics and applications, starters and speed control. Learn every formula with its condition. Most questions are on proportionality: "what happens to X if Y is doubled".

4.1 Electromechanical Energy Conversion ★★☆

- **Energy balance (motor)**: Electrical energy input = **mechanical energy output + increase in stored field energy + energy losses**. A generator works the other way. The conversion is **reversible**, so the same machine can run as a motor or a generator.
- The **coupling field** in practical rotating machines is **magnetic**, not electric. Magnetic energy density at B = 1.5 T is **≈ 9 × 10^5 J/m³**. Electric energy density at the breakdown field of air (3 × 10^6 V/m) is only **≈ 40 J/m³**.
- Basic laws: **e = Blv** (generator action, **Fleming's right-hand rule**) and **F = BIl** (motor action, **Fleming's left-hand rule**). Both actions occur at the same time in every machine.
- **Field energy** $W_f = \int i\, d\lambda$. **Co-energy** $W'_f = \int \lambda\, di$. For a **linear** (unsaturated) system, $\mathbf{W_f = W'_f = \tfrac{1}{2} Li^2 = \tfrac{1}{2} \Phi^2 \mathcal{R}}$.
- **Most of the field energy is stored in the air gap**. The gap reluctance is far higher than the iron reluctance (μ_r of iron ≈ 1000s), and energy density = $\mathbf{B^2/2\mu_0}$. For the same B, iron stores $1/\mu_r$ as much per unit volume.
- **Singly excited system**: one winding, usually on the stator. Examples: relay, solenoid, electromagnet, **reluctance motor**, VR stepper motor. Torque is produced only by **reluctance variation (saliency)**.
- **Doubly excited system**: two windings (stator and rotor). Examples: **synchronous machine, DC machine**, loudspeaker. Torque = alignment torque ($i_1 i_2\, dM/d\theta$) + reluctance torque (if salient).
- **Reluctance torque** exists only if the reluctance changes with rotor position (**salient poles**). In a cylindrical rotor it is **zero**. In salient-pole synchronous machines it varies as **sin 2δ**.
- Mechanical force always acts in the direction that **reduces reluctance**, i.e. **increases inductance** (the system moves towards maximum co-energy).
- Force per unit area on an iron surface = $\mathbf{B^2/2\mu_0}$ N/m² (magnetic pressure). This is the lifting force of an electromagnet.

KEY FORMULAE

- Force: $f = \mathbf{+\partial W'_f(i, x)/\partial x}$ at **i constant** = $\mathbf{-\partial W_f(\lambda, x)/\partial x}$ at **λ constant**
- Torque: $T = \partial W'_f/\partial\theta$. Singly excited, linear: $\mathbf{T = \tfrac{1}{2} i^2\, dL/d\theta}$ (= $-\tfrac{1}{2} \Phi^2\, d\mathcal{R}/d\theta$)
- Doubly excited: $T = \tfrac{1}{2} i_1^2\, dL_{11}/d\theta + \tfrac{1}{2} i_2^2\, dL_{22}/d\theta + \mathbf{i_1 i_2\, dM/d\theta}$
- Efficiency η = output/(output + losses) = 1 − losses/input
- Maximum efficiency when **variable (copper) loss = constant loss**

Loss	Sub-type	Depends on	Nature	Reduced by
Copper (I^2R) loss	Armature/winding, field, brush contact	**Load current²**	**Variable** (field Cu loss in shunt machine is constant)	Thicker conductors
Core (iron) loss	Hysteresis: $P_h = \eta_h B_m^{1.6} f V$	**f, $B_m^{1.6}$** (Steinmetz)	**Constant**	**Silicon steel / CRGO** (narrow B-H loop)
	Eddy current: $P_e = K_e B_m^2 f^2 t^2 V$	**f^2, B_m^2, t^2**	**Constant**	**Laminations** (thin t), high resistivity (Si)
Mechanical	Friction (bearing, brush), windage	Speed	Constant (near-constant-speed machines)	Good bearings, fans
Stray load loss	Leakage-flux eddy losses, skin effect, armature reaction distortion	Load	Variable (≈ **1%** of output by convention for DC machines)	Good design

- **Stray losses** (in DC machine testing) = **iron loss + mechanical loss**. **Constant losses of a shunt machine** = iron + mechanical + **shunt field copper loss**.
- A transformer has **no mechanical loss** (no moving parts). This is why its efficiency is the **highest of all electrical machines** (95–99.5%).

MEMORY TRICK

"**Co-energy at Constant Current**": use W′ with + sign when i is held constant, and W with − sign when λ (flux) is held constant. Both give the same force.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In a linear magnetic system, field energy and co-energy are? → **Equal**.
- Most of the magnetic energy in a machine is stored in? → **The air gap**.
- Reluctance torque is present in? → **Salient-pole machines only** (zero for cylindrical rotor).
- Eddy current loss is proportional to? → **$f^2 B_m^2 t^2$**.
- Condition for maximum efficiency of any machine? → **Variable loss = constant loss**.

4.2 Single-Phase Transformer – Construction & Equivalent Circuit ★★★

- A transformer is a **static** device. It works on **mutual induction** (Faraday's law) and transfers power between circuits at the **same frequency**. It changes V and I but **not frequency or power** (ideally).
- It **cannot work on DC**. With no changing flux there is no back EMF, so the current is limited only by the small winding resistance and the **winding burns**.
- Rating is in **kVA, not kW**. Cu loss depends on current and core loss on voltage. Neither depends on load pf.
- **Core material**: **CRGO** (cold-rolled grain-oriented) silicon steel, **≈ 3% (3–4.5%) Si**. Silicon raises resistivity (less eddy loss) and lowers hysteresis loss. Laminations are **0.23–0.35 mm** thick for 50 Hz, insulated by varnish or oxide (carlite) coating. Amorphous-metal cores give very low no-load loss (energy-efficient distribution transformers).
- Flux in CRGO is kept along the **rolling direction**. Normal B_m = **1.5–1.7 T** (CRGO), ~1.1–1.4 T for hot-rolled steel.
- **Stepped (cruciform) core** with circular coils improves the **space (stacking) factor** and reduces mean turn length (less copper).
- **LV winding is placed next to the core** and HV outside. This saves insulation (the core is at earth potential). **Sandwich (interleaved) disc windings** are used in shell type to reduce leakage reactance.
- Leads and joints: core joints are **interleaved (staggered)** to reduce reluctance and magnetising current. Core is **earthed at one point only**.

Feature	Core type	Shell type
Arrangement	**Windings surround the core** (on limbs)	**Core surrounds the windings** (on central limb)
Magnetic circuit	Single magnetic path	**Two parallel (divided) paths**; central limb carries full flux, outer limbs half
Laminations	L-shaped	E and I shaped
Coil type	Cylindrical (concentric)	Sandwich / disc
Leakage reactance	Higher	**Lower**
Mechanical strength (SC forces)	Less	**Better** (windings braced by core)
Maintenance / repair	**Easy**	Difficult
Suited for	**High voltage**, most power & distribution transformers	**Low voltage, high current** (e.g. furnace transformers)

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KEY FORMULAE

- EMF: $E = 4.44\, f\, N\, \Phi_m$ = 4.44 f N B_m A_i (RMS). EMF per turn = 4.44 f Φ_m
- 4.44 = $\sqrt{2}\,\pi$ = form factor 1.11 × 4 (sinusoidal flux)
- Transformation ratio $K = N_2/N_1 = E_2/E_1 = I_1/I_2$ (K > 1 step-up, K < 1 step-down)
- $\Phi_m \approx V/(4.44\, f\, N)$: flux is fixed by **V/f**, not by load
- Referred to primary: $R_{01} = R_1 + R_2/K^2$, $X_{01} = X_1 + X_2/K^2$. Referred to secondary: $R_{02} = R_2 + K^2 R_1$
- Impedance referred from side 2 to side 1: $Z' = Z \times (N_1/N_2)^2$
- Per-unit: $Z_{pu} = Z_{actual} \times S_{base}/V_{base}^2$. Change of base: $Z_{pu,new} = Z_{pu,old} \times (S_{new}/S_{old}) \times (V_{old}/V_{new})^2$
- SC current (infinite bus) = I_{rated}/Z_{pu}. Example: Z = 5% gives $I_{sc} = \mathbf{20 \times I_{rated}}$

- **Ideal transformer**: zero winding resistance, **zero leakage flux**, infinite core permeability (zero magnetising current), zero core loss. Input VA = output VA and $V_1 I_1 = V_2 I_2$.
- **Practical transformer**: has R_1, R_2, leakage reactances X_1, X_2, core loss and finite magnetising current. **Leakage flux** links only its own winding and is represented by a **series reactance**.
- **No-load current I_0** is **2–5% of rated current** (≤ 1–2% in large transformers). It has two components:
  - $I_w = I_0 \cos \varphi_0$: active/core-loss component, **in phase with V_1**, supplies hysteresis and eddy loss.
  - $I_\mu = I_0 \sin \varphi_0$: magnetising component, **in phase with Φ** (lags V_1 by 90°), and the much larger part.
- No-load pf cos φ_0 is **very low, 0.1–0.3 lagging**. φ_0 is close to 90° (typically 75°–85°).
- Because of core saturation the magnetising current is **non-sinusoidal**, rich in the **3rd harmonic** (peaky wave).
- **Magnetising inrush current** on switching (worst at voltage zero, plus residual flux) can be **6–10 × rated**. It is rich in the **2nd harmonic**, which is used for harmonic restraint in differential protection.
- In the **exact equivalent circuit**, the shunt branch $R_0 \parallel X_0$ sits between the primary and secondary series impedances. In the **approximate** circuit it is moved to the supply terminals (I_0 is small, so the error is negligible).
- **Per-unit impedance has the same value on both sides** of a transformer. Typical %Z: distribution transformers **≈ 4–5%**, large power transformers **≈ 8–15%**. X/R is large in big units.

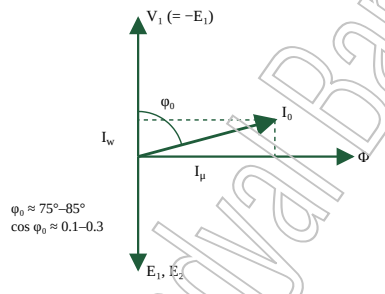

Fig 4.1 No-load phasor diagram of a transformer (Φ as reference; E_1, E_2 lag Φ by 90°).

Cooling code	Meaning	Typical use
AN / AF	Air natural / air forced (dry type)	Indoor, small, fire-prone places
ONAN	**O**il **N**atural circulation, **A**ir **N**atural (radiators)	Distribution transformers and smaller power transformers
ONAF	Oil natural, **air forced (fans)** on radiators	Medium power. Fans raise rating by about 25–33%
OFAF	**Oil forced (pumps)**, air forced	Large power transformers
OFWF	Oil forced, **water forced** (oil–water heat exchanger)	Very large units, hydro stations, limited space
ODAF / ODWF	Oil **directed** through windings	Very large generator transformers

- Code letters (IEC 60076-2 / IS 2026): 1st = internal coolant (**O** mineral oil, **K** fire-resistant liquid such as ester/silicone), 2nd = its circulation (**N** natural, **F** forced, **D** directed), 3rd = external coolant (**A** air, **W** water), 4th = its circulation.
- **Transformer oil** (mineral, IS 335) has two functions: **insulation and cooling**. Breakdown voltage (2.5 mm gap): **≥ 30 kV** for new oil as received, about **≥ 60 kV** after filtration. **Flash point ≥ 140 °C**. It must be moisture-free. Even ~50 ppm water sharply reduces BDV. **Askarel (PCB)** was a non-flammable synthetic but is now **banned** (toxic).
- **Conservator**: a cylindrical drum above the tank, connected by pipe. It allows **oil expansion/contraction** and keeps the main tank full, which reduces the oil surface exposed to air.
- **Breather**: contains **silica gel** that dries the air entering the conservator. Dry gel is **blue** and turns **pink** when saturated (cobalt chloride indicator). Non-toxic gel goes orange → green. Usually fitted to transformers above about 50 kVA.
- **Buchholz relay**: a **gas-actuated** relay fitted in the **pipe between the main tank and the conservator**. The **upper float** gives an **alarm** for slow gas accumulation from incipient faults (insulation failure, core hot-spots, inter-turn faults). The **lower float/flap trips** the breaker on a severe fault (oil surge). It is only possible in **oil-immersed transformers with a conservator**. It is usually provided above ~ **500 kVA** (some texts say 750 kVA).
- Other fittings: **explosion vent / pressure relief device**, oil and winding temperature indicators (OTI/WTI), **bushings** (porcelain, condenser type above ~33 kV), tap changer, radiators.

MEMORY TRICK

"**W**atts with **V**, **μ** with **Φ**": I_w is in phase with V_1 (supplies watts), and I_μ is in phase with Φ (creates flux). Cooling code order = **Inside medium, its Motion, Outside medium, its Motion** (O-N-A-N).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- The no-load current of a transformer is about? → **2–5% of rated current**, at pf **0.1–0.3 lagging**.
- Laminating the core reduces? → **Eddy current loss** (silicon reduces hysteresis loss).
- A transformer connected to a DC supply of rated voltage will? → **Burn out** (no back EMF).
- Buchholz relay is located? → **Between main tank and conservator**. It protects against **internal incipient faults**.
- Colour of silica gel when it has absorbed moisture? → **Pink**.
- Shell-type transformers are preferred for? → **Low-voltage, high-current** applications.

4.3 OC & SC Tests, Regulation & Efficiency ★★★

Feature	Open-Circuit (No-load) test	Short-Circuit test
Supply given to	**LV side** at **rated voltage** (HV open)	**HV side** at **reduced voltage, 5–10% (up to ~15%) of rated** (LV shorted)
Reason for side	Rated LV voltage is easily available. Current is measurable	Rated HV current is small, so meters of low current range suffice
Wattmeter reads	**Core (iron) loss P_i** (Cu loss negligible)	**Full-load copper loss** (core loss negligible because flux is very low)
Parameters found	**R_0, X_0** (shunt branch)	**R_{eq}, X_{eq}, Z_{eq}** (series branch)
Wattmeter type	**Low-pf wattmeter**	Normal (UPF) wattmeter
pf of test	Very low (0.1–0.3)	Low-medium (0.2–0.5)

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KEY FORMULAE

- OC: $\cos\varphi_0 = W_0/(V_1 I_0)$. $I_w = I_0 \cos\varphi_0$, $I_\mu = I_0 \sin\varphi_0$. $\mathbf{R_0 = V_1/I_w}$, $\mathbf{X_0 = V_1/I_\mu}$
- SC: $\mathbf{Z_{eq} = V_{sc}/I_{sc}}$, $\mathbf{R_{eq} = W_{sc}/I_{sc}^2}$, $X_{eq} = \sqrt{(Z_{eq}^2 - R_{eq}^2)}$
- Cu loss at another current: $P_{cu} = W_{sc} \times (I/I_{sc})^2$. At fraction x of full load: $\mathbf{P_{cu} = x^2 P_{cu,fl}}$
- % Regulation = $(V_{2,nl} - V_{2,fl})/V_{2,nl} \times 100$. Some texts divide by the rated (full-load) voltage
- Approximate: $\mathbf{\% R \approx \varepsilon_r \cos\varphi \pm \varepsilon_x \sin\varphi}$ (+ lagging, − leading), where ε_r = %R, ε_x = %X. At fraction x of load multiply by x
- More exact: $\% R = \varepsilon_r \cos\varphi \pm \varepsilon_x \sin\varphi + (\varepsilon_x \cos\varphi \mp \varepsilon_r \sin\varphi)^2/200$
- Efficiency: $\mathbf{\eta = x S \cos\varphi / (x S \cos\varphi + P_i + x^2 P_{cu,fl})}$
- Max efficiency: $\mathbf{x^2 P_{cu,fl} = P_i}$, so $\mathbf{x = \sqrt{(P_i/P_{cu,fl})}}$. kVA at η_{max} = rated kVA × $\sqrt{(P_i/P_{cu,fl})}$
- All-day (energy) efficiency = **output kWh / input kWh over 24 h**

- **Regulation** is **positive for lagging and UPF** loads. It becomes **zero** at a **leading pf** where $\mathbf{\tan\varphi = \varepsilon_r/\varepsilon_x}$ ($\cos\varphi = X/Z$, i.e. $\varphi = 90° - \theta$). It is **negative** (terminal voltage rises on load) at more leading pf (Ferranti-like effect).
- **Maximum regulation** occurs when the **load pf angle φ = impedance angle θ** (lagging), i.e. **$\cos\varphi = R/Z$**. Its value = **ε_z** (% impedance).
- At **UPF**, regulation ≈ **ε_r** (% resistance) only.
- Good transformer regulation is **low** (≈ 2–5%). Large transformers have high X/R, so regulation at lagging pf is dominated by reactance.
- **Maximum efficiency occurs when Cu loss = core loss**. The **load at which η_{max} occurs is independent of pf**, but the **value of η_{max} is highest at UPF**.
- Efficiency at a given kVA falls as pf falls. η = 0 at no load (output zero), and η is also 0 at short circuit.
- **Power transformers** run near full load all the time and are designed for **η_{max} near full load**, with $P_i \approx P_{cu,fl}$.
- **Distribution transformers** are energised 24 h but lightly loaded most of the day. Core loss runs all day, so they are designed with **low core loss** and **η_{max} at about 50–75% load** ($P_i < P_{cu,fl}$). **All-day efficiency** is the relevant measure for them.
- For an all-day efficiency calculation: core loss × 24 h is constant, and Cu loss = $\Sigma\,(x_k^2 P_{cu,fl} \times \text{hours}_k)$.

Test	Purpose	Key point
Sumpner's (back-to-back) test	Heat run (temperature rise), efficiency at full load	Needs **two identical transformers**. Primaries are in parallel at rated V ($W_1 = \mathbf{2P_i}$). Secondaries are in **series opposition** with an auxiliary low voltage injecting rated current ($W_2 = \mathbf{2P_{cu}}$). Only the losses are drawn from the supply
Polarity test	Relative polarity of windings (essential before paralleling or 3-ph connection)	Join one HV and one LV terminal and apply V to HV. Voltmeter reading $V_1 - V_2$ means **subtractive** polarity, $V_1 + V_2$ means **additive**. Subtractive is standard in India
Ratio test	Turns ratio	Voltmeters on both sides at no load
Separation of core loss	Separates P_h and P_e	Run at constant **V/f**. Plot P_i/f against f, which is a straight line A + Bf
Insulation resistance / HV tests	Insulation	Megger (IR), separate-source and induced overvoltage tests, impulse test

- **Effect of frequency and voltage** ($B_m \propto V/f$):
  - With **V/f constant**, B_m is constant. Then $\mathbf{P_h \propto f}$ and $\mathbf{P_e \propto f^2}$, so $P_i = Af + Bf^2$.
  - With **V constant, f increased**, B_m falls. **$\mathbf{P_e \propto V^2}$ is unchanged (independent of f)**. **$\mathbf{P_h \propto V^{1.6}/f^{0.6}}$ decreases**. So core loss and magnetising current decrease, but leakage reactance (∝ f) increases and regulation worsens.
  - With **V constant, f reduced** (e.g. a 60 Hz transformer on 50 Hz), B_m rises by 20%. This causes **over-fluxing, saturation**, large I_0 and heating.
  - With **f constant, V increased**, both P_h ($\propto V^{1.6}$) and P_e ($\propto V^2$) increase.
- **Over-fluxing** protection responds to the **V/f ratio**.

MEMORY TRICK

"**Open the High, Short the Low**": OC test keeps HV open (supply on LV); SC test shorts LV (supply on HV). OC gives **Core** loss and SC gives **Copper** loss. Each test finds the loss of the branch it does not short.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Iron loss 400 W, full-load Cu loss 1600 W. Load for maximum efficiency? → **x = √(400/1600) = 0.5, i.e. half load**.
- Regulation can be zero or negative at? → **Leading power factor**.
- Maximum regulation occurs when? → **Load pf angle = impedance angle (cos φ = R/Z)**.
- Sumpner's test is used to find? → **Temperature rise (heat run) and efficiency**. Two identical transformers are needed.
- A transformer designed for 60 Hz is used on 50 Hz at the same voltage. Core flux? → **Increases by 20%** (over-fluxing).
- All-day efficiency is important for? → **Distribution transformers**.

4.4 Three-Phase Transformers, Parallel Operation & Autotransformer ★★★

- A single **3-phase unit** costs about **15% less** than a bank of three 1-ph units, weighs less and needs less space. Its drawback is that one fault puts the whole unit out, and a **spare is costly**. A bank of 1-ph units is used where transport is difficult and only one spare unit is needed.
- The 3-ph core type has **3 limbs**, with no return limb needed because the fluxes sum to zero. 5-limb cores are used for very large units (lower height) and give a path for zero-sequence flux.
- **Clock notation** (IS 2026 / IEC 60076-1): HV in capitals, LV in small letters. The HV line-voltage phasor is at **12 o'clock**. The clock number gives the LV phasor position, each hour = **30°**, LV **lagging** HV for clockwise positions. **Yd1**: LV lags HV by 30°. **Dyn11**: LV leads HV by 30° (−330° = +30°). "n" means the neutral is brought out.

Vector group	Phase displacement	Examples	Parallel with
Group 1	**0°** (clock 0)	Yy0, Dd0, Dz0	Group 1 only
Group 2	**180°** (clock 6)	Yy6, Dd6, Dz6	Group 2 only
Group 3	**−30°** (LV lags, clock 1)	Yd1, Dy1, Yz1	Group 3. Group 4 only with changed external phase connections
Group 4	**+30°** (LV leads, clock 11)	**Dyn11**, Yd11, Yz11	Group 4. Group 3 only with changed external phase connections

- **Never parallel Group 1/2 with Group 3/4**. The 30° displacement causes a huge circulating current.

Connection	Phase shift	Merits / demerits	Typical use
Y-y	0° or 180°	Phase voltage = $V_L/\sqrt{3}$, so least insulation. **3rd-harmonic voltages and oscillating neutral** problem. Unbalanced load is poor. Needs a **tertiary delta** or earthed neutral	Small HV units, interconnecting (with tertiary)
Δ-d	0° or 180°	No 3rd-harmonic problem. Handles unbalanced load. **Can run in V-V if one unit fails**. No neutral, and more insulation	LV, high-current systems
Y-d	±30°	No harmonic problem (Δ carries 3rd harmonic current). Common for **step-down** at the receiving end of a transmission line	Receiving substation. **YNd1/YNd11 generator transformers** (star HV, delta on generator side)
D-y	±30°	Neutral available on LV for **3-ph 4-wire** supply. Unbalanced load is fine	**Distribution transformers (Dyn11, 11 kV/433 V)**, step-up at generating stations in some systems
Zigzag (z)	±30° (Yz), 0°/180° (Dz)	Handles unbalance, suppresses harmonics	**Earthing (grounding) transformers**, rectifiers

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KEY FORMULAE

- **Open-delta (V-V)** capacity = √3 V I. Ratio to Δ-Δ (3VI) = 1/√3 = **57.7%**. Ratio to the combined rating of the two units (2VI) = √3/2 = **86.6%** (utilisation factor)
- In V-V at UPF load, the two units work at pf **cos 30° = 0.866**. At load pf angle φ they work at cos(30° − φ) and cos(30° + φ)
- **Scott-T**: teaser turns = **$(\sqrt{3}/2)\ N_1 = 0.866\ N_1$**, connected to the **centre tap** of the main transformer. The 3-ph neutral is at **1/3 of the teaser winding** measured from the centre-tap end
- Parallel load sharing (equal ratios): **$S_A = S_L \times Z_B/(Z_A + Z_B)$**, $S_B = S_L \times Z_A/(Z_A + Z_B)$, i.e. **inversely ∝ impedance** (phasor values)
- Autotransformer, k = V_2/V_1 = **low voltage/high voltage (< 1)**: Cu weight (auto) = **(1 − k)** × Cu weight (two-winding). **Saving = k × Cu of two-winding**
- Auto: VA **transformed** (inductively) = **(1 − k) V_1I_1**. VA **conducted** (directly) = **k V_1I_1** (step-down, V_1I_1 = output VA)
- Two-winding transformer reconnected as auto: $S_{auto} = S_{2W}/(1 - k)$. Example: 2400/240 V, 50 kVA as a 2640/2400 V auto gives k = 2400/2640 and **S = 550 kVA (11×)**

- **Open-delta (V-V)** is used as a temporary arrangement when one unit of a Δ-Δ bank fails, or for small loads expected to grow (the third unit is added later).
- **Scott-T connection** converts **3-phase to 2-phase** (90° apart) or the reverse. It uses two 1-ph transformers: a **main** (centre-tapped) and a **teaser** (86.6% tap). Uses: **electric arc furnaces**, 2-ph loads, 2-ph motors, and drawing a balanced 3-ph load for **traction** (two single-phase sections). When the two 2-ph loads are equal, the 3-ph side is **balanced**.
- Other conversions: **3-ph to 6-ph** (rectifiers) by double-star, double-delta or diametrical connection. 3-ph to 1-ph needs special arrangements.

Condition for parallel operation	Essential?	If not met
Same polarity	**Essential**	Dead short circuit
Same phase sequence (3-ph)	**Essential**	Short circuit in each cycle
Same phase displacement (vector group) (3-ph)	**Essential**	Large circulating current
Same voltage ratio (rated voltages)	Desirable (small difference tolerable)	**Circulating current even at no load**
Same **per-unit (%) impedance**	Desirable	Load not shared in proportion to kVA ratings. Unit with lower Z_{pu} overloads
Same **X/R ratio**	Desirable	Units work at different pf. Total output < sum of ratings

- A **transformer with lower % impedance takes more load**. To share load in proportion to rating, the **% impedances (on own base) must be equal**.
- Ratio of kVA ratings should preferably not exceed **3:1** for paralleling.
- **Autotransformer**: a **single winding** with part common to both circuits. It is economical when **k is near 1** (ratio up to about 2–3). Advantages: **less copper, smaller size, lower losses, higher efficiency, better regulation** (lower leakage reactance).
- Disadvantages: **no electrical isolation**. Lower impedance gives **higher fault current**. If the common winding opens, the **full primary voltage appears across the load**. Not used where the ratio is large (e.g. 11 kV/415 V).
- Uses: **auto-transformer starters** for induction and synchronous motors, **variac** (variable autotransformer), boosters, **interconnecting transformers (ICTs)** between EHV systems such as 400/220 kV and 220/132 kV, and voltage stabilisers.
- **Tertiary (third) winding**: usually **delta-connected**, fitted on **Y-y** and auto (YNa0) transformers. It **suppresses 3rd harmonics**, **stabilises the neutral** under unbalance (gives a path for zero-sequence current), supplies station auxiliaries, and connects **shunt reactors / synchronous condensers / capacitors**. Rating is typically about **1/3 of the main winding**.
- **Tap changers** are placed on the **HV winding** (lower current, finer voltage steps, HV is the outer winding so taps are easy to reach), usually near the **neutral end or middle**.
  - **Off-load (off-circuit) tap changer**: operated only when **de-energised**. Distribution transformers typically have **±5% in 2.5% steps** (5 positions).
  - **On-load tap changer (OLTC)**: changes taps **without interrupting load** using a transition **impedance (resistor – Jansen high-speed type, or reactor)** to limit the circulating current between taps. Typical range **+10% to −15% (or ±10%)** in steps of **1.25%**. Controlled by an AVR (automatic voltage regulating relay).
- **Instrument transformers** (CT, PT/VT) are dealt with in the Measurements chapter. Key facts: **CT secondary is never open-circuited** (dangerous high voltage and core saturation). Standard CT secondary is **1 A or 5 A**, and standard PT secondary is **110 V** (line).

MEMORY TRICK

"**Open Delta = 58 / 87**": 57.7% of the original Δ-Δ, 86.6% of the two units. Autotransformer: "**k saved, (1 − k) used**", where k = low/high.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Three 1-ph transformers of 100 kVA each in Δ-Δ. One is removed. Capacity in V-V? → **173.2 kVA (57.7% of 300)**.
- Copper saving in a 400/200 V autotransformer compared with a two-winding transformer? → **k = 0.5, so 50% saving**.
- Scott-T connection converts? → **3-phase to 2-phase**. Teaser tapping at **86.6%**.
- Can Dyn11 and Yy0 transformers be paralleled? → **No** (30° phase displacement).
- Delta tertiary winding in Y-y transformers is used to? → **Suppress 3rd harmonics and stabilise the neutral**.
- Two transformers in parallel share load in which proportion? → **Inversely as their impedances**.

4.5 DC Generators ★★★

Part	Material	Function / fact
Yoke (frame)	**Cast iron** (small machines), **cast steel / rolled steel** (large)	Mechanical support and **return path for flux**
Pole core & pole shoe	**Laminated** steel (1–1.25 mm)	Pole shoe spreads flux and supports the field coils. Laminated to reduce **eddy loss from tooth ripple**
Field winding	Copper	Shunt: **many turns, thin wire, high R**. Series: **few turns, thick wire, low R**
Armature core	**Silicon steel laminations 0.35–0.5 mm**	Carries conductors in slots. Rotates, so flux in it **alternates** → iron loss
Armature winding	Copper, double-layer	EMF induced is **AC** inside the armature
Commutator	**Hard-drawn copper segments** insulated by **mica**	**Mechanical rectifier** (AC → DC in generator). In a motor it acts as an inverter (DC → AC in the conductors)
Brushes	**Carbon / electro-graphite. Copper-graphite** for low-voltage, high-current machines	Collect current. Carbon gives **high contact resistance (helps commutation)**, is self-lubricating and wears the commutator less. Brush drop ≈ **1 V per brush (2 V total)**
Bearings	Ball/roller	—

Feature	Lap winding	Wave winding
Parallel paths (simplex)	**A = P**	**A = 2** (independent of P)
Multiplex (m-plex)	A = mP	A = 2m
Brushes needed	= P	2 minimum (P usually fitted)
Suited for	**Low voltage, high current**	**High voltage, low current**
Special requirement	**Equaliser rings** (avoid circulating currents due to unequal path EMFs)	**Dummy coils** (mechanical balance), if needed
Pitch	$Y_b - Y_f = \pm 2$ (progressive if $Y_b > Y_f$)	Average pitch $Y_A = (Z \pm 2)/P$ must be an integer
Other name	Parallel / multiple winding	Series / two-circuit winding

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KEY FORMULAE

- EMF: $E_g = P\Phi ZN/(60A)$ volts (Φ in Wb per pole, Z = total conductors, N in rpm). For a given machine $E \propto \Phi N$
- Shunt generator: $I_a = I_L + I_{sh}$, $E_g = V + I_aR_a + V_{brush}$, $I_{sh} = V/R_{sh}$
- Series generator: $I_a = I_{se} = I_L$, $E_g = V + I_a(R_a + R_{se})$
- Critical speed: $N_c = N \times R_{sh}/R_c$ (R_c = critical resistance at speed N). $R_c \propto$ speed
- Armature reaction per pole ($I_c = I_a/A$, θ_m = brush shift in mechanical degrees): demagnetising AT = $Z I_c \theta_m/360$, cross-magnetising AT = $Z I_c (1/2P - \theta_m/360)$
- Reactance voltage (linear commutation) = $L \times 2I_c/T_c$ (T_c = commutation period)
- Generator maximum efficiency at $I_a^2R_a$ = constant losses, i.e. $I = \sqrt{(W_c/R_a)}$

Type	Field connection	External characteristic	Applications
Separately excited	Field from an independent DC source	Slight drop (I_aR_a + armature reaction). **Wide, stable voltage control**	**Ward-Leonard drives**, testing labs, electroplating
Shunt	Field across armature (high R)	**Drooping**. Voltage falls with load and the curve **turns back** at breakdown. SC current is small	**Battery charging**, lighting, excitation of alternators
Series	Field in series with armature and load	**Rising** (V increases with load until saturation)	**Boosters** (in DC feeders to compensate line drop), series arc lighting (old)
Cumulative compound	Series flux **aids** shunt flux	**Over**-compound ($V_{fl} > V_{nl}$), **flat/level** ($V_{fl} = V_{nl}$) or **under**-compound ($V_{fl} < V_{nl}$)	Over: **long feeders** (compensates line drop). Flat: short-distance supply, hotels, offices
Differential compound	Series flux **opposes** shunt flux	**Steeply drooping** (nearly constant current)	**Arc welding**

- **Long-shunt**: shunt field across (armature + series field). **Short-shunt**: shunt field across the armature only.
- Degree of compounding is adjusted by a **diverter** (low resistance) across the series field.
- **OCC (magnetisation curve)**: E_0 against I_f at constant speed. It starts at the **residual voltage** (a few % of rated) and is linear, then shows a **knee (saturation)**. OCC at another speed: $E \propto N$ for the same I_f.
- **Critical field resistance R_c** = slope of the **tangent to the initial (linear) part of the OCC** through the origin. If $R_{sh} > R_c$ the generator **fails to build up**. The no-load voltage is where the field-resistance line cuts the OCC.
- **Critical speed**: the speed at which the given R_{sh} becomes the critical resistance. Below it, no build-up.
- For a **series generator** there is a **critical load resistance**. Build-up fails if the **load resistance is greater** than critical (an open load gives no build-up).

Conditions for voltage build-up (self-excited)	Remedy if failed
1. **Residual magnetism** must be present in the poles	**Flash the field** from a battery or external DC
2. Field winding connected so that its mmf **aids the residual flux**	Reverse the field connections (or the direction of rotation)
3. Shunt field resistance **less than critical resistance**	Reduce field rheostat resistance
4. Speed **above critical speed**	Increase prime-mover speed
5. Series generator: load resistance below critical (load connected)	Connect / reduce load resistance

- **Armature reaction** is the effect of armature mmf on the main field flux.
  - With brushes on the **GNA** (geometric neutral axis), the effect is purely **cross-magnetising**. It **distorts** the field (strengthens one pole tip, weakens the other) and **shifts the MNA** (magnetic neutral axis). The MNA moves **forward (in the direction of rotation) in a generator** and **backward in a motor**.
  - If the brushes are shifted to the new MNA (forward in a generator, backward in a motor), a **demagnetising** component also appears, which **reduces the net flux**.
  - Ill effects: lower EMF (saturation of pole tips causes a net flux reduction even with no brush shift), **sparking**, extra iron loss, and higher voltage between commutator segments (**flashover** risk).
- **Remedies**:
  - **Compensating winding**: placed in **slots in the pole faces**, in **series with the armature**, with current **opposite** to the adjacent armature conductors. It **neutralises cross-magnetising AR under the pole faces**. It is costly and used in **large machines with heavy/fluctuating loads** (rolling mills).
  - **Interpoles (commutating poles)**: narrow poles **midway between main poles (on the GNA)**, in **series with the armature**. They neutralise the AR in the commutating zone and induce an EMF that cancels the **reactance voltage**. Polarity: **generator – same as the next main pole ahead** (in direction of rotation). **Motor – same as the main pole just behind** (preceding).
  - High-reluctance pole tips, chamfered poles, and laminated poles with staggered laminations.
- **Commutation**: reversal of current in a coil while it is short-circuited by the brush. Ideal is **linear commutation**. **Self-inductance of the coil** opposes the reversal (**reactance voltage**), which delays the reversal (under-commutation) and causes **sparking at the trailing brush edge**.
  - **Resistance commutation**: uses **high-resistance carbon brushes**.
  - **Voltage (EMF) commutation**: injects an opposing EMF into the commutating coil by **interpoles** (most common) or by **brush shift** (beyond the MNA).
- **Parallel operation of DC generators**: same **polarity** (+ to +), equal **terminal voltage** at the time of connection, and preferably **drooping** characteristics. **Shunt generators** share load stably. To transfer load, raise the field excitation of one machine. **Series and over-compound generators** are unstable in parallel and need an **equaliser bar** (a low-resistance connection joining the series fields so they stay at the same potential).
- Shunt generator SC: armature short-circuit makes V = 0, so $I_{sh} = 0$ and the current is only from residual EMF. It is **self-protecting**.

MEMORY TRICK

"**Lap = Large current, Wave = Wide (high) voltage**." Interpole polarity: "**Generator Goes ahead, Motor looks back**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- 4-pole, lap-wound, 500 conductors, Φ = 0.02 Wb, 1200 rpm. EMF? → $E = 4 \times 0.02 \times 500 \times 1200/(60 \times 4)$ = **200 V** (wave: 400 V).
- Critical field resistance is obtained from? → **Slope of the tangent to the initial part of the OCC**.
- A shunt generator fails to build up voltage. The first check? → **Residual magnetism** (flash the field).
- Generator used for arc welding? → **Differential compound**. As a booster? → **Series**.
- Compensating winding is connected in? → **Series with the armature, in pole-face slots**.
- Equaliser rings are used in? → **Lap winding**. Equaliser bar is used for? → **Parallel operation of compound/series generators**.

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4.6 DC Motors – Characteristics & Applications ★★★

KEY FORMULAE

- Back EMF: $E_b = P\Phi ZN/(60A) = V - I_aR_a$. $I_a = (V - E_b)/R_a$
- Power equation: $V I_a = E_b I_a + I_a^2R_a$. **Mechanical power developed = E_bI_a**
- Torque: $T_a = P\Phi ZI_a/(2\pi A) = 0.159\ P\Phi ZI_a/A$ N·m, so $T \propto \Phi I_a$
- $T_a = 9.55\ E_bI_a/N$ N·m (N in rpm). Shaft torque T_{sh} = 9.55 × output/N ($T_{sh} < T_a$ by the lost torque)
- Speed: $N = (V - I_aR_a)/(k\Phi)$, i.e. $N \propto E_b/\Phi$. Speed ratio $N_2/N_1 = (E_{b2}/E_{b1}) \times (\Phi_1/\Phi_2)$
- Series motor (unsaturated, $\Phi \propto I_a$): $T \propto I_a^2$, $N \propto 1/I_a$, $N \propto 1/\sqrt{T}$. After saturation $T \propto I_a$
- Maximum mechanical power: $E_b = V/2$ ($I_a = V/2R_a$). Efficiency is then **50%** (below 50% overall), so it is never used in practice
- % Speed regulation = $(N_0 - N_{fl})/N_{fl} \times 100$
- Maximum efficiency: $I_a^2R_a$ **(variable loss) = constant losses**

- **Back EMF** makes the motor **self-regulating**. When load increases, speed falls slightly, E_b falls, and I_a rises to produce more torque. At start, **N = 0 so E_b = 0** and $I_a = V/R_a$ (very large).
- E_b is typically **≈ 90–95% of V** at full load (I_aR_a drop ≈ 2–5% in large machines, more in small ones).
- **Direction of rotation** is reversed by reversing **either armature or field** connections, **not both**. Reversing the supply polarity does **not** reverse a shunt/series motor. A series motor can therefore run on AC (**universal motor**).
- **Shunt motor**: Φ ≈ constant, so it is a **nearly constant-speed** motor (speed regulation ≈ **5–15%**). $T \propto I_a$ (straight line). Starting torque moderate (≈ 1.5–2 × FL, limited by the starter).
- **Field open in a shunt motor**: Φ falls to the residual value, so the speed **rises dangerously (runaway)** on light load. On heavy load the motor may stall and draw excessive current. Hence the field circuit carries no fuse/switch, and the starter has an NVR.
- **Series motor**: high **starting torque** ($T \propto I_a^2$), variable speed. At no load I_a → small, Φ → small and N → **dangerously high**. So it is **never started or run on no load** and is **never belt-driven** (belt may slip off). It is always **direct-coupled or geared** to the load.
- Series motor **power ≈ constant** over a wide range (T × N ≈ const). It is called a **constant-power** or **variable-speed** motor.
- **Cumulative compound motor**: characteristics lie between shunt and series. It has a **definite no-load speed** (safe on no load), good starting torque, and suits loads with **sudden heavy loading** (with a flywheel).
- **Differential compound motor**: speed may **rise with load**, so it is **unstable**. At start the series field may dominate and the motor can start in the reverse direction. It is **rarely used** (only some experimental/research work).

Characteristic	Shunt (separately excited)	Series	Cumulative compound
T vs I_a	**Straight line** through origin (slight droop due to AR)	**Parabola** ($T \propto I_a^2$), then straight after saturation	Between; steeper than shunt
N vs I_a	**Slightly drooping, nearly constant**	**Rectangular hyperbola** ($N \propto 1/I_a$)	Drooping more than shunt; finite no-load speed
N vs T (mechanical)	Slightly drooping	Hyperbolic; **$N \propto 1/\sqrt{T}$**	Between
Starting torque	Medium	**Highest**	High
No-load operation	Safe	**Dangerous**	Safe

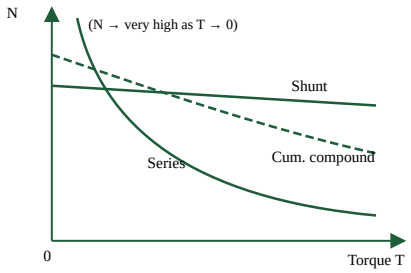

Fig 4.2 Speed–torque characteristics of DC motors (qualitative).

Motor	Applications
Shunt	Near-constant speed: **lathes, machine tools, centrifugal pumps, fans, blowers, conveyors**, drilling, spinning and weaving machines, printing presses
Series	High starting torque: **electric traction (trains, trams, trolley buses), cranes, hoists, lifts**, conveyors with heavy start, starter motors of automobiles, vacuum cleaners and mixers (universal version)
Cumulative compound	Intermittent/sudden heavy loads: **presses, shears, punching machines, rolling mills**, elevators, heavy planers, reciprocating compressors, ice machines
Differential compound	Hardly any (unstable)
Separately excited	Ward-Leonard drives, **paper mills, rolling mills, steel mills** (wide, precise speed control)

- **Losses in a DC machine**: armature Cu ($I_a^2R_a$, variable), field Cu (shunt = constant, series = variable), **brush contact loss** (≈ 2 V × I_a), iron loss and mechanical loss (constant at constant speed), stray load loss (≈ 1%). Efficiency of DC machines is about **85–95%**.

Test	Type	Machines	Key point / merit / limitation
Brake test	**Direct** (load) test	**Small** motors	Output = (W − S) × r × ω (belt and pulley). Heat dissipation is a problem, so it is not for large machines
Swinburne's test	**Indirect, no-load**	**Shunt and compound** (flux constant). **Not for series**	Constant losses are measured at no load and η is **predetermined at any load**. Economical. It **ignores the change in iron loss** due to armature reaction, and the rise in resistance due to temperature, at load
Hopkinson's test (back-to-back / regenerative)	**Regenerative, full load**	**Two identical shunt machines** coupled mechanically and electrically	The supply provides **only the losses**. Tested at full load for a long time (temperature rise and commutation). Needs two identical machines
Field's test	Indirect	**Two identical series machines** (traction motors)	Motor drives generator. Series fields are in the same circuit so excitation is equal. Generator output is wasted in resistance
Retardation (running-down) test	Indirect	Shunt machines	Gives **stray (iron + mechanical) losses** from the speed–time curve of a coasting machine

MEMORY TRICK

"**SWinburne = Shunt Without load**; **HOPkinson = Hand-in-hand pair** (two shunt machines); **FIELD = Fully series pair**." A series motor is "**Strong start, Scary at no-load**".

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- 220 V shunt motor, R_a = 0.5 Ω, I_a = 20 A. Back EMF? → **220 − 10 = 210 V**.
- A DC series motor should never be started without load because? → **Speed becomes dangerously high**.
- Condition for maximum power output of a DC motor? → **E_b = V/2**.
- Swinburne's test cannot be performed on? → **DC series motor**.
- Motor best suited for electric traction? → **DC series motor**. For a lathe? → **DC shunt**. For a punching press? → **Cumulative compound**.
- Torque of a series motor below saturation is proportional to? → **I_a^2**.

4.7 Starting & Speed Control of DC Motors ★★★

- **Need for a starter**: at start E_b = 0, so **I_a = V/R_a**. With R_a ≈ 0.5 Ω on 220 V this is ≈ 440 A, i.e. 15–20 × full load. Such a current would damage the commutator and brushes (sparking), cause a large dip in supply voltage, and give a mechanical shock. The starter inserts **resistance in series with the armature** and cuts it out step by step as E_b builds up.
- The starting current is usually limited to **about 1.5–2 × full-load current**.
- **Very small motors (below about 0.5 kW / fractional hp)** may be started DOL because their R_a is relatively high and their inertia is low.

Starter	Used for	Terminals	NVR (hold-on) coil connection	Remark
2-point	**Series motors**	L, A (line, armature)	In series with the motor circuit. Acts as a **no-load release**	Handle drops to OFF if the load is lost (prevents overspeed)
3-point	**Shunt and compound** motors	**L, F, A**	**In series with the shunt field**	**Drawback**: when field current is reduced for speed control (field weakening), the NVR may release and **trip the motor**. Not for wide field-control range
4-point	Shunt/compound with **field (speed) control**	L, F, A, + separate line connection	**Directly across the supply** (with a protective resistance), **independent of field current**	Drawback: it does **not** trip if the field circuit opens, so field-failure protection is lost

- **NVR (No-Volt Release)**: an electromagnet holding the starter handle in the ON position. On supply failure (or field failure in a 3-point starter) it releases, and a spring returns the handle to OFF. This prevents restarting with no resistance when supply returns.
- **OLR (Over-Load Release)**: a coil in the line circuit. On overload it lifts an armature that **short-circuits the NVR coil**, which releases the handle.
- Modern drives use **automatic (contactor) starters**, time-delay or current-limit acceleration, and thyristor/chopper soft starting.

KEY FORMULAE

- $N \propto (V - I_a R_a)/\Phi$. Control variables: **Φ (field), R (armature circuit), V (armature voltage)**
- Armature resistance control (same torque, so I_a same): $N_2/N_1 = [V - I_a(R_a + R)]/(V - I_a R_a)$
- Field control: $N_2/N_1 \approx (E_{b2}/E_{b1})(i_{f1}/i_{f2})$ (linear magnetic circuit)
- Efficiency of armature-resistance control ≈ **N/N_0** (e.g. half speed gives ≈ 50% efficiency)
- Plugging current ≈ **$(V + E_b)/R \approx 2V/R$**, so extra resistance is needed

Method	Speed range	Load type	Merits	Demerits
Flux (field) control: field rheostat in series with shunt field	**Above base speed** only (weakening Φ raises N)	**Constant power** (T ∝ 1/N)	Simple, cheap, **efficient** (small field current, low loss)	Upper speed limited by commutation, armature reaction and mechanical stability (typically up to ≈ 2–3 × base)
Armature resistance (rheostatic) control	**Below base speed** only	**Constant torque**	Simple. Gives speeds down to near zero	**Large power loss** in resistance (poor efficiency). **Poor speed regulation** (speed changes with load). Used for short-time duties, e.g. cranes, hoists
Armature voltage control (multiple-voltage, Ward-Leonard, choppers/rectifiers)	**Below base speed** (0 up to base)	**Constant torque**	Smooth, wide, efficient. Good regulation	Needs a variable-voltage source (costly)

- **Base speed** = speed at **rated armature voltage and rated (full) field**. Below base speed use armature-voltage control (constant torque). Above base speed use field weakening (constant power).
- **Ward-Leonard system**: a separately excited DC motor whose armature is fed by a **separately excited DC generator** driven by an induction/synchronous motor (an **M-G set**, so 3 machines). The generator field is varied (and reversed).
  - Merits: **very wide, smooth, stepless speed control in both directions**, inherent **regenerative braking**, fast reversal, good speed regulation.
  - Demerits: **high initial cost** (3 machines of the same rating), more floor space, **low overall efficiency** (≈ 60–70%), noise and maintenance.
  - Uses: **elevators/lifts, mine winders, rolling mills, paper machines, large cranes, excavators**.
  - **Ward-Leonard–Ilgner system**: adds a **flywheel** on the M-G set (with a slip-ring induction motor) to smooth heavy **fluctuating loads** (rolling mills, mine hoists).
  - The modern static equivalent is the **thyristor converter / chopper-fed DC drive**.
- **Series motor speed control**:
  - **Field diverter**: a variable resistance in **parallel with the series field**. It diverts current, so Φ falls and **speed rises** (above normal).
  - **Armature diverter**: a resistance in **parallel with the armature**. For constant load torque this raises the field current, so **speed falls** (below normal).
  - **Tapped field control**: fewer field turns give less Φ, so **speed rises**. Used in **electric traction**.
  - **Paralleling field coils**: field coils regrouped in series/parallel give different flux levels (fan motors).
  - **Series-parallel control** (traction, 2 or more motors): at start the motors are in **series** (each gets V/2, low speed and less starting loss). For high speed they are switched to **parallel** (each gets V). Often combined with rheostats, called **series-parallel starting**.
  - Variable-resistance in series with the motor (rheostatic control), as for shunt motors.

Electric braking	How it is done	Features
Plugging (reverse-current braking)	**Armature supply connections reversed** (field unchanged) with an extra series resistance	V and E_b act together, so current ≈ 2V/R. **Most rapid braking** and gives a braking torque even at zero speed. **Most wasteful**: power taken from supply plus KE is dissipated. The motor must be disconnected at **zero speed** or it will **reverse**. Used in rolling mills, elevators, printing presses, machine tools
Dynamic (rheostatic) braking	Armature **disconnected from supply** and connected across a **braking resistor**. Field remains excited (shunt) or field connections reversed (series, self-excited)	Motor acts as a generator. KE is dissipated as heat in the resistor. **Braking torque falls with speed**, so it **cannot hold the load at standstill** (mechanical brake needed). Simple and cheap

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Regenerative braking	Occurs when the load drives the motor **above no-load speed** so that $E_b > V$	Armature current reverses and **energy is returned to the supply**, which makes it the **most efficient** method. Used in **traction (down gradients), hoists/cranes lowering loads**, and electric vehicles. **Cannot bring the motor to rest**. **Not directly possible with series motors** (E_b cannot exceed V because field current reverses with I_a). They must be reconnected as separately excited. Supply must be able to receive power

- Order of energy efficiency: **regenerative > dynamic > plugging**. Order of braking speed: **plugging > dynamic**.
- Electric braking **supplements, not replaces**, mechanical brakes. A mechanical brake is required to **hold a load at standstill**.

MEMORY TRICK

"**Field Fast, Armature Slow**": field weakening gives speeds above base (constant **P**ower), and armature resistance/voltage gives speeds below base (constant **T**orque). Braking: "**Plug = Punish** (wastes most), **Regen = Return** (saves most)".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In a 3-point starter, the NVR coil is connected in? → **Series with the shunt field**.
- Why is a 4-point starter preferred for a motor with field control? → **NVR is independent of field current**, so it does not trip during field weakening.
- Speeds above base speed in a DC shunt motor are obtained by? → **Field (flux) control** (constant power).
- Ward-Leonard method of speed control is a type of? → **Armature voltage control**.
- Braking in which energy is fed back to the supply? → **Regenerative braking** ($E_b > V$).
- Quickest method of electric braking? → **Plugging**.

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5. Electrical Machines II – Induction & Synchronous Machines

> **NOTE**
> Rotating AC machines usually give **8–12 of the 60 questions** in a PSU/SSC JE-level paper. The most-asked areas are slip and frequency numericals, the power-flow ratio 1 : s : (1 − s), T_{max} and s_m, the star-delta starter (1/3), the single-phase motor types, pitch and distribution factors, regulation methods (pessimistic or optimistic), synchronisation conditions, V-curves and insulation classes. Learn the formulae and the "which is greater" facts. Most numericals take one line.

5.1 Three-Phase Induction Motor – Principle & Construction ★★★

- **Principle**: a 3-phase supply to a 3-phase distributed winding produces a **rotating magnetic field (RMF)**. This field induces an EMF and a current in the rotor conductors (**transformer action**). The rotor current reacts with the field and produces torque, so the rotor follows the field (**Lenz's law**).
- The induction motor is a **singly excited** machine: only the stator is fed. It is a "rotating transformer" whose secondary is short-circuited.
- RMF concept: **Galileo Ferraris** (1885) and **Nikola Tesla** (1888). Tesla patented the polyphase induction motor.
- **Magnitude of the RMF** for a 3-phase winding = **1.5 Φ_m** (3/2 × maximum flux of one phase). It is constant in magnitude and rotates at synchronous speed. For an m-phase winding it is **(m/2) Φ_m**.
- Direction of rotation of the RMF follows the **phase sequence**. Interchanging **any two supply leads** reverses the motor.
- If one phase is lost while running (**single phasing**), the motor keeps running but draws heavy current and overheats. It will **not start** from rest on single phasing.
- **Synchronous speed** N_s = **120f/P** rpm. The rotor can never reach N_s as a motor, because there would be no relative motion, no induced EMF and no torque.
- **Slip** s = $(N_s - N)/N_s$. Slip speed = $N_s - N$.
- Typical full-load slip: **2–5%** (large motors about 1–2%, small motors up to 5–6%). No-load slip is about **0.5–1%**.
- **Rotor frequency** f_r = **s f**. At standstill (s = 1) f_r = f. At full load it is typically **1–3 Hz**.
- Rotor EMF at slip s: E_{2s} = **sE_2**. Rotor reactance: X_{2s} = **sX_2**. Both are maximum at standstill.
- The rotor field rotates at sN_s relative to the rotor and at **N_s relative to the stator**. The stator and rotor fields are therefore **always stationary with respect to each other**, which is why steady torque is produced.
- The number of rotor poles is automatically equal to the number of stator poles in a squirrel cage. A slip-ring rotor must be **wound for the same number of poles** as the stator.
- **Stator**: **0.35–0.5 mm silicon-steel laminations** to reduce eddy loss. Semi-closed slots are used in small motors and open slots in large ones.
- **Air gap** is kept **as small as mechanically possible** (about **0.2–2 mm**). A larger gap needs more magnetising current, which lowers the **power factor**. Magnetising current is **25–40% of I_{FL}** in an induction motor, against 2–5% in a transformer.
- No-load power factor is very low, about **0.1–0.2 lagging**. Full-load pf is **0.8–0.9 lagging**. Efficiency is **85–97%**, rising with rating.
- **Skewing** (rotor slots inclined by about one slot pitch) gives **reduced magnetic hum/noise**, **prevents cogging (magnetic locking)**, **reduces crawling** (slot harmonics) and smoother torque. On the debit side it slightly **increases rotor resistance and leakage reactance**.
- The rotor bars of a cage are **not insulated** from the core, because the induced voltage is very low. They are aluminium (die-cast) or copper.
- Stator and rotor slot numbers are never made equal ($S_s \neq S_r$), so that **cogging** is avoided.
- An induction motor is also called an **asynchronous motor**. It runs at nearly constant speed (a shunt characteristic).

Feature	Squirrel cage	Slip ring (wound rotor / phase-wound)
Rotor	Bars short-circuited by **end rings**	3-phase winding, **star-connected**, ends brought to **3 slip rings**
Share of motors in use	**≈ 90%** or more	≈ 10%
External resistance	Not possible	Possible, through brushes
Starting torque	Low to moderate (1.5–2 T_{FL})	**High**; can equal T_{max} at start
Starting current	High (5–7 × I_{FL})	Low (limited by rotor resistance)
Speed control	Only from the stator side	Rotor side as well (resistance, slip power recovery)
Cost, maintenance	**Cheap, robust, little maintenance**	Costlier; brushes and slip rings need upkeep
Efficiency, pf	Higher	Slightly lower
Explosive atmosphere	**Suitable** (no sparking)	Not suitable (brush sparking)
Applications	Pumps, fans, blowers, lathes, compressors	Cranes, hoists, lifts, elevators, mill drives (high T_{st})

- **Deep-bar and double-cage rotors** give high starting torque with good running efficiency. At start $f_r = f$, so the current crowds into the high-resistance outer cage. When running, f_r is low and the current flows mainly in the low-resistance inner cage, which has high reactance.
- In a double cage, the **outer cage has high R and low X** and the **inner cage has low R and high X**.

> **KEY FORMULAE**
> - $N_s = 120f/P$; $s = (N_s - N)/N_s$; $N = N_s(1 - s)$
> - $f_r = sf$; $E_{2s} = sE_2$; $X_{2s} = sX_2$
> - Rotor current $I_2 = sE_2/\sqrt{(R_2^2 + (sX_2)^2)}$; rotor pf = $R_2/\sqrt{(R_2^2 + (sX_2)^2)}$
> - RMF (3-phase) = 1.5 Φ_m

Poles (50 Hz)	2	4	6	8	10	12
N_s (rpm)	**3000**	**1500**	**1000**	**750**	**600**	**500**
Typical full-load N (rpm)	2880–2950	1440–1470	960–980	720–735	≈ 580	≈ 485

> **MEMORY TRICK**
> To find the poles from a rated speed, **take the next higher synchronous speed**. A motor marked 1440 rpm on 50 Hz has N_s = 1500, so **P = 4** and s = 60/1500 = **4%**, giving f_r = 0.04 × 50 = **2 Hz**.

> **PREVIOUSLY ASKED / LIKELY QUESTIONS**
> - A 6-pole, 50 Hz induction motor runs at 960 rpm. What is the rotor frequency? → s = 0.04, so **f_r = 2 Hz**.
> - What is the magnitude of the RMF of a 3-phase winding? → **1.5 Φ_m**.
> - Why is the air gap of an induction motor kept small? → To reduce the **magnetising current** and so improve the **power factor**.
> - Why are rotor slots skewed? → To reduce **noise and cogging** and to give smoother torque.
> - How is the direction of an induction motor reversed? → **Interchange any two supply phases**.
> - Which motor is used in cranes and hoists? → The **slip-ring induction motor**.

5.2 Equivalent Circuit, Power Flow & Torque ★★★

- **Per-phase equivalent circuit** (referred to the stator):
  - $R_1 + jX_1$ (stator), then the shunt branch $R_c \parallel jX_m$ (core loss and magnetising), then jX_2' and **R_2'/s**.
  - R_2'/s splits into R_2' (rotor Cu loss) + **$R_2'(1 - s)/s$**, the **load resistance** that represents mechanical output.
- This is the same as a transformer circuit with a variable load resistance. The magnetising branch **cannot be shifted to the input terminals as accurately** as in a transformer, because I_0 is large. The approximate circuit (magnetising branch moved to the terminals) does this only for rough work; the IEEE-recommended circuit keeps X_m after $R_1 + jX_1$.

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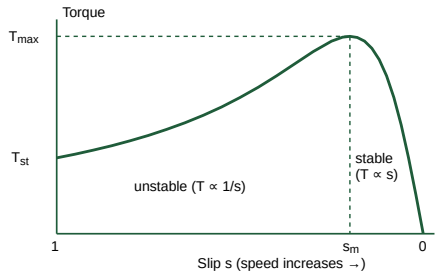
- At no load $s \approx 0$, so $R_2'/s \to \infty$ (open circuit). At standstill $s = 1$, so the load resistance is zero (short circuit, as in the blocked-rotor test).

Quantity	Expression	Ratio
Air-gap power (rotor input) P_g	$3I_2^2R_2/s$	**1**
Rotor copper loss P_{c2}	$3I_2^2R_2 = sP_g$	**s**
Gross mechanical power P_m	$3I_2^2R_2(1-s)/s = (1-s)P_g$	**1 − s**

- **Power flow**: stator input → minus stator Cu loss and core loss (mainly the stator core, since rotor iron loss is negligible at the low f_r) → **air-gap power** → minus rotor Cu loss → gross mechanical power → minus friction and windage → **shaft output**.
- **Rotor efficiency** (ignoring mechanical loss) = $P_m/P_g = \mathbf{1 - s \approx N/N_s}$. Operating at high slip is therefore inefficient. This is why rotor-resistance speed control wastes power.
- Torque from air-gap power: $\mathbf{T = P_g/\omega_s} = P_m/\omega$. Air-gap power in watts is called the "torque in synchronous watts".
- **Torque equation** (stator impedance neglected): $T = (3/\omega_s) \cdot s\, E_2^2\, R_2/(R_2^2 + s^2X_2^2)$. Also $T \propto \varphi\, I_2 \cos\varphi_2$.
- Since $E_2 \propto V$, $\mathbf{T \propto V^2}$ at a given slip. A 10% voltage dip gives about a **19% torque drop** ($0.9^2 = 0.81$).
- **Maximum (breakdown or pull-out) torque** occurs when $R_2 = sX_2$, so $\mathbf{s_m = R_2/X_2}$.
- $\mathbf{T_{max} = 3E_2^2/(2\omega_s X_2)}$. It is **independent of rotor resistance R_2**, $\propto \mathbf{V^2}$ and $\propto \mathbf{1/X_2}$. Changing R_2 only changes the **slip (speed) at which T_{max} occurs**.
- T_{max} is typically $\mathbf{2–3 \times T_{FL}}$. Full-load slip lies well within the stable region.
- **Starting torque** ($s = 1$): $T_{st} = (3/\omega_s)E_2^2R_2/(R_2^2 + X_2^2)$. T_{st} is maximum ($= T_{max}$) when $\mathbf{R_2 = X_2}$, i.e. $s_m = 1$.
- **Effect of increasing R_2**:
  - s_m increases, so T_{max} shifts towards standstill.
  - T_{st} increases (up to $R_2 = X_2$) and starting current decreases.
  - T_{max} is unchanged.
  - Full-load slip rises and efficiency falls.
- **Torque–slip curve**:
  - At **low slip** ($sX_2 \ll R_2$): $\mathbf{T \propto s}$, a straight line. This is the stable running region.
  - At **high slip** ($sX_2 \gg R_2$): $\mathbf{T \propto 1/s}$, a rectangular hyperbola. This is the unstable region.
- **Stable region**: $0 < s < s_m$, where $dT/ds > 0$ (equivalently $dT/dN < 0$). **Unstable region**: $s_m < s < 1$.
- **Regions of operation**:
  - **Motoring**: $0 < s < 1$.
  - **Generating**: $\mathbf{s < 0}$ ($N > N_s$). Torque reverses and power is fed back to the supply.
  - **Braking (plugging)**: $\mathbf{s > 1}$, between 1 and 2. The rotor runs against the RMF (two supply phases reversed). Slip at the instant of plugging is $\approx \mathbf{2 - s}$.
- The maximum torque in generating mode is **slightly greater** than in motoring mode when R_1 is considered.
- With stator impedance included: $s_m = R_2'/\sqrt{(R_1^2 + (X_1 + X_2')^2)}$ and $T_{max} = 3V^2/[2\omega_s(R_1 + \sqrt{(R_1^2 + (X_1 + X_2')^2)})]$.
- **Under V/f constant**, T_{max} stays constant (neglecting R_1). At reduced frequency with constant V the flux rises and the core saturates.



Torque–slip curve (motoring region) drawn for $s_m = 0.2$, which gives $T_{st}/T_{max} = 2(0.2)/(1 + 0.04) \approx 0.38$

KEY FORMULAE

- $P_g : P_{c2} : P_m = 1 : s : (1 - s)$; $P_{c2} = sP_g$
- $T = P_g/\omega_s = 60P_g/(2\pi N_s)$
- $s_m = R_2/X_2$; $T_{max} = 3E_2^2/(2\omega_s X_2)$
- $T/T_{max} = 2\, s\, s_m/(s^2 + s_m^2)$
- $\mathbf{T_{st}/T_{max} = 2s_m/(1 + s_m^2) = 2a/(1 + a^2)}$, where $a = R_2/X_2$
- $T_{FL}/T_{max} = 2\, s_f\, s_m/(s_f^2 + s_m^2)$
- $T_{st}/T_{FL} = (I_{st}/I_{FL})^2 \approx s_f$ (since $T \propto I_2^2R_2/s$)

- **Worked example**: rotor input = 10 kW and s = 4%.
  - Rotor Cu loss = **400 W**.
  - Gross mechanical power = **9.6 kW**.
- **Worked example**: a 4-pole, 50 Hz motor with P_g = 15.7 kW.
  - $\omega_s = 2\pi \times 1500/60 = 157.1$ rad/s.
  - $T = 15.7\ \text{kW}/157.1\ \text{rad/s} \approx$ **100 N·m**.
- **Worked example**: $R_2 = 0.02\ \Omega$ and $X_2 = 0.1\ \Omega$.
  - $s_m = 0.2$.
  - $T_{st}/T_{max} = 0.4/1.04 =$ **0.385**.
  - For $T_{st} = T_{max}$, the total rotor resistance must be 0.1 Ω, so add **0.08 Ω** per phase.

MEMORY TRICK

"Resistance shifts, Reactance limits". R_2 shifts the location (s_m) of the torque peak but not its height. X_2 and V decide the height ($T_{max} \propto V^2/X_2$).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- The maximum torque of an induction motor is independent of? → **Rotor resistance**.
- If the supply voltage is halved, what happens to the starting torque? → It becomes **one-fourth** ($T \propto V^2$).
- Slip at maximum torque? → $\mathbf{R_2/X_2}$.
- The ratio of rotor Cu loss to rotor input equals? → **Slip**.
- An induction machine operates as a generator when slip is? → **Negative** ($N > N_s$).
- Condition for maximum starting torque? → $\mathbf{R_2 = X_2}$.

5.3 No-Load & Blocked Rotor Tests, Circle Diagram ★★☆

Point	No-load (running light) test	Blocked-rotor (short-circuit) test
Analogous to	**Open-circuit test** of a transformer	**Short-circuit test** of a transformer
Condition	Rated V and f, no load, $s \approx 0$	Rotor locked ($s = 1$), **reduced voltage** (≈ 10–20% of rated) to circulate rated current
Wattmeter reads	**Core loss + friction & windage** (+ small stator Cu loss $I_0^2R_1$)	**Full-load copper loss** (core loss negligible at low V)
Gives	**Magnetising branch** R_0, X_0; no-load pf ≈ 0.1–0.2	**Equivalent R_{eq}, X_{eq}, Z_{eq}** referred to the stator; short-circuit current at rated V
Separating losses	Plot P_0 against V and extrapolate to V = 0. The intercept gives **friction & windage**.	$R_2' = R_{eq} - R_1$ (R_1 by DC test × 1.1–1.3 for skin effect)

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- Large motors are best blocked-rotor tested at **reduced frequency (≈ 25% of rated)**. This gives a rotor resistance that is realistic for normal running, since the running f_r is low (IEEE 112).
- In the blocked-rotor test X_1 and X_2' are usually taken as **equal** (wound rotor, design classes A/D). Class B uses a 0.4 : 0.6 split and class C 0.3 : 0.7.
- **Circle diagram** (Heyland diagram):
  - With constant V and constant parameters, the locus of the stator current is a **circle**.
  - It is drawn from the **no-load and blocked-rotor test** data and the stator resistance.
  - Its **diameter ≈ V/X** ($X = X_1 + X_2'$).
- Lines in the circle diagram:
  - **Output line**: joins the no-load point O′ to the blocked-rotor point A.
  - **Torque line**: separates stator Cu loss from rotor Cu loss.
  - Maximum output and maximum torque are found where **tangents parallel to the output line and the torque line** touch the circle.
- From the circle diagram one can read:
  - full-load current, pf, slip and efficiency
  - maximum output and maximum torque
  - starting torque

Phenomenon	Cause	Symptom	Remedy
Crawling	Space harmonics in the air-gap flux, especially the **7th** (forward, at $N_s/7$). The 5th rotates backward.	The motor runs steadily at a low speed of **≈ 1/7 of N_s** and fails to accelerate. Mainly a squirrel-cage problem.	Proper chording and distribution, skewing
Cogging (magnetic locking)	**Stator slots = rotor slots** (or an integral multiple). Reluctance is minimum when teeth align.	The motor **refuses to start** at all.	**$S_s \neq S_r$**, skewing

- **Losses**:
  - Constant: stator core loss, friction & windage.
  - Variable: stator and rotor Cu loss, stray load loss.
  - Rotor iron loss is negligible because f_r is low.
- Efficiency η = output/(output + losses). It is maximum when **variable loss = constant loss**.
- **Induction generator** (asynchronous generator):
  - It is driven **above N_s**, so s is negative.
  - It **cannot supply its own reactive power**. It always **draws magnetising (lagging) VAr** from the grid or from a **capacitor bank** (self-excited induction generator, SEIG).
  - It needs no synchronisation.
  - Its output frequency is fixed by the grid, not by the speed.
  - Typical uses: **wind turbines**, mini or micro hydro.
  - Disadvantage: it operates only at a **leading pf** as seen from the generator side, i.e. it absorbs VAr.
- **DFIG** (doubly-fed induction generator): a slip-ring machine with a back-to-back converter in the rotor circuit, rated at only about **25–30% of the machine rating**. It is the most common variable-speed wind generator.
- An SEIG fails to build up if the capacitance is below the critical value, or if the residual magnetism is lost.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- The no-load test of an induction motor gives? → **Core loss + friction & windage loss**.
- The blocked-rotor test gives? → **Copper loss and equivalent impedance**.
- Crawling is due to? → **The 7th (and 5th) space harmonics**; the motor runs at about $N_s/7$.
- Cogging occurs when? → **The number of stator slots equals the number of rotor slots**.
- An induction generator needs which external supply? → **Reactive power** (from the grid or capacitors).

5.4 Starting & Speed Control of Induction Motors ★★★

- At start s = 1. The motor behaves like a short-circuited transformer and draws **5–7 × I_{FL}** at a low pf (≈ 0.2–0.4). The starter must limit this current.
- Every starter also provides **overload protection** (thermal relay) and **no-volt (under-voltage) release**.
- Reduced-voltage starting reduces the current but reduces the torque by the **square** of the voltage ratio.

Starter	Line current at start (vs DOL)	Starting torque (vs DOL)	Remarks
DOL (direct-on-line)	I_{sc} = 5–7 × I_{FL}	$T_{st} = (I_{sc}/I_{FL})^2 s_f T_{FL}$	Small motors, conventionally **up to ≈ 5 hp (3.7 kW)**. Simplest and cheapest, with the highest T_{st}.
Star–delta	**1/3**	**1/3**	The motor must be **delta-connected in running** with 6 terminals. Phase voltage is 1/√3 at start. About 5–20 hp. Gives an open-transition current surge.
Autotransformer (tapping x)	**x^2**	**x^2**	Taps of 50/65/80%. Suits large and star- or delta-connected motors. Motor current = x × I_{sc}, line current = x^2 × I_{sc}.
Stator resistance/reactor	**x**	**x^2**	Poorest torque per ampere; wastes energy.
Rotor resistance	Reduced	**Increased** (up to T_{max})	**Slip-ring motors only**. Gives a high starting torque with low current.
Soft starter	Adjustable (≈ 2–4 × I_{FL})	Ramped	Uses **anti-parallel thyristors** (SCR) for voltage ramp-up. Smooth, with no mechanical shock. Bypass contactor after start.
VFD	≈ 1–1.5 × I_{FL}	Up to ≈ 1.5 × T_{FL} from zero speed	Best starting; also controls speed.

- **Star–delta** is equivalent to an autotransformer starter with **x = 1/√3 = 0.577 (57.7%)**.
- **Example**: I_{sc} = 6 I_{FL} and s_f = 4%.
  - DOL T_{st} = 36 × 0.04 = **1.44 T_{FL}**.
  - Star–delta T_{st} = 1.44/3 = **0.48 T_{FL}**; line current at start = **2 I_{FL}**.
- **Example**: to limit the line current to 2 × I_{FL} when I_{sc} = 6 × I_{FL}, use an autotransformer with x^2 = 1/3, so **x ≈ 57.7%**.
- **Speed control methods**. Speed N = 120f(1 − s)/P can be changed through f, P or s.

Method	Side	Key facts
V/f constant (VVVF/VFD)	Stator	■ Keeps the **air-gap flux constant**, so T_{max} is constant. This is the **constant-torque region** below base speed. ■ Above base speed V stays at rated and f rises (**field weakening, constant-power region**). ■ It is the most efficient method and is widely used (PWM inverters with IGBTs). ■ At low frequency, a **voltage boost** compensates the IR drop.
Pole changing	Stator	■ Squirrel cage only, because a cage adapts to any pole number. ■ Gives discrete speeds. ■ (a) Separate windings. ■ (b) **Consequent-pole method** (Dahlander): reversing half the coils gives a **2 : 1** speed ratio. ■ (c) **Pole Amplitude Modulation (PAM)**: one winding, **close ratios** such as 8/10 poles.

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Stator voltage control	Stator	■ $T \propto V^2$. ■ Narrow range, below rated speed only. ■ High slip means poor efficiency. ■ Suitable for **fans and pumps** ($T \propto N^2$). Used in AC voltage controllers (thyristor) and fan regulators.
Supply frequency alone	Stator	■ Changing f alone with constant V changes the flux. ■ It is not used alone, so f and V are varied together.
Rotor resistance	Rotor	■ Slip-ring motors only. ■ Speed falls as R_2 increases, while T_{max} stays the same. ■ Wasteful, since the rotor loss is sP_g. ■ Gives poor speed regulation. ■ Gives speeds below N_s only.
Slip power recovery – Kramer	Rotor	■ Slip power is rectified and returned (the original Kramer returned it to the shaft through a DC motor). ■ The **static Kramer** (diode bridge + line-commutated inverter) returns it to the supply. ■ Gives **sub-synchronous speeds only**.
Slip power recovery – Scherbius	Rotor	■ Slip power is returned to the **supply**. ■ The **static Scherbius** (cycloconverter or back-to-back converter) allows both **sub- and super-synchronous** operation.
Injecting EMF in the rotor	Rotor	■ An EMF at slip frequency is injected into the rotor. ■ An EMF in phase with E_{2s} raises the speed; one in anti-phase lowers it. ■ This is the basis of the Kramer and Scherbius systems.
Cascade (concatenation)	Both	■ Two motors on a common shaft. The rotor of the slip-ring main motor feeds the stator of the auxiliary motor. ■ Cumulative: $\mathbf{N = 120f/(P_1 + P_2)}$. Differential: $\mathbf{N = 120f/(P_1 - P_2)}$. ■ Together with $120f/P_1$ and $120f/P_2$, this gives 4 **speeds**.

- **Cascade example**: a 6-pole and a 4-pole motor on 50 Hz give four speeds.
  - Main motor alone: **1000** rpm.
  - Auxiliary motor alone: **1500** rpm.
  - Cumulative: **600** rpm.
  - Differential: **3000** rpm.
- **Braking** methods for induction motors:
  - **Plugging**: reverse two phases. This is the quickest method but involves heavy heating.
  - **Dynamic (DC injection) braking**: DC is fed to the stator.
  - **Regenerative braking**: occurs when $N > N_s$, as in hoists lowering a load or descending trains.

KEY FORMULAE

- Star–delta: $I_{st,line} = (1/3)\, I_{sc,DOL}$; $T_{st} = (1/3)(I_{sc}/I_{FL})^2\, s_f\, T_{FL}$
- Autotransformer: $T_{st} = x^2(I_{sc}/I_{FL})^2\, s_f\, T_{FL}$
- Cascade: $N = 120f/(P_1 \pm P_2)$
- V/f = constant, so Φ = constant

MEMORY TRICK

"Star-Delta = One-Third Twice": one-third the current AND one-third the torque. An autotransformer at tap x gives x^2 for both. A stator resistor gives x for the current but x^2 for the torque, the worst bargain.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In a star–delta starter the starting torque is what fraction of DOL? → **1/3**.
- Which speed control method keeps the flux constant? → **V/f control**.
- The consequent-pole method gives which speed ratio? → **2 : 1**.
- Speed of the cumulative cascade of 8-pole and 4-pole motors on 50 Hz? → 6000/12 = **500 rpm**.
- A static Kramer drive allows which speeds? → **Below synchronous speed only**.
- Which starter suits only slip-ring motors? → **Rotor resistance starter**.

5.5 Single-Phase Induction Motors ★★☆

- A single-phase winding produces a **pulsating (alternating)** field, not a rotating one. Such a motor is **not self-starting**: net starting torque is **zero**.
- **Double revolving field theory** (Ferraris):
  - A pulsating field of maximum Φ_m is equivalent to **two fields of $\Phi_m/2$ each**, rotating in opposite directions at N_s.
  - At standstill the two torques are equal and opposite.
  - Once the rotor is spun in either direction, the forward torque exceeds the backward torque and the motor continues.
- Slip with respect to the forward field = **s**. Slip with respect to the backward field = **2 − s**.
- **Cross-field theory** is the alternative explanation.
- To start, a second (**auxiliary/starting**) winding is displaced **90° electrical** in space, with a current phase-shifted in time. This creates a rotating field.
- For the same frame size, single-phase motors have lower output, efficiency and pf than 3-phase motors. They are made in **fractional kW** sizes.
- **Centrifugal switch** opens the auxiliary winding at about **70–80% of N_s**. If it fails to open, the auxiliary winding burns out.
- Reversal: reverse the connections of **either the main or the auxiliary winding**, but not both.

Type	How the phase split is produced	Phase angle between currents	T_{st} (× T_{FL})	Typical applications
Resistance split-phase	Auxiliary winding of **high R / low X** (thin wire); centrifugal switch	≈ **25–30°**	1.5–2 (moderate)	Washing machines, blowers, small grinders, wood-working tools
Capacitor-start	**Electrolytic capacitor** in the auxiliary circuit; switched out by the centrifugal switch	≈ **80–90°**	**3–4.5 (high)**	Compressors, pumps, refrigerators, air conditioners
Capacitor-start capacitor-run (two-value capacitor)	Large electrolytic capacitor for start, small **oil/paper (or film) capacitor** permanently connected for run	≈ 90°	High	Best running pf and efficiency, quiet. Refrigerators, **AC compressors**, large pumps.
Permanent split capacitor (PSC)	A single capacitor (film/paper) permanently connected; **no centrifugal switch**	< 90° at start	**Low** (0.5–1)	**Ceiling fans**, table fans, blowers, coolers. Speed control by voltage (regulator).
Shaded-pole	A **copper shading ring** on part of each salient pole delays the flux	Small	**Very low** (≈ 0.25–0.5)	Small table fans, exhaust fans, hair dryers, record players, toys, advertising displays

- **Shaded-pole motor**:
  - Rotation is from the **unshaded part towards the shaded part**. It is normally **not reversible**.
  - Lowest efficiency (≈ 5–35%) and lowest cost.
  - Up to about 1/20–1/6 hp.

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- The capacitor in a **ceiling fan** is typically about 2–4 μF (PSC). If it fails, the fan will not start but will run when pushed.
- **Universal motor**:
  - A **series-wound commutator motor** that runs on **both AC and DC** at about the same speed.
  - Laminated yoke and poles.
  - Very **high speed (up to 20,000–30,000 rpm)** and high starting torque.
  - Uses: **mixers, vacuum cleaners, portable drills, sewing machines, hair dryers**.
  - Speed falls sharply with load.
  - Like a DC series motor, it is never run unloaded.
- **Repulsion motor**: a brushes-shorted commutator. It has high starting torque.
- **Stepper motor**:
  - Moves in discrete steps for each pulse and works in open loop.
  - Variable reluctance step angle $\beta = (N_s - N_r)/(N_s N_r) \times 360°$, or $\boldsymbol{\beta = 360°/(m \times N_r)}$, where m = phases and N_r = rotor teeth.
  - Resolution = 360°/β steps per revolution.
  - Types: VR, permanent magnet and **hybrid**, which has the smallest steps (1.8°).
  - Uses: printers, CNC, disk drives, robotics.
- **Reluctance motor**: a salient (cut-teeth) cage rotor that starts as an induction motor and pulls into synchronism. Constant speed; used in **timers, clocks, signalling devices**.
- **Hysteresis motor**:
  - Smooth hardened-steel (high hysteresis) cylinder rotor.
  - Constant torque from standstill to synchronous speed.
  - **Noiseless**, and self-starts and runs synchronously.
  - Uses: **electric clocks, tape recorders, record players, gyros**.

MEMORY TRICK

"Fan = PSC; Fridge = Capacitor-Start; Mixer = Universal; Clock = Hysteresis or Reluctance". A shaded-pole motor always turns **toward the shade**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Why is a single-phase induction motor not self-starting? → Its field is **pulsating, not rotating**; by double revolving field theory the net starting torque is zero.
- Slip of the rotor with respect to the backward field? → **2 − s**.
- Which single-phase motor has the highest starting torque? → **Capacitor-start** (or the two-value capacitor motor).
- In which direction does a shaded-pole motor rotate? → From the **unshaded to the shaded** part of the pole.
- Which motor runs on both AC and DC? → The **universal motor** (series).
- Which motor is used in electric clocks? → The **hysteresis** (or reluctance) synchronous motor.

5.6 Synchronous Generator (Alternator) ★★★

- **Rotating field, stationary armature** is standard for all but very small machines. The reasons:
  - High-voltage armature insulation is easier on the stator, up to **11–33 kV** generated.
  - Only **two slip rings carrying low-voltage DC** excitation (≈ 125–600 V) are needed.
  - The armature is easier to cool.
  - The rotor is lighter and can run at high speed.
  - Output is taken from fixed terminals with no sliding contacts.
- Frequency **f = PN/120**. Synchronous speeds at 50 Hz: 2-pole 3000 rpm, 4-pole 1500 rpm.
- The stator is laminated silicon steel. The armature winding is usually a **double-layer, short-pitched, distributed** winding.
- **Brushless excitation** uses an AC exciter with rotating diodes, with no slip rings. **Static excitation** uses thyristors fed through slip rings.

Feature	Cylindrical (non-salient / round) rotor	Salient-pole rotor
Prime mover	**Steam/gas turbines** (turbo-alternators)	**Hydraulic turbines**, diesel engines
Speed	**High**: 3000/1500 rpm	**Low**: 100–1000 rpm
Poles	**2 or 4**	**Many** (e.g. 12–60 or more)
Diameter / length	**Small diameter, long axial length**	**Large diameter, short length**
Air gap	Uniform	Non-uniform
Reactance	$X_d = X_q = X_s$	$\boldsymbol{X_d > X_q}$
Damper windings	Not needed (solid rotor acts as a damper)	**Needed** (pole-face bars)
Windage loss / noise	Less	More
Mounting	Horizontal	Usually vertical (hydro)
Cooling	Hydrogen (large units), water-cooled stator	Air
Typical SCR	**0.5–0.7**	**1.0–1.5**

KEY FORMULAE

- $E_{ph} = \mathbf{4.44\, f\, \Phi\, T_{ph}\, K_p\, K_d}$ (T_{ph} = turns per phase in series; $K_w = K_p K_d$)
- Pitch (coil-span) factor $K_p = \mathbf{\cos(\alpha/2)}$, where α = angle by which the coil falls short of full pitch (electrical)
- Distribution (breadth) factor $K_d = \mathbf{\sin(m\beta/2)/[m \sin(\beta/2)]}$, where m = slots/pole/phase and β = slot angle = 180°/(slots per pole)
- n^{th} harmonic: $K_{pn} = \cos(n\alpha/2)$; $K_{dn} = \sin(nm\beta/2)/[m \sin(n\beta/2)]$
- To **eliminate the nth harmonic**: α = 180°/n, i.e. coil span = **(1 − 1/n) of the pole pitch** (short by 1/n)
- $Z_s = R_a + jX_s$; $X_s = X_l$ (leakage) + X_{ar} (armature reaction)
- $E = V + I_a Z_s$ (generator) ; % Regulation = $(E_0 - V)/V \times 100$
- **SCR** = I_f for rated V on OC ÷ I_f for rated I_a on SC = $\mathbf{1/X_s}$ **(pu, saturated)**
- Cylindrical: $\mathbf{P = (EV/X_s) \sin \delta}$; $P_{max} = EV/X_s$ at **δ = 90°**
- Salient: $P = (EV/X_d) \sin\delta + (V^2/2)(1/X_q - 1/X_d) \sin 2\delta$

- **Pitch factor examples**:
  - Full pitch: $K_p = 1$.
  - 5/6 pitch (α = 30°): $K_p = \cos 15° = \mathbf{0.966}$.
  - 4/5 pitch (α = 36°): $K_p = \cos 18° = \mathbf{0.951}$.
- **Short pitching**: 4/5 pitch eliminates the **5th** harmonic and 6/7 pitch eliminates the **7th**. A **5/6 pitch** largely suppresses both the 5th and 7th, which is why it is common. Short pitching also **saves copper** in the end connections.
- **Distribution factor example**: 36 slots, 4 poles, 3-phase.
  - m = 3 and β = 180°/9 = 20°.
  - $K_d = \sin 30°/(3 \sin 10°) = 0.5/0.521 = \mathbf{0.96}$.
- **Third-harmonic** (and triplen) voltages **do not appear in the line voltage of a star-connected** alternator. Alternators are therefore star-connected. A delta connection would let triplen currents circulate.
- K_p and K_d are always ≤ 1.
- **Armature reaction** (effect of armature flux on the main field) depends on the load pf, for a generator:
  - **UPF**: **cross-magnetising** (distorting).
  - **Zero pf lagging**: purely **demagnetising**, so the terminal voltage falls.
  - **Zero pf leading**: purely **magnetising**, so the terminal voltage rises.
  - Intermediate pf gives partly cross-magnetising and partly de- or magnetising reaction.
- In a **synchronous motor** the effects reverse: a lagging current magnetises and a leading current demagnetises.
- **OC test** gives the OCC (E vs I_f), the magnetisation curve that saturates. **SC test** gives the SCC (I_{sc} vs I_f), which is a **straight line** because the machine is unsaturated with near zero-pf lagging current.
- From the OC and SC tests: $\mathbf{Z_s = E_{oc}/I_{sc}}$ at the same field current. R_a comes from a DC test, and $X_s = \sqrt{(Z_s^2 - R_a^2)}$.

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- **SCR** (typical 0.5–1.5):
  - A higher SCR means lower X_s, **better voltage regulation**, a **higher stability limit** (larger P_{max}) and higher fault current.
  - It also means a larger air gap and a bigger, costlier machine.
- Synchronous reactance X_s is about **1–2 pu** for turbo-alternators. Sub-transient $X''_d \approx 0.1–0.2$ pu, with $\mathbf{X''_d < X'_d < X_d}$.

Voltage regulation method	Basis	Result
EMF / synchronous impedance method	Uses the unsaturated Z_s from OC/SC; the whole effect is treated as a reactance	**Pessimistic**: gives a **higher** regulation than actual
MMF (Ampere-turn / Rothert's) method	Treats the whole effect (including leakage) as an armature-reaction MMF	**Optimistic**: gives a **lower** regulation than actual
ZPF / Potier triangle method	Uses the OCC plus a **zero-pf (lagging) full-load curve**; separates leakage reactance (Potier reactance X_p) from armature reaction	**Most accurate**
ASA (American Standards Association) method	A modified MMF method that accounts for saturation	Accurate, close to actual
Two-reaction (Blondel)	Used for salient-pole machines	Accounts for $X_d \neq X_q$

- Regulation is **positive** for lagging and UPF loads. It can be **negative** (V rises on load) for **leading pf** loads.
- **Two-reaction theory** (Blondel):
  - The armature MMF is resolved into a **d-axis** component (along the field poles) and a **q-axis** component (interpolar).
  - Because the d-axis reluctance is low, $\mathbf{X_d > X_q}$. Typical $X_q \approx 0.6–0.7\ X_d$.
- **Slip test** (for X_d and X_q):
  - The rotor is driven slightly off synchronous speed with the field open, and a reduced 3-phase voltage is applied to the stator.
  - $\mathbf{X_d = V_{max}/I_{min}}$ and $\mathbf{X_q = V_{min}/I_{max}}$.
- **Reluctance power** (the sin 2δ term) exists even with **zero excitation** in a salient-pole machine, because of the saliency. It is maximum at δ = 45°. The total P_{max} of a salient machine occurs at **δ < 90°**.
- **Load angle (power angle) δ**: the angle between E and V. For a generator **E leads V**; for a motor E lags V. Steady-state stability limit: δ = 90° for a cylindrical rotor.
- Synchronising power coefficient $P_s = dP/d\delta = (EV/X_s)\cos\delta$. It is maximum at δ = 0 and zero at δ = 90°.
- **Example**: E = 1.2 pu, V = 1.0 pu, X_s = 1.0 pu. P_{max} = **1.2 pu**; at P = 0.6 pu, sin δ = 0.5, so δ = **30°**.

MEMORY TRICK

"EMF Exaggerates, MMF Minimises, Potier is Precise". The EMF method is pessimistic (high), the MMF method optimistic (low), and ZPF/Potier the most accurate. For the slip test: **"d = max/min, q = min/max"** (voltage over current).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Pitch factor of a coil short-pitched by 30°? → cos 15° = **0.966**.
- To eliminate the 5th harmonic, the coil span should be? → **4/5 of the pole pitch** (short by 36° electrical).
- Armature reaction at zero pf lagging in an alternator? → **Demagnetising**.
- Which regulation method gives a pessimistic result? → The **synchronous impedance (EMF) method**.
- Short-circuit ratio equals? → The **reciprocal of the saturated X_s in pu**.
- Which rotor is used for hydro alternators? → **Salient-pole** (low speed, many poles, large diameter).

5.7 Parallel Operation & Synchronous Motor ★★★

- Alternators are run in parallel for **reliability**, meeting load growth, maintenance without shutdown, and running units near full load (best efficiency).
- **Conditions for synchronising** an incoming alternator to the bus:
  - Equal **terminal voltage** (RMS).
  - Equal **frequency**.
  - Same **phase sequence**.
  - Voltages **in phase** (phase angle zero) at the instant of closing.
  - Similar waveform is desirable.
- The **phase sequence** is checked only once, at installation. A wrong sequence produces a dead short circuit.
- **Dark lamp method**:
  - Three lamps are connected between the corresponding phases (R–R′, Y–Y′, B–B′).
  - Close the switch at the middle of the **dark period**.
  - If the lamps flicker **together**, the sequence is correct. If they brighten and darken **one after another**, the phase sequence is wrong.
  - Drawback: a lamp can look dark even with about 1/3 rated voltage across it.
- **Bright lamp method** (lamps cross-connected): close at maximum brightness.
- **Two-bright, one-dark** (Siemens–Halske) method: one lamp is straight-connected and two are cross-connected. Close when the **straight lamp is dark and the other two are equally bright**. The direction in which the brightness rotates shows whether the incoming machine is fast or slow.
- **Synchroscope**: the pointer rotates **clockwise = incoming machine fast** and anticlockwise = slow. Close the breaker when the pointer is **at 12 o'clock** (in phase) and moving slowly in the "fast" direction. Synchronising today is usually **automatic synchronising with a check-synch relay (25)**.
- **Infinite bus**: a system so large that its **voltage and frequency stay constant** whatever the load or excitation of one machine. Its **impedance is zero and its inertia infinite**.
- Effects on a machine connected to an infinite bus:
  - **Changing the excitation** changes only the **reactive power (kVAr) and pf**. Active power is unchanged.
  - An **over-excited** generator delivers lagging VAr (lagging pf). An **under-excited** one absorbs VAr (leading pf).
  - **Increasing the prime-mover input** (steam or water, via the governor) increases **active power (kW)** and the load angle δ. Frequency does not change on an infinite bus.
- **Load sharing**:
  - kW sharing is decided by the **governor speed–droop characteristics**. A machine with a smaller droop takes more of a load change.
  - kVAr sharing is decided by **excitation** (AVR droop).
- If the prime mover of a paralleled alternator fails, the machine **runs as a synchronous motor** drawing power from the bus (**motoring**). The **reverse power relay (32)** protects against this.
- **Synchronous motor**:
  - Runs only at $\mathbf{N_s = 120f/P}$ (zero speed regulation) whatever the load, up to the pull-out torque.
  - Doubly excited: AC to the stator, DC to the rotor.
  - **Not self-starting**: the rotor poles cannot follow a field rotating at N_s from rest because of inertia, so the average torque is zero.
- **Starting methods**:
  - (1) **Damper winding** (induction start): the motor starts as a squirrel-cage motor. The field is applied (with the field winding first shorted through a resistor) near N_s, and the motor pulls into step.
  - (2) **Pony motor**: a small induction motor brings the machine up to speed.
  - (3) A DC exciter used as a motor.
  - (4) **Variable-frequency supply**.
  - (5) **Slip-ring (synchronous induction) motor**.
- The field winding is **never left open** during starting, because a dangerously high voltage would be induced in it.
- **V-curves**:
  - A plot of **armature current I_a against field current I_f** at constant load.
  - The minimum I_a occurs at **UPF** (normal excitation).
  - Curves for higher loads lie **above and slightly to the right**.

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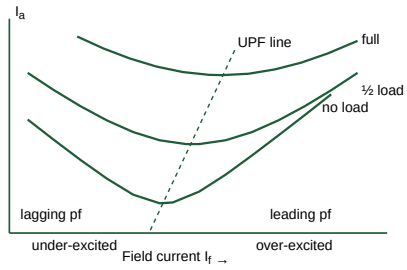
- **Inverted-V curves**: a plot of **pf against I_f**. The peak (UPF) is at normal excitation.
- **Under-excited → lagging pf**. **Normal** → UPF. **Over-excited → leading pf**, so the motor supplies reactive power.
- **Synchronous condenser (compensator)**: an over-excited synchronous motor running **at no load**, used for pf improvement and voltage control. Its pf is nearly zero leading, so it behaves like a variable capacitor. It can also be under-excited to absorb VAr.
- **Hunting**:
  - Oscillation of the rotor about its equilibrium position (δ oscillates) after sudden load changes or supply disturbances.
  - It is suppressed by **damper (amortisseur) windings**: copper bars in the pole faces short-circuited at the ends. Induced currents oppose relative motion.
  - A flywheel also helps.
- Effect of increasing load on a synchronous motor: the speed stays the same and δ increases. If δ goes beyond about 90° (cylindrical), the motor **pulls out of step**.
- Applications of synchronous motors:
  - **Constant-speed loads**: reciprocating compressors, rolling mills, cement mills, large pumps, MG sets.
  - **pf correction** in industries.
  - **Electric clocks** and timers (small reluctance or hysteresis types).



V-curves of a synchronous motor. The minimum I_a on each curve is at UPF, and these minima shift right as load rises.

Point	Synchronous motor	Induction motor
Speed	**Constant (N_s)** at all loads	Slightly less than N_s; falls with load
Self-starting	**No**	**Yes**
Excitation	Doubly excited (needs a DC source)	Singly excited
Power factor	**Adjustable**: lagging, unity or leading	Always **lagging** (poor at light load)
Torque vs voltage	$T \propto V$ ($P = EV \sin\delta/X_s$)	**$T \propto V^2$**, more sensitive to voltage dips
Speed control	Only by changing frequency	Several methods
Hunting	Possible	None
Efficiency	Higher (at large ratings, low speeds)	Slightly lower
Cost / complexity	Costlier; economical above ≈ 50–100 kW at low speeds	Cheap, simple, robust
Use for pf correction	**Yes** (synchronous condenser)	No

MEMORY TRICK

"Steam → Watts, Field → VArs". The governor or prime mover sets the kW, and the excitation sets the kVAr. For a motor, **"Over → Leads"**: an over-excited synchronous motor takes a leading current, like a capacitor.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Increasing the excitation of an alternator on an infinite bus changes? → Only its **reactive power / pf**.
- Which condition for synchronisation is checked only once? → **Phase sequence**.
- An over-excited synchronous motor operates at which pf? → **Leading**.
- What is a synchronous condenser? → An **over-excited synchronous motor on no load**.
- What prevents hunting? → **Damper windings**.
- Which parameters are plotted in a V-curve? → **Armature current vs field current** (at constant load).

5.8 Losses, Efficiency & Special Machines ★★☆

Category	Loss	Depends on	Reduced by
Constant (no-load / rotational)	**Hysteresis** (core)	$\propto B_m^{1.6} f$ (Steinmetz)	Silicon steel, CRGO
	Eddy current (core)	$\propto B_m^2 f^2 t^2$	Thin laminations (0.35–0.5 mm)
	Mechanical: friction (bearings, brushes), windage	Speed	Good bearings, fan design
Variable (load)	Armature/stator and rotor **copper loss**	$\propto I^2R$ (load^2)	Bigger conductors
	Stray load loss	Load (leakage flux, harmonics)	Taken as ≈ 0.5–1% of output
	Brush contact loss (slip-ring/commutator)	$\propto I$ (drop ≈ 1–2 V per brush set)	—
Excitation	Field Cu loss of a synchronous machine	Constant for constant I_f	—

- **Efficiency** η = output/input = output/(output + losses).
- **Maximum efficiency** occurs when **variable loss = constant loss**. The load at η_{max} is $x = \sqrt{(P_{const}/P_{cu,FL})}$ of full load.
- **Example**: a 10 kW induction motor has total losses of 1.2 kW at full load. η = 10/11.2 = **89.3%**.
- **Example**: an alternator has constant loss = 2 kW and full-load Cu loss = 8 kW. η_{max} occurs at $x = \sqrt{(2/8)} = 0.5$, i.e. half load.
- **Example**: a 4-pole, 50 Hz induction motor has stator input 50 kW, stator losses 2 kW, slip 3% and mechanical loss 1 kW.
  - P_g = 48 kW.
  - Rotor Cu loss = 1.44 kW.
  - P_m = 46.56 kW and output = 45.56 kW.
  - η = 45.56/50 = **91.1%**.
- Large alternators have an efficiency of about **98–99%**. Induction motors range from 85 to 97%.
- Temperature rise limits the rating of a machine. **Rating is limited by heating**, i.e. by the insulation class.
- Life of insulation roughly **halves for every 8–10 °C rise** above its rated temperature (Montsinger's rule).

Insulation class (IS 1271 / IEC 60085)	Y	A	E	B	F
Max temperature (°C)	**90**	**105**	**120**	**130**	**155**
Typical materials	Cotton, paper, silk (unimpregnated)	Same, impregnated or in oil	Enamels, polyester films	Mica, glass fibre with organic binders	Mica/glass with epoxy/polyester binders

- Most modern LV motors use **Class F insulation with Class B temperature rise**, which gives a thermal margin. IEC 60085 also uses the numerical designations 200, 220 and 250 °C above Class H.

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- **Duty types** (IEC 60034-1 / IS 12824):

Duty	Description	Example
S1	**Continuous running** at constant load (thermal equilibrium reached)	Pumps, fans
S2	**Short-time duty** (e.g. 10, 30, 60, 90 min); rest long enough to cool	Sluice gates, valve actuators
S3	**Intermittent periodic duty**; starting does not affect temperature	Cyclic loads
S4	Intermittent periodic with **starting**	Cranes, hoists
S5	Intermittent periodic with starting and **electric braking**	Lifts, rolling mill auxiliaries
S6	**Continuous-operation periodic duty** (load and no-load periods, no rest)	Saws, conveyors
S7	Continuous-operation periodic with electric braking	Reversing drives
S8	Continuous-operation periodic with related **load/speed changes**	Multi-speed drives
S9, S10	Non-periodic load and speed variations; discrete constant loads	Drives on VFDs

- **IP code** (IEC 60529 / IS 12063), written IP XY:
  - First digit (**0–6**) = protection against **solid objects and dust**. 5 = dust-protected, **6 = dust-tight**.
  - Second digit (**0–8**, with 9 added later for high-pressure hot jets) = protection against **water**. 4 = splashing, 5 = jets, 7 = temporary immersion, 8 = continuous immersion.
- Typical IP values: **IP55** for a TEFC (totally enclosed fan-cooled) industrial motor, IP23 for a drip-proof motor, and IP68 for submersible pumps.
- **Cooling code**: IC 411 = TEFC.

Special machine	Key facts	Applications
BLDC motor	■ PM rotor and wound stator. ■ **Electronic commutation** via an inverter, using **Hall-effect sensors** (or sensorless back-EMF) for rotor position. ■ **Trapezoidal** back EMF. ■ No brushes, so it is efficient, long-lived and quiet with low EMI. ■ It is effectively an "inside-out" DC motor.	**Energy-efficient ceiling fans** (≈ 28–35 W vs ≈ 70–75 W), computer fans, drones, EVs, HVAC
PMSM	■ PM rotor with a **sinusoidal** back EMF. ■ Sinusoidal currents under **field-oriented (vector) control**. ■ Runs synchronously and has no rotor Cu loss. ■ High power density.	EV traction, servo drives, robotics, washing machines
Linear induction motor (LIM)	■ An "unrolled" induction motor. ■ The primary produces a **travelling field** with synchronous velocity $v_s = 2\tau f$ (τ = pole pitch). ■ A large air gap gives a **low pf and efficiency**. ■ Suffers **end effects**. ■ Thrust is produced directly, with no gears.	**Maglev**, some metro systems, conveyors, sliding doors, cranes
Switched reluctance motor	■ Salient poles on both stator and rotor, with no windings or magnets on the rotor. ■ Torque comes from reluctance alignment. ■ Rugged. ■ Suffers torque ripple and noise.	Vacuum cleaners, EV research, mining
AC servo motor	■ A 2-phase induction motor with a **high-R rotor** (high R/X) for a linear, drooping T–N curve and no single-phasing run. ■ Uses a control winding plus a reference winding.	Control systems, position control

- LIM example: $\tau = 0.2$ m on 50 Hz gives $v_s = 2 \times 0.2 \times 50 =$ **20 m/s** (72 km/h).
- A BLDC motor needs **rotor position feedback**, from Hall sensors or an encoder. A stepper motor works **open loop**.

CODE UPDATE

Motor efficiency classes follow **IEC 60034-30-1**: **IE1** (standard), **IE2** (high), **IE3** (premium), **IE4** (super-premium), with IE5 defined in IEC TS 60034-30-2. The Indian equivalent is **IS 12615:2018**.

MEMORY TRICK

Insulation classes in order: **"Y A E B F H C"**, read as "**Y**oung **A**nd **E**nergetic **B**oys **F**ind **H**igh **C**areers". The temperatures are **90, 105, 120, 130, 155, 180, > 180**: two steps of +15, then +10, +25, +25.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Maximum permissible temperature for Class F insulation? → **155 °C**.
- Class B insulation temperature limit? → **130 °C**.
- Maximum efficiency of a machine occurs when? → **Copper (variable) loss = constant loss**.
- Duty cycle S1 means? → **Continuous duty**.
- How does a BLDC motor sense rotor position? → **Hall-effect sensors** (electronic commutation).
- Synchronous speed of a LIM? → $v_s = 2\tau f$.

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6. Power Systems I – Generation, Transmission & Load Flow

> **NOTE**
> Power Systems has the most weight in PSU electrical papers. Expect 8–12 MCQs from this chapter and Chapter 7 together. The most common questions are on load-curve factors, hydro and wind power formulae, the HVDC vs HVAC comparison, ABCD constants, the Ferranti effect, SIL, corona, sag, string efficiency, cable dielectric stress, per-unit change of base, and the comparison of Ybus and load-flow methods. Learn every formula in the boxes and the typical values.

6.1 Power Generation Concepts ★★★

Thermal (steam) power plant

- Thermal plants work on the **Rankine cycle**, which has four steps: **pump** (isentropic compression) → **boiler** (heat added at constant pressure) → **turbine** (isentropic expansion) → **condenser** (heat rejected at constant pressure).
- Overall efficiency is **≈ 30–40%**. Old subcritical units give about 30–35%. Supercritical and ultra-supercritical units give about **38–42%+**.
- **Heat rate** = heat input per kWh generated. η = **860 / heat rate (kcal/kWh)**. A heat rate of about 2,400 kcal/kWh means η ≈ 36%.
- The **largest single loss** is in the **condenser**, which rejects about 50% of the heat input to cooling water.
- A **supercritical** unit works above the critical point of water (**221.2 bar, 374.15 °C**). Its boiler is **once-through** and has **no drum**. Indian supercritical unit sizes are 660 and 800 MW.
- Indian coal has a **high ash content (≈ 30–45%)** and a low calorific value. This is why ash handling and ESPs matter so much. Of the total ash, about 80% is **fly ash** and about 20% is bottom ash.
- Thermal plants start slowly (hours) and serve as **base-load plants**; the site needs coal, water and ash-disposal land.

Component	Function (frequent MCQ)
Economiser	Heats the **feedwater** using the **flue gases** leaving the boiler
Air preheater	Heats the **combustion air** using flue gas. It is placed after the economiser and is the last heat-recovery unit before the ESP and chimney
Superheater	Raises steam above its saturation temperature. This raises efficiency and reduces moisture, which would erode the turbine blades
Reheater	Reheats steam coming out of the HP turbine before it goes to the IP/LP turbine
Condenser	Condenses the exhaust steam and keeps a **vacuum** (≈ 0.07–0.1 bar abs). This increases the enthalpy drop across the turbine, so η rises
Cooling tower	Cools the condenser water. It is needed where river water is not enough
ESP (electrostatic precipitator)	Removes **fly ash** from the flue gas. Efficiency is **> 99%** and it works on high DC voltage (corona)
Deaerator	Removes dissolved O_2 and CO_2 from the feedwater to prevent corrosion
Feedwater heaters	**Regenerative** heating with steam bled from the turbine, which raises η
Draught fans (FD/ID)	The FD fan pushes air into the furnace. The ID fan pulls flue gas out to the chimney

Hydro-electric plant

- Power output P = **9.81 η Q H kW**, where Q is in m^3/s, H is the net head in m and η is the overall efficiency. Overall efficiency of a hydro plant is high, about **85–90%**.
- The **surge tank** protects the penstock from **water hammer** when the load drops suddenly. It is placed as close to the powerhouse as practicable.
- The **draft tube** is used only with **reaction turbines**. It recovers kinetic energy at the exit and lets the turbine sit above the tailrace. A **Pelton** turbine does **not** need one.
- **Specific speed** $N_s = N\sqrt{P} / H^{5/4}$, where P is in kW and H in m. Low N_s suits high head; high N_s suits low head.
- Hydro plants **start quickly** (minutes), have **no fuel cost** and low running cost, but a high capital cost and a long gestation period. **Storage-type** plants are used for peak load and **run-of-river** plants for base load.
- A **pumped storage** plant pumps water up during off-peak hours and generates during the peak. Its cycle efficiency is about **70–80%** and it is a **peak-load** plant.
- MNRE classifies small hydro as: micro up to 100 kW, mini 101 kW–2 MW, and **small hydro up to 25 MW**. Large hydro has been treated as renewable in India since **2019**.
- **Cavitation** occurs in **reaction turbines** (Francis, Kaplan) and is checked using Thoma's cavitation factor σ.

Turbine	Type	Head (approx.)	Specific speed (approx., metric)	Flow
Pelton	**Impulse** (tangential flow)	**High**, > 250–300 m	**Low**, ≈ 10–35 (single jet)	Small Q
Francis	**Reaction** (radial → mixed flow)	**Medium**, ≈ 30–300 m	≈ 60–300	Medium Q
Kaplan / Propeller	Reaction (**axial flow**)	**Low**, < 30 m	**High**, ≈ 300–1000	Large Q

- Textbooks quote slightly different head ranges. The order Pelton (high head) > Francis (medium) > Kaplan (low) never changes. **Kaplan** blades are **adjustable** and give good part-load efficiency. Propeller blades are fixed. **Francis** is the **most widely used** turbine.

Nuclear power plant

- Energy comes from **fission** of U-235 by a **slow (thermal) neutron**. Each fission releases about **200 MeV** and 2–3 neutrons.
- **Natural uranium** contains about **0.7% U-235**. Light-water reactors use **enriched** uranium (≈ 3–5%).
- The **multiplication factor** k = 1 means the reactor is critical (steady). k > 1 is supercritical and k < 1 is subcritical.
- **Fertile** materials are U-238 → Pu-239 and **Th-232 → U-233**. India has large **thorium** reserves, which is the basis of the third stage of its programme.
- India follows Dr Homi Bhabha's **three-stage programme**: Stage I is PHWR (natural U), Stage II is fast breeder reactors (Pu), and Stage III is thorium-based (U-233).
- India's first nuclear plant is **Tarapur** (Maharashtra, 1969), which uses **BWRs**. Most Indian reactors are **PHWRs**: Rawatbhata (RAPS), Kalpakkam (MAPS), Narora, Kakrapar, Kaiga. **Kakrapar-3** is the first **indigenous 700 MWe PHWR**. **Kudankulam** (Tamil Nadu) has **VVER-1000** reactors (a Russian PWR design) of 1000 MW each. The Prototype Fast Breeder Reactor (PFBR, 500 MWe) is at **Kalpakkam**.
- NPCIL operates the plants and **AERB** is the safety regulator. Nuclear plants have a high capital cost and very low fuel cost, so they are **base-load plants**.

Reactor part	Purpose	Materials
Moderator	Slows fast neutrons down to thermal energy	**Heavy water (D_2O)**, light water, **graphite**, beryllium
Control rods	Absorb neutrons to control the reaction rate	**Cadmium, boron**, hafnium
Coolant	Carries heat to the steam generator	H_2O, D_2O, CO_2, **liquid sodium** (in an FBR)
Reflector	Sends escaping neutrons back into the core	Graphite, beryllium
Shielding	Protects people from radiation	Concrete, lead

Reactor	Fuel	Moderator / coolant	India example
PHWR	**Natural U**	D_2O / D_2O (pressurised)	RAPS, Kaiga, Kakrapar (most Indian reactors)
BWR	Enriched U	H_2O / H_2O (boils in the core, no separate steam generator)	**Tarapur 1 & 2**
PWR (VVER)	Enriched U	H_2O / H_2O (pressurised, uses a steam generator)	**Kudankulam**

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FBR	Pu + U-238 (MOX)	**No moderator** / liquid sodium	FBTR, PFBR (Kalpakkam)

Gas turbine, diesel and renewables

- Gas turbines work on the **Brayton (Joule) cycle**: isentropic compression → heat added at constant pressure → isentropic expansion → heat rejected at constant pressure.
- The **compressor absorbs about 60–70%** of the turbine work. Open-cycle η is about **25–35%**. Efficiency rises with the pressure ratio and the turbine inlet temperature, and also with regeneration, intercooling and reheating.
- A **combined cycle** runs a Brayton (gas) cycle and uses its exhaust in an HRSG to drive a Rankine (steam) cycle. Efficiency is about **50–60%**.
- Gas turbines **start quickly** (minutes), so they are used as **peak-load** plants.
- **Diesel** plants come in small sizes and are used for standby, emergency and isolated loads. η is about 35–40%, but fuel cost is high.
- **Solar PV** works on the photovoltaic effect. The silicon band gap is **≈ 1.1 eV**. One cell gives V_{oc} ≈ **0.5–0.6 V**. Commercial modules are about **15–22%** efficient.
- **STC** (Standard Test Conditions) are **1000 W/m², 25 °C, AM 1.5**.
- **Fill factor** FF = $P_{max}/(V_{oc}\, I_{sc})$ ≈ 0.7–0.85. I_{sc} ∝ irradiance, while V_{oc} varies only logarithmically with it. Output **falls as temperature rises** (≈ 0.4–0.5%/°C). An **MPPT** tracks the maximum power point.
- **Wind power** $P = ½\, \rho\, A\, V^3\, C_p$. Power is ∝ **V³** and ∝ **D²**, so doubling V gives 8× the power and doubling the blade length gives 4×. Air density ρ ≈ 1.225 kg/m³.
- The **Betz limit** is $C_{p,max} = 16/27 =$ **59.3%**. Practical turbines reach C_p ≈ 0.35–0.45.
- Typical wind speeds: **cut-in ≈ 3–4 m/s**, rated ≈ 12–15 m/s, **cut-out ≈ 25 m/s**. Common wind generators are the DFIG, PMSG (direct drive) and squirrel-cage induction generator (fixed speed). Turbines are either HAWT or VAWT (Darrieus, Savonius).

Load curve terms

Term	Definition	Range
Demand factor	Maximum demand ÷ connected load	**≤ 1**
Load factor	Average load ÷ maximum demand (for the period)	**< 1** (higher is better)
Diversity factor	Sum of individual maximum demands ÷ maximum demand of the whole system	**≥ 1** (higher is better)
Plant capacity factor	Average demand ÷ plant rated capacity = actual energy ÷ maximum possible energy (capacity × 8760 h)	< 1
Utilisation factor	Maximum demand ÷ rated plant capacity	≤ 1
Plant use factor	Actual energy ÷ (capacity × hours actually in operation)	≤ 1
Reserve capacity	Plant capacity − maximum demand	—

- The area under the **load curve** is the **energy** (kWh). Average load = area ÷ hours.
- The **load duration curve** plots the same loads arranged in **descending order**, so its area is the same energy.
- A **high load factor** and **high diversity factor** give a lower cost per unit generated, because fixed charges are spread over more kWh.

Base-load plants	Peak-load plants
Run all the time at high load factor. Low running cost, high capital cost: **nuclear**, large **coal** thermal, **run-of-river hydro**	Run for a few hours, need **quick start**, low capital cost: **gas turbine**, **pumped storage**, storage hydro, diesel

Tariffs

- **Two-part tariff** (**Hopkinson demand** tariff) = a fixed charge per kW of **maximum demand** plus a running charge per kWh.
- **Three-part tariff** (**Doherty**) = a fixed charge per consumer plus a charge per kW of maximum demand plus a charge per kWh.
- **Block-rate** tariff: the energy rate per unit is lower for each higher block of consumption.
- **Flat-rate** tariff: a different fixed rate per kWh for each type of consumer.
- **Power-factor tariffs**: a kVA maximum demand tariff, or a kWh + kVARh tariff. Both penalise low pf.
- **ToD/ToU** tariff: the rate depends on the time of day, to flatten the load curve.

India – power sector facts

- Total installed capacity was about **500 GW at the end of 2025** and **≈ 550 GW by mid-2026**, and is growing fast. **Coal** is still the **largest single source** (≈ 40–45% of capacity) and gives about 70–75% of the energy generated. **Solar** (≈ 160+ GW in 2026) is the largest renewable source.
- **Non-fossil** capacity (RE + large hydro + nuclear) crossed **50%** of the total in **June–July 2025**, five years ahead of the 2030 NDC target, and was ≈ 54% by mid-2026. The national goal is **500 GW non-fossil capacity by 2030**.
- The **Ministry of Power**, **CEA** (planning and standards) and **CERC/SERCs** (tariff regulators) run the sector.
- **POWERGRID** is the Central Transmission Utility. **Grid-India** (formerly POSOCO) runs the NLDC and RLDCs.
- India has **five regional grids** (N, E, W, NE, S), all working as **one synchronous national grid** since **31 Dec 2013** ("One Nation–One Grid–One Frequency"). Nominal frequency is **50 Hz**.

KEY FORMULAE

- $P_{hydro} = 9.81\, \eta\, Q\, H$ kW ; $N_s = N\sqrt{P} / H^{5/4}$
- $P_{wind} = ½\, \rho\, A\, V^3\, C_p$; $C_{p,max} = 16/27 = 0.593$
- $\eta_{thermal} = 860 /$ heat rate (kcal/kWh)
- Capacity factor = Load factor × Utilisation factor
- Annual energy = Max demand × LF × 8760 kWh

MEMORY TRICK

"Pelton is High, Kaplan is Low, Francis is in the middle." Head goes down P → F → K, while specific speed and flow go up P → F → K. For the load factors, only **D**iversity is ≥ 1: **D**iversity = **D**ivide the sum of individual maxima.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Economiser uses flue gas to heat? → **Feedwater**.
- Turbine for a head of 500 m? → **Pelton** (impulse).
- Maximum fraction of wind power that can be extracted? → **59.3% (Betz)**.
- Diversity factor is always? → **≥ 1**.
- Moderator used in Indian PHWRs? → **Heavy water**.
- Two-part tariff is also called? → **Hopkinson demand tariff**.

6.2 AC & DC Transmission Concepts ★★★

- Standard Indian AC voltages are 11, 22, 33, 66, 110, **132, 220, 400, 765 kV**. **765 kV** is the highest AC voltage in commercial use. A **1200 kV UHVAC** test station has been built at **Bina** (MP).
- Indian HVDC levels are **±500 kV** and **±800 kV** (UHVDC). There is also **±320 kV VSC** (Pugalur–Thrissur, India's first VSC-HVDC).
- **Rihand–Dadri ±500 kV** was India's first long-distance commercial bipolar HVDC link. Other links: Talcher–Kolar ±500 kV, **Biswanath Chariali–Agra ±800 kV** (multi-terminal), Champa–Kurukshetra ±800 kV, Raigarh–Pugalur ±800 kV.
- **Vindhyachal** was India's first **back-to-back** HVDC station.
- **Why high voltage?** For the same power, loss, distance and pf, the current is I ∝ 1/V, so the conductor area and volume ∝ **1/V²**. Line loss is ∝ $1/V^2$ and the percentage voltage drop is ∝ $1/V^2$. Conductor volume is also ∝ **$1/\cos^2\varphi$**.
- The drawbacks of higher voltage are larger insulators, larger clearances and taller towers, more **corona**, and costlier transformers and switchgear.

System (overhead, same max voltage to earth)	Conductor volume relative to DC 2-wire (mid-point earthed) = 1
DC 2-wire, mid-point earthed	1
DC 3-wire	**5/16 = 0.3125** (least)
1-φ 2-wire AC, mid-point earthed	$2/\cos^2\varphi$
3-φ 3-wire AC	**$0.5/\cos^2\varphi$**
3-φ 4-wire AC (neutral half size)	$0.583/\cos^2\varphi$ (7/12)

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- Among AC systems, **3-φ 3-wire needs the least copper** → used for transmission (ratios per V.K. Mehta; they change on a "same voltage between conductors" basis).

HVDC vs HVAC

Point	HVDC	HVAC
Conductors	2 (bipolar), or 1 with earth return	3
Reactance / stability limit	**None**, so there is no stability limit on distance	The $P = V_1V_2\sin\delta/X$ limit applies
Charging current / Ferranti effect	**Nil**	Present, and limits long cables
Skin effect	**Absent**	Present
Corona and radio interference	Lower	Higher
Right of way / tower	**Smaller**, cheaper line	Larger
Linking two systems	Can link **asynchronous** systems or systems at **different frequencies**	Systems must be synchronous
Power flow control	Fast and precise (by converter firing angle)	Hard to control
Terminal cost	**High** (converters, filters)	Low
Converter needs	Converters **absorb reactive power** (≈ 50–60% of the DC power) and produce **harmonics**, so filters are needed	—
Circuit breaking	**Difficult**, because there is no current zero	Easy
Voltage transformation	Not possible directly	Easy (transformer)

- **Break-even distance** (where HVDC becomes cheaper than HVAC): about **500–800 km** for overhead lines and about **30–50 km** for **underground or submarine cables**. Above this distance, the lower line cost of HVDC more than pays for its costly terminals.

HVDC link	Features
Monopolar	One conductor (usually **negative**, which gives less corona and RI), with return through **earth or sea**
Bipolar	Two conductors, one + and one −. The mid-point is earthed. **Most common**. If one pole fails, the other runs as a monopole and carries about half the power
Homopolar	Two or more conductors of the **same polarity** (usually negative) with earth return. Less insulation is needed, but earth current flows continuously
Back-to-back	Rectifier and inverter are at the **same station** and there is no DC line. Used for **asynchronous tie** between grids
Multi-terminal	More than two converter stations

Converter basics

- **LCC** (line-commutated converter) uses **thyristors**. In **rectifier** mode α < 90°; in **inverter** mode α > 90°, and the inverter is controlled by its extinction angle γ.
- **VSC** uses **IGBTs**. It needs no reactive power support, can black-start, controls P and Q independently and suits multi-terminal schemes and weak grids.
- For a 6-pulse bridge, $V_{dc} = \mathbf{1.35\ V_{LL}\cos\alpha}$ (= $(3\sqrt{2}/\pi)\ V_{LL}\cos\alpha$).
- A **12-pulse** converter (two 6-pulse bridges fed through Y–Y and Y–Δ transformers) cancels the **5th and 7th** harmonics. AC-side characteristic harmonics are **12k ± 1** (11, 13, 23, 25…). DC-side harmonics are **12k**.
- The **smoothing reactor** reduces DC ripple, limits fault current and prevents commutation failure. AC filters also supply part of the converter's reactive power.

Conductors

- **Kelvin's law**: the most economical conductor size is the one for which the **annual cost of energy lost = annual interest and depreciation on the part of the capital cost that varies with conductor size**.
- **ACSR** (aluminium conductor steel reinforced) has a **galvanised steel core** for strength and **aluminium** for conductivity. It is written as Al/steel strands, for example **30/7** or 54/7.
- Indian ACSR code names: Dog (11–66 kV), Panther (132 kV), Zebra (220 kV), **Moose** (400 kV twin/quad).
- **AAAC** (all-aluminium alloy, Al–Mg–Si) is stronger than AAC, resists corrosion better, has a lower weight per strength and lower losses than ACSR, and suits coastal areas.
- **AAC** is used for short spans. **HTLS** (high-temperature low-sag) conductors are used for **re-conductoring**, which raises ampacity on the same towers.
- Stranded conductors have **$3n^2 - 3n + 1$** strands for n layers including the centre strand (7, 19, 37, 61…). Overall diameter = (2n − 1)d.
- **Bundled conductors** use two or more sub-conductors per phase. They **raise the GMR**, which **reduces L and X**, raises C and therefore **raises SIL** and power capacity. They also **reduce the surface gradient**, so **corona loss and radio interference are lower**.
- Bundles in India: **twin or quad** at 400 kV, **quad or hexa** at 765 kV. Spacers hold the sub-conductors apart.
- **Skin effect** makes AC current crowd towards the conductor surface, which raises the effective R. It **increases** with frequency, conductor diameter, permeability and conductivity. It is **absent in DC** and negligible below about 1 cm diameter at 50 Hz.
- Skin depth $\delta = \sqrt{(2\rho/\omega\mu)}$, which is ≈ **9.3 mm for copper at 50 Hz**. Stranding, ACSR and hollow or tubular conductors reduce skin effect.
- **Proximity effect** is current crowding caused by the field of nearby conductors. It is **significant in cables** (close spacing) and negligible in overhead lines.
- **Transposition** means swapping the phase positions at regular intervals, giving one full cycle in three sections. It **balances L and C** of the three phases (making the line symmetrical) and **reduces interference** with nearby communication lines.

MEMORY TRICK

"DC wins on distance, AC wins on terminals." HVDC has no X, no charging current, no skin effect and no stability limit, but it needs costly converters, harmonic filters and Q support, and a DC breaker is hard to build. The break-even distances are **500–800 km in the air and 30–50 km under water**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Conductor volume for a given power is proportional to? → **$1/V^2$** (and $1/\cos^2\varphi$).
- HVDC link with earth return and two conductors of the same polarity? → **Homopolar**.
- Link used to connect two asynchronous grids at one station? → **Back-to-back HVDC**.
- Bundled conductors reduce? → **Inductive reactance and corona**.
- Kelvin's law equates? → **Annual energy-loss cost = annual interest + depreciation on the variable capital cost**.
- Skin effect is absent in? → **DC**.

6.3 Line Parameters & Performance ★★★

Resistance, inductance and capacitance

- R = ρl/A. Stranding increases the effective length by about **1–2%**. AC resistance is greater than DC resistance because of skin effect. R rises with temperature: $R_2 = R_1[1 + \alpha(t_2 - t_1)]$.
- **Inductance** of a single-phase two-wire line (loop): L = **0.4 ln(D/r′) mH/km** (= 4×10^{-7} ln(D/r′) H/m). Each conductor contributes 0.2 ln(D/r′) mH/km.
- For a solid round conductor, **$r' = r\,e^{-1/4} = 0.7788\ r$**. This is the GMR (self-GMD), and it accounts for the internal flux linkage. The internal inductance is $\mu_0/8\pi$ = 0.05 mH/km per conductor.
- **3-φ line**, inductance per phase: L = **0.2 ln(GMD/GMR) mH/km**.
  - Symmetrical spacing D: GMD = D.
  - Unsymmetrical but transposed: **$GMD = (D_{ab}\,D_{bc}\,D_{ca})^{1/3}$**.
  - Larger spacing gives larger L. A larger conductor or bundle gives smaller L.
- **GMR of bundles** (d = spacing between sub-conductors): 2 sub-conductors $\sqrt{(r'd)}$; 3 sub-conductors $(r'd^2)^{1/3}$; 4 sub-conductors $1.09(r'd^3)^{1/4}$. For **capacitance**, use the actual radius r instead of r′.
- **Capacitance**, line-to-neutral: $C_n = \mathbf{2\pi\varepsilon_0/ln(D/r)\ F/m} \approx 0.0556/\ln(D/r)$ μF/km. Here the **actual radius r** is used, not r′.

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- Capacitance between the two conductors of a single-phase line is $C_{ab} = \pi\varepsilon_0/\ln(D/r)$, which is **half** of C_n.
- **Earth** increases the line capacitance slightly (method of images). Its effect is usually neglected.
- Typical values for an overhead line: L ≈ 1 mH/km, X ≈ **0.3–0.4 Ω/km**, C ≈ 0.01 µF/km. Wave velocity $1/\sqrt{(LC)} \approx$ **3×10^5 km/s** (speed of light). In a cable, velocity ≈ $c/\sqrt{\varepsilon_r}$, which is about half.

Classification of lines

Line	Length (Nagrath–Kothari / most PSU keys)	Model
Short	**< 80 km** (V.K. Mehta: up to 50 km, < 20 kV)	Only series R + jX. **Capacitance neglected**
Medium	**80–250 km** (Mehta: 50–150 km, 20–100 kV)	Lumped C: **nominal T** or **nominal π** (end-condenser method is also used)
Long	**> 250 km** (Mehta: > 150 km, > 100 kV)	**Distributed** parameters, rigorous solution with γ and Z_c

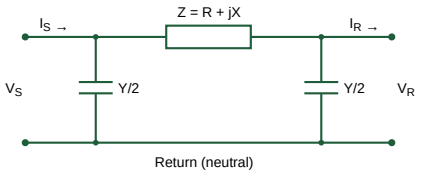

Fig. 6.1 Nominal-π model of a medium line. Half of the total shunt admittance is placed at each end. In the nominal-T model, the whole of Y sits at the middle and Z/2 is placed on each side.

ABCD (generalised) constants

$V_S = A\,V_R + B\,I_R$; $I_S = C\,V_R + D\,I_R$

Model	A = D	B (Ω)	C (S)
Short	1	Z	**0**
Nominal T	1 + YZ/2	**Z(1 + YZ/4)**	Y
Nominal π	1 + YZ/2	Z	**Y(1 + YZ/4)**
Long (exact)	**cosh γl**	**Z_c sinh γl**	**sinh γl / Z_c**

- For every passive, linear, bilateral network, **AD − BC = 1** (reciprocity). A line is a **symmetrical** network, so **A = D**.
- **A and D** are dimensionless, **B** is in ohms and **C** is in siemens.
- The magnitude of A is about 1 and is **< 1** for a real line, which is why the voltage rises at no load.
- Open-circuit receiving voltage $V_{R0} = V_S/A$. Short-circuit sending current $I_S = D\,V_S/B$.
- For two networks in cascade, the overall matrix is $[ABCD]_1 \times [ABCD]_2$.
- Propagation constant **$\gamma = \sqrt{(zy)} = \alpha + j\beta$**, where α is attenuation and β is the phase constant (rad/km). **$Z_c = \sqrt{(z/y)}$**. For a lossless line, γ = jβ with β = ω√(LC), and A = cos βl, B = jZ_c sin βl.
- Wavelength at 50 Hz ≈ **6000 km**, so a **quarter-wave line ≈ 1500 km**.

Regulation, efficiency and power flow

- Voltage regulation = $(|V_{R,NL}| - |V_{R,FL}|)/|V_{R,FL}| \times 100$, where $V_{R,NL} = |V_S|/|A|$.
- For a **short line**, % regulation ≈ $(I\,R\cos\varphi \pm I\,X\sin\varphi)/V_R \times 100$. Use + for lagging pf and − for leading pf.
- Regulation is **maximum** when the load angle φ = θ, the impedance angle (tan φ = X/R, lagging).
- Regulation is **zero** at a **leading** pf with tan φ = R/X. It is **negative** at a more leading pf.
- Transmission efficiency = $P_R/(P_R$ + losses) and is typically > 94–98%.
- Power transfer over a lossless line is $P = (V_S V_R/X)\sin\delta$. The maximum (**steady-state stability limit**) is at **δ = 90°**.
- Receiving-end **power circle diagram**:
  - Centre is at $-(|A||V_R|^2/|B|)\angle(\beta - \alpha)$, where β and α are here the angles of B and A.
  - Radius = **$|V_S||V_R|/|B|$**.
  - $P_{R,max} = |V_S||V_R|/|B| - (|A||V_R|^2/|B|)\cos(\beta - \alpha)$, reached when δ = β.
  - The diagram is used to find the compensation (Q) needed to hold a given voltage.

Ferranti effect and surge impedance

- **Ferranti effect**: at **no load or light load**, the **receiving-end voltage is greater than the sending-end voltage**. It happens in **long lines and cables**.
- Cause: the capacitive **charging current** flowing through the series inductance produces a drop in phase opposition to V_R, so voltage rises along the line.
- For a lossless line on open circuit, $V_R = V_S/\cos\beta l$. The rise is ≈ $\omega^2 LCl^2/2$ per unit, so it is ∝ **l^2** and ∝ **f^2**.
- It is controlled by **shunt reactors** and by switching off shunt capacitors.
- **Surge impedance** $Z_c = \sqrt{(L/C)}$ for a lossless line. It is **independent of length** and purely resistive.
- Typical Z_c: overhead line ≈ **400 Ω** (300–500 Ω; lower for bundled conductors, ≈ 250–300 Ω). Cable ≈ **40–50 Ω** (high C, low L).
- **SIL = V_{LL}^2/Z_c** (MW, with V in kV). This is the load at which the line is "flat".
  - At SIL the voltage is **constant along the whole line**, the pf is unity everywhere, and Q generated by C equals Q absorbed by L: ½CV^2 = ½LI^2.
  - Load > SIL: the line absorbs Q and the voltage sags. Load < SIL: the line generates Q and the voltage rises.
- Typical SIL: 132 kV ≈ 50 MW; 220 kV ≈ 130 MW; **400 kV ≈ 500–600 MW** (twin); **765 kV ≈ 2200–2300 MW** (quad). SIL ∝ **V^2** and is raised by bundling and by series capacitors.

Corona

- **Corona** = ionisation of air around a conductor when the surface gradient exceeds the **breakdown strength of air ≈ 30 kV/cm peak (21.1 kV/cm rms)** at NTP (25 °C, 76 cm Hg).
- Signs: **violet glow**, **hissing**, **ozone**, power loss, radio interference, non-sinusoidal (third-harmonic) current.
- Air density factor **$\delta = 3.92\,b/(273 + t)$**, with b in cm Hg and t in °C. δ = 1 at 25 °C and 76 cm Hg.
- **Disruptive critical voltage** (phase-to-neutral rms): **$V_d = g_0\,\delta\,m_0\,r\ln(D/r)$ kV**, with g_0 = 21.1 kV/cm rms and r in cm.
  - Irregularity factor m_0 = 1 for a polished conductor, 0.98–0.93 for a rough one, and **0.87–0.80 for stranded** conductors.
- **Visual critical voltage** $V_v = g_0\,\delta\,m_v\,r\,(1 + 0.3/\sqrt{(\delta r)})\ln(D/r)$. Since $V_v > V_d$, the glow appears only after loss has started.
- **Peek's formula** for corona loss: $P_c = (242.2/\delta)(f + 25)\sqrt{(r/D)}(V_p - V_d)^2 \times 10^{-5}$ kW/km/phase.
  - Loss ∝ **$(f + 25)$**, ∝ **$(V - V_d)^2$** and ∝ √(r/D).
  - Peterson's formula is used when V_p/V_d is close to 1.
- **Corona increases with**: **foul weather** (rain, fog, snow – lowers V_d, loss rises several-fold), low air density (**high altitude**, high temperature), rough/stranded surface, small diameter, close spacing, higher voltage.
- **Ways to reduce corona**: a larger conductor diameter (ACSR, hollow conductors), **bundled conductors**, larger spacing and **corona/grading rings** at fittings.
- **Advantages of corona**: it reduces the steepness of travelling-wave surges, acting as a "safety valve", and it increases the virtual diameter of the conductor, which reduces the electrostatic stress.
- **Disadvantages of corona**: energy loss, **ozone corroding** the conductor, interference, and harmonic currents.
- Corona loss is **lower on DC** lines than on AC lines of comparable voltage.

Sag and tension

- For **supports at equal levels**, sag **$S = wL^2/(8T)$**, where L = span, w = weight per unit length and T = horizontal tension. If l is the **half-span**, $S = wl^2/(2T)$. This parabolic form is valid for small sags; the exact curve is a catenary.
- Conductor length ≈ $L + 8S^2/(3L)$.
- **Supports at unequal levels** (difference h): the lowest point is at $x_1 = L/2 - Th/(wL)$ from the lower support and $x_2 = L/2 + Th/(wL)$ from the higher support. Then $S_1 = wx_1^2/2T$ and $S_2 = wx_2^2/2T$.
- **Ice loading** (radial thickness t, conductor diameter d): $w_i = \rho_{ice} \times \pi t(d + t)$ per unit length, with ice density ≈ 915 kg/m³.

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- **Wind loading** $w_w = p \times (d + 2t)$, where p is the wind pressure per unit projected area. The wind force acts **horizontally**.
- Effective loading $\mathbf{w_e = \sqrt{[(w + w_i)^2 + w_w^2]}}$. The resulting sag $S = w_e L^2/8T$ is **inclined** at $\theta = \tan^{-1}[w_w/(w + w_i)]$ to the vertical. **Vertical sag = S cos θ**.
- Working tension = ultimate tensile strength ÷ **factor of safety (≈ 2)**.
- Sag **rises with temperature**, ice and wind. Too little sag → high tension (snapping in winter); too much → taller towers, less clearance.
- **Stringing chart** = curves of **sag and tension vs temperature** for a span, used during stringing to set the correct sag at the ambient temperature.
- **Vibration dampers** (Stockbridge type) control **aeolian vibration**, which is high-frequency and low-amplitude and caused by steady low wind. **Galloping** is low-frequency, high-amplitude motion caused by ice-coated conductors in wind.

KEY FORMULAE

- $L = 0.2 \ln(GMD/GMR)$ mH/km per phase ; $r' = 0.7788\ r$
- $C_n = 2\pi\varepsilon_0/\ln(D/r)$ F/m $= 0.0556/\ln(D/r)$ μF/km
- $\gamma = \sqrt{(zy)}$; $Z_c = \sqrt{(z/y)} \approx \sqrt{(L/C)}$; $v = 1/\sqrt{(LC)}$
- $SIL = V_{LL}^2/Z_c$; $P = V_S V_R \sin\delta / X$
- $V_d = 21.1\ \delta\ m_0\ r \ln(D/r)$ kV (rms, phase) ; $\delta = 3.92b/(273 + t)$
- $S = wL^2/8T$ (equal supports) ; $w_e = \sqrt{[(w + w_i)^2 + w_w^2]}$

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In a nominal-π line, which constant equals Z? → **B**.
- For a transmission line, AD − BC = ? → **1**.
- Ferranti effect is due to? → **Line capacitance (charging current) at light load**.
- Typical surge impedance of an overhead line? → **≈ 400 Ω**. Of a cable? → **≈ 40–50 Ω**.
- Corona loss increases in? → **Foul weather and at higher frequency (∝ f + 25)**.
- If the span is doubled with T constant, the sag becomes? → **4 times**.

6.4 Insulators & Cables ★★★

Overhead line insulators

Type	Use
Pin	Distribution lines **up to 33 kV**; economical up to about 25 kV. Above that it becomes too bulky. Safety factor = puncture strength ÷ flashover voltage ≈ **10**
Suspension (disc string)	**Above 33 kV**. Each disc is rated about **11 kV**. The string is flexible and a damaged disc is easy to replace. Voltage can be increased by adding discs. Needs taller towers
Strain (tension)	**Dead ends, sharp corners, river crossings**. Two or more strings are used in parallel, mounted horizontally
Shackle (spool)	**LV distribution**. Mounted horizontally or vertically; also used at dead ends and turns on LT lines
Post	**Substations** (busbars, isolators). Line-post insulators are used on compact lines
Stay (egg/guy)	In the stay wire, at least about 3 m above ground

- Typical number of discs per string: 33 kV ≈ 3; 66 kV ≈ 5; **132 kV ≈ 9**; **220 kV ≈ 14–15**; **400 kV ≈ 23–24**. Tension strings carry one or two more discs.
- **Porcelain** (glazed): dielectric strength ≈ 60 kV/cm, high compressive strength, most widely used. Its weakness is internal cracks, which cannot be seen.
- **Toughened glass**: higher dielectric strength (≈ 140 kV/cm), and a **damaged disc shatters** so it is easy to spot. Its weakness is that moisture condenses on the surface more easily.
- **Composite / polymer** (FRP rod with silicone-rubber sheds): **light**, **hydrophobic**, excellent in **polluted areas** and resistant to vandalism. Its weakness is ageing under UV and tracking.
- Pollution design uses the **specific creepage distance** (older IEC 60815): light **16**, medium 20, heavy 25 and very heavy **31 mm/kV**.
- **Flashover** is an arc through the **air around** the insulator. The insulator is usually not damaged.
- **Puncture** is breakdown **through the body** of the insulator and damages it permanently. Insulators are designed so that **puncture strength >> flashover voltage**, which means flashover happens first.
- An **arcing horn** moves the flashover arc away from the insulator surface so the insulator is not damaged.

String efficiency

- Each disc behaves as a capacitor C (its self capacitance). Each metal link pin also has a **shunt capacitance mC** (often written kC) to the earthed tower.
- The shunt capacitance currents all add up in the lower discs. This makes the **voltage across the units unequal**: the **disc nearest the line conductor carries the largest voltage** and the disc nearest the cross-arm the least.
- **String efficiency** $\mathbf{\eta = V/(n \times V_n)}$, where V is the voltage across the whole string, n is the number of discs and V_n is the voltage across the disc nearest the line.
- η is **100%** on **DC** (no capacitive currents) and for an ideal equal sharing. η **falls as n rises** and as m rises.
- For a 3-disc string (top to bottom): $V_2 = V_1(1 + m)$ and $\mathbf{V_3 = V_1(1 + 3m + m^2)}$. Total $V = V_1 + V_2 + V_3$.
- **Ways to improve η**:
  - (1) **Longer cross-arm**, which reduces m. The practical limit is tower cost; m < 0.1 is desired.
  - (2) **Capacitance grading**: discs of different capacitance, with the **least at the top** (cross-arm) and the most near the line. This can give equal voltage in theory.
  - (3) **Static shielding** with a **guard ring** connected to the line conductor. Its capacitance to each link pin supplies the charging current, cancelling the shunt currents to the tower.

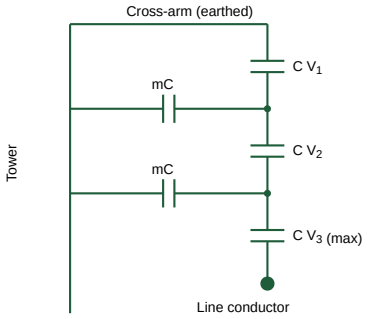

Fig. 6.2 Equivalent circuit of a 3-disc string. The shunt capacitance mC from each link pin to the tower makes $V_3 > V_2 > V_1$.

Underground cables

- **Construction** (inside → out): **conductor** → **insulation** (paper/PVC/XLPE) → **metallic sheath** (lead/Al, keeps out moisture) → **bedding** (protects sheath from armour) → **armouring** (steel wire/tape) → **serving**.
- **Insulation resistance** $R_i = (\rho/2\pi l) \ln(R/r)$. It is ∝ **1/length** (the leakage paths are in parallel), which is the opposite of conductor resistance.
- **Capacitance** (single core) $C = 2\pi\varepsilon_0\varepsilon_r\, l / \ln(R/r) = \mathbf{0.024\ \varepsilon_r/\log_{10}(R/r)}$ **μF/km**. Cable capacitance is **much higher** than that of an overhead line.
- **Dielectric stress** at radius x: $g = V/[x \ln(R/r)]$.
  - **Maximum at the conductor surface**: $g_{max} = V/[r \ln(R/r)]$.
  - Minimum at the sheath: $g_{min} = V/[R \ln(R/r)]$.
  - $g_{max}/g_{min} = R/r$.
- **Most economical core size**: for a given sheath radius R, g_{max} is least when **r = R/e**, so **ln(R/r) = 1** and $g_{max} = V/r$. The conductor diameter is then $d = 2V/g_{max}$, and $D = 2.718\ d$.
- **Grading of cables** makes the stress more uniform, which allows a smaller cable for the same voltage.
  - **Capacitance grading** uses layers of dielectric with different ε_r, with the **highest ε nearest the core** ($\varepsilon_1 > \varepsilon_2 > \varepsilon_3$). For equal g_{max} in every layer, $\varepsilon_1 r_1 = \varepsilon_2 r_2 = \varepsilon_3 r_3$. Its drawback is that suitable materials are hard to get.
  - **Intersheath grading** uses one homogeneous dielectric divided by thin **metallic intersheaths** held at intermediate potentials (from tappings on the supply transformer). Its drawbacks are that the intersheaths can be damaged during laying and they carry charging-current losses.

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- **Capacitance of a 3-core cable**, where C_s = core-to-sheath and C_c = core-to-core capacitance:
  - Per-phase capacitance to neutral $\mathbf{C_N = C_s + 3C_c}$.
  - Measurement test 1: all three cores bunched, measured against the sheath, gives $C_1 = 3C_s$.
  - Measurement test 2: two cores joined to the sheath, measured against the third core, gives $C_2 = C_s + 2C_c$.
  - So $C_N = 3C_2/2 - C_1/6$.
- **Charging current** $I_c = V_{ph}\,\omega\,C_N$. In long AC cables (a few tens of km), charging current uses up the cable's rating. This is why **long submarine links use HVDC**.
- **Dielectric loss** = $V^2\,\omega\,C$ **tan δ**. The loss angle δ = 90° − φ, where φ is the angle between V and the insulation current. A **low tan δ means healthy insulation**. Rising tan δ indicates moisture or ageing. It is measured with a **Schering bridge**.
- Typical tan δ: XLPE ≈ 0.0005–0.001; PILC ≈ 0.003–0.01. Typical ε_r: XLPE ≈ 2.3, paper ≈ 3.5–4, PVC ≈ 5–8.

Cable type	Voltage (typical)	Key feature
Belted	**Up to 11 kV** (some texts say 22 kV)	The cores are insulated and an overall belt of paper surrounds them. Field has a **tangential** component, which causes leakage along the paper layers and voids
Screened H-type (Hochstadter)	Up to **66 kV**	Each core has a metallised paper screen and the screens are in contact with the sheath. The field is **radial only**, so there is no tangential stress and no voids
S.L. type	Up to 66 kV	Each core has its own **lead sheath**, so bending is easier and the risk of a core-to-core fault is lower
Pressure cables (oil-filled, gas-pressure)	**Above 66 kV**	Oil or gas under pressure removes voids and ionisation
XLPE	LV up to 400 kV+ (now standard)	Continuous rating **90 °C**, short circuit **250 °C**. Low tan δ, light, easy jointing
PVC	LV, up to about 3.3/6.6 kV	Continuous rating **70 °C**, short circuit 160 °C

KEY FORMULAE

- $\eta_{string} = V / (n \times V_n)$; 3 discs: $V_3 = V_1(1 + 3m + m^2)$
- $R_{ins} = (\rho/2\pi l)\ln(R/r)$; $C = 2\pi\varepsilon_0\varepsilon_r/\ln(R/r)$ per m
- $g_{max} = V/[r\ln(R/r)]$; economical r = R/e
- $C_N = C_s + 3C_c$; $P_{diel} = V^2\omega C \tan\delta$

MEMORY TRICK

In a string, **the disc nearest the "Line" takes the Largest voltage**. Strings grade "small C on top, big C at the bottom". In a cable, **the stress is highest at the core**, so the highest-ε layer goes at the core. Remember **r = R/e** for the economic core.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Pin insulators are used up to? → **33 kV**.
- In a suspension string, which disc is most stressed? → **The one nearest the line conductor**.
- String efficiency on DC? → **100%**.
- Maximum dielectric stress in a single-core cable occurs at? → **The conductor surface**.
- Insulation resistance of a cable varies with length as? → **Inversely**.
- Belted cables are used up to? → **11 kV**.

6.5 Compensation & Voltage Control ★★★

P–f and Q–V links

- The voltage drop is ΔV ≈ (RP + XQ)/V. For transmission lines X >> R, so **voltage magnitude depends mainly on reactive power Q**. This is the **Q–V** link, and it is **local**.
- The **load angle δ and frequency depend on active power P**. This is the **P–f (P–δ)** link, and it is **system-wide**.

Compensating devices

Device	Key points
Series capacitor	Cuts net X to $(X_L - X_C)$, so **power transfer and stability rise**; self-regulating with load current. Degree of compensation X_C/X_L ≈ **25–70%** (never 100% – resonance). Risk: **sub-synchronous resonance (SSR)** with turbo-generator shafts (Mohave, USA, 1970–71). Protected by MOV, spark gap, bypass breaker
Shunt reactor	Absorbs Q at light load. Controls the **Ferranti rise** and switching overvoltages on long EHV lines. Can be a line reactor or a bus reactor
Shunt capacitor	Supplies Q, which improves pf and raises the local voltage. Cheap. Its output is **Q ∝ V²**, so it gives **less Q when the voltage is low**, just when it is most needed. Switched in steps
Synchronous condenser	Synchronous motor **at no load**: **over-excited** supplies Q, under-excited absorbs Q. **Stepless** control, adds inertia and fault level; costly, has losses and maintenance – economic only in large sizes
SVC (static VAR compensator)	Shunt **TCR + TSC** (+ fixed capacitor or filters) using thyristors. Fast. At its limit it behaves like a fixed capacitor, so Q ∝ V²
STATCOM	Shunt **voltage-source converter** (IGBT). Gives **rated current even at low voltage** (Q ∝ V at limit); faster than SVC, **smaller footprint**, better fault-time support
TCSC	Thyristor-controlled **series** capacitor. Gives variable series compensation, damps power oscillations and can mitigate SSR
SSSC / **UPFC**	The SSSC is a series VSC. The **UPFC** combines series and shunt converters and controls **V, X and δ**. It is the **most versatile** FACTS device
Tap-changing **transformer**	**Off-load** or **on-load** (OLTC, on the **HV winding** – lower current); typically ±10% in 1.25% steps. Redistributes Q but **does not generate it**
Booster / phase-shifting transformer	A booster injects voltage in series for regulation. A phase shifter controls the flow of **active power** through the angle

- **FACTS** (Flexible AC Transmission Systems) is a concept introduced by **N.G. Hingorani** (EPRI, 1988).
- Shunt compensation mainly controls **voltage**. Series compensation mainly raises **transfer capability and stability**.

Load frequency control (LFC) and AGC

- **Speed regulation (droop)** R = −Δf/ΔP (Hz/MW or pu). It is typically **4–5%** for steam/hydro sets in India (range 3–6%). A 5% droop means a 5% fall in frequency drives the set from no load to full load.
- Sets in parallel with **equal % droop** share load **in proportion to their ratings**. Otherwise each set's share is ∝ 1/R.
- **Primary control** (governor, a few seconds) **arrests** the change in frequency but leaves a **steady-state error** $\Delta f_{ss} = -\Delta P_D/\beta$.
  - β = D + 1/R is the **area frequency response characteristic (AFRC)**, and D is the load damping constant.
  - Without governor action, $\Delta f_{ss} = -\Delta P_D/D$.
- **Secondary control** (AGC, an **integral** controller, minutes) brings Δf back to **zero** and restores the scheduled tie-line flow.
- **Tertiary control** (economic dispatch, about 15 min and longer) re-sets the generation schedule.
- **ACE** (area control error) = $\Delta P_{tie} + B\,\Delta f$, where B is the frequency bias, usually set equal to β. The AGC drives ACE to zero, so each area absorbs its own load change ("non-interference").
- Tie-line power change $\Delta P_{tie} = (2\pi T_{12}/s)(\Delta f_1 - \Delta f_2)$. Two areas with primary control only: $\Delta f = -\Delta P_D/(\beta_1 + \beta_2)$.
- In India, the **IEGC** sets the normal frequency band (49.90–50.05 Hz). Grid-India and the RLDCs manage frequency and AGC.

Power factor correction

- **Causes of low pf**: **induction motors** at light load (pf ≈ 0.2–0.3 at no load, 0.8–0.9 at full load), lightly loaded transformers, arc/induction furnaces, welding sets, lamp chokes, over-voltage (more magnetising current).

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- **Disadvantages of low pf** (I ∝ 1/cosφ for the same P): higher kVA rating of equipment, larger conductors, **copper loss ∝ 1/cos²φ**, poor regulation, reduced handling capacity, tariff penalties.
- **Correction devices**: static capacitors (most common, low loss, no moving parts); **synchronous condensers** (large installations); **phase advancers**, which are mounted on the induction motor shaft and supply exciting ampere-turns to the rotor circuit at slip frequency.
- Required capacitor rating: $\mathbf{Q_C = P(\tan\varphi_1 - \tan\varphi_2)}$ **kVAR**.
- Per-phase capacitance: star-connected $C_Y = Q_C/(\omega V_L^2)$. Delta-connected $\mathbf{C_\Delta = C_Y/3}$, but each capacitor must withstand the line voltage.
- **Most economical pf**: let x = annual tariff charge per kVA of maximum demand and y = annual cost (interest + depreciation) per kVAR of capacitor. Then $\mathbf{\sin\varphi_2 = y/x}$, so $\cos\varphi_2 = \sqrt{[1 - (y/x)^2]}$. The economic pf is independent of the original pf.
- Correcting all the way to unity is **uneconomical**: each extra kVAR gives a smaller and smaller reduction in kVA. The usual target is **0.95–0.98 lagging**.

KEY FORMULAE

- $\Delta V \approx (PR + QX)/V$; $P = V_1V_2\sin\delta/(X_L - X_C)$ with series C
- $\Delta f_{ss} = -\Delta P_D/(D + 1/R)$; $ACE = \Delta P_{tie} + B\Delta f$
- $Q_C = P(\tan\varphi_1 - \tan\varphi_2)$; economical $\sin\varphi_2 = y/x$
- Shunt capacitor: $Q = V^2\omega C$ ($\propto V^2$)

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Voltage at a bus is mainly controlled by? → **Reactive power (Q)**.
- Series capacitive compensation may cause? → **Sub-synchronous resonance**.
- Device used to absorb reactive power on a lightly loaded EHV line? → **Shunt reactor**.
- Over-excited synchronous motor at no load is called? → **Synchronous condenser**.
- The integral controller in LFC makes? → **Steady-state frequency error zero**
- kVAR to raise pf of P kW from φ_1 to φ_2? → $\mathbf{P(\tan\varphi_1 - \tan\varphi_2)}$.

6.6 Distribution Systems & Per-Unit ★★★

- Indian distribution: **primary** distribution is at **11 kV** (also 33, 22, 6.6 kV). **Secondary** distribution is at **400/230 V, 3-φ 4-wire** (IS 12360; older value 415/240 V).
- The usual distribution transformer is **Dyn11** (Δ on HV, star with earthed neutral on LV).
- **Feeder**: connects a substation to a distribution area and has **no tappings**, so the **current is the same** along its length. It is designed for **current-carrying capacity**.
- **Distributor**: has **tappings** for consumers along its length, so the current varies. It is designed mainly for **voltage drop**.
- **Service mains**: the small cable from the distributor to the consumer's terminals.
- Permissible voltage variation (IE Rules 1956, Rule 54): LV **±6%**; HV +6%/−9%; EHV +10%/−12.5%. Frequency ±3% (Rule 55); grid frequency is now governed by the IEGC band.

Scheme	Features
Radial	Fed from one end only. **Simplest and cheapest**. A fault cuts off every consumer beyond it. Consumers at the far end get poor voltage. Used for short lines and low-density areas
Ring main	The distributor forms a closed loop, so each consumer is fed by **two paths**. Less voltage fluctuation and **more reliable**. Used in urban areas, with ring main units (RMUs)
Interconnected (mesh)	A ring fed by two or more substations. **Most reliable**, with better use of reserve capacity, but costly and with complex protection

DC and AC distributors

- **Fed at one end, uniformly loaded** (i A/m, loop resistance r Ω/m, length l): total drop = $\mathbf{irl^2/2}$, which equals ½ × total load current × total resistance. **Minimum potential** is at the **far end**.
- **Fed at both ends with equal voltages, uniformly loaded**: **minimum potential at the mid-point** and maximum drop = $\mathbf{irl^2/8}$. This is **one-fourth** of the drop when fed from one end.
- **Concentrated loads, fed at both ends**: assume a current I_A from end A and write KVL from A to B ($V_A - V_B = \Sigma$ drops). The point of minimum potential is where the current reverses direction.
- **Ring distributor** = distributor fed at both ends with equal voltages (interconnector → Thevenin). **DC 3-wire** saves copper; unbalance handled by a **balancer set**.
- **AC distributors**: the drop includes I(R cosφ + X sinφ). Load pf may be referred to the **receiving-end voltage** or to the **voltage at each load point**, and the two methods give different phasor sums.
- **3-φ 4-wire**: the neutral carries the **unbalance current** (zero for a balanced load, apart from triplen harmonics). Single-phase loads are connected between a phase and neutral. The neutral is usually sized at half the phase conductor (full size where non-linear loads are heavy).

Per-unit system

- **Per-unit value = actual value ÷ base value**, and it has no dimension. Choose any two bases (normally $\mathbf{MVA_{base}}$, common to the whole system, and $\mathbf{kV_{base}}$, which changes across transformers in the ratio of their turns). The other bases follow.
- For a 3-φ system with V_L and 3-φ MVA as bases, the **pu value of an impedance is the same** as the single-phase value with phase quantities.
- When the voltage bases on the two sides are in the transformer's turns ratio, the transformer's **pu impedance is the same whichever side it is referred to**, and the ideal transformer disappears from the circuit.
- **Advantages**: machine parameters fall in a **narrow range** whatever the rating (errors easy to spot); makers quote pu/% values; √3 factors disappear; suits computer studies.
- Typical pu values (on own rating):
  - Transformer leakage reactance ≈ **0.04–0.15 pu**, with larger units at the higher end.
  - Generator: $X_d \approx 1.0–2.0$ pu; $X'_d \approx 0.2–0.35$; $X''_d \approx 0.1–0.25$.
  - Transformer no-load current ≈ 0.01–0.05 pu (smaller for large units).

KEY FORMULAE

- $I_{base} = kVA_{base}/(\sqrt{3}\, kV_{base})$; $Z_{base} = (kV_{base})^2/MVA_{base}$ Ω
- $Z_{pu} = Z_{actual}(\Omega) \times MVA_{base}/(kV_{base})^2$
- $\mathbf{Z_{pu,new} = Z_{pu,old} \times (MVA_{new}/MVA_{old}) \times (kV_{old}/kV_{new})^2}$
- % Z = $Z_{pu} \times 100$; $S_{pu} = V_{pu}\, I_{pu}^*$
- Distributor uniformly loaded: one end $irl^2/2$; both ends (equal V) $irl^2/8$

MEMORY TRICK

Change of base: **"MVA goes Up top (new/old), kV goes Down (old/new)²"**. Z_{pu} is ∝ MVA and ∝ 1/kV². Distributors: **"one end = ½, both ends = ⅛"**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Feeders are designed mainly on the basis of? → **Current-carrying capacity**.
- Distributors are designed mainly on the basis of? → **Voltage drop**.
- Most reliable distribution scheme? → **Interconnected (ring fed from several substations)**.
- A 0.1 pu reactance on 50 MVA is expressed on 100 MVA (same kV) as? → **0.2 pu**.
- Uniformly loaded distributor fed at both ends: point of minimum potential? → **Mid-point**.

6.7 Bus Admittance Matrix & Load Flow ★★★

Y_{bus} and Z_{bus}

- Nodal equation: $\mathbf{I_{bus} = Y_{bus}\, V_{bus}}$. For n buses (excluding ground/reference), Y_{bus} is **n × n**.
- **Formation by inspection**:
  - **Diagonal Y_{ii}** (self or driving-point admittance) = **sum of all admittances connected to bus i**, including shunt (line-charging y/2, shunt capacitors and reactors).
  - **Off-diagonal Y_{ij}** (mutual or transfer admittance) = **$-y_{ij}$**, the negative of the admittance between buses i and j. It is **0** if the buses are not connected.
- Formal method: $\mathbf{Y_{bus} = A^T\, y\, A}$, where A is the bus incidence matrix and y the primitive admittance matrix (singular transformation).

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- **Properties of Y_{bus}:**
  - **Sparse**: a typical bus connects to only 3–4 others, so > 90% of the elements are zero in large systems. This allows sparsity techniques and optimal ordering.
  - **Symmetric** ($Y_{ij} = Y_{ji}$) if there are **no phase-shifting transformers**. Off-nominal tap transformers modelled as π keep it symmetric; phase shifters make it **unsymmetric**.
  - Usually **diagonally dominant**, and complex with large imaginary parts.
  - If there is **no shunt element** to ground (no line charging, no shunt loads), each row sums to zero. Y_{bus} is then **singular** and Z_{bus} does not exist. A connection to ground makes it non-singular.
- **Adding or removing a line** between i and j changes only $\mathbf{Y_{ii}, Y_{jj}, Y_{ij}, Y_{ji}}$, so the matrix is easy to modify.
- $\mathbf{Z_{bus} = Y_{bus}^{-1}}$. It is a **full (dense)** matrix and is symmetric if Y_{bus} is. Its diagonal elements Z_{ii} are the **Thevenin impedances** at each bus.
- Z_{bus} is built directly by the $\mathbf{Z_{bus}}$ **building algorithm** (adding branches and links one at a time). It is used mainly in **short-circuit (fault) studies**.
- Y_{bus} is preferred for **load flow**.

Load flow – bus classification

- Each bus has four quantities: **P, Q, |V|, δ**. Two are specified and two are found by the load flow.

Bus type	Specified	Unknown (found)	Remarks
Slack / swing / reference	**|V|, δ** (δ = 0°)	P, Q	**Only one** per system. It supplies the **losses**, which are not known in advance. Usually the largest generator bus
PV (generator / voltage-controlled)	**P, |V|**	Q, δ	Q limits are checked. If one is violated, Q is fixed at the limit and the bus becomes a **PQ** bus. Also used for synchronous condensers and SVCs
PQ (load)	**P, Q**	|V|, δ	**Most buses** (≈ 80–85%) are PQ buses

- Load flow equations are **non-linear algebraic** equations: $P_i - jQ_i = V_i^* \Sigma Y_{ij} V_j$. They are solved by iteration.
- For n buses with one slack bus and m PV buses, the number of unknowns (and equations in polar NR) is **2(n − 1) − m**. These are the δ of n − 1 buses and the |V| of n − 1 − m buses. This is also the **Jacobian order**.

Solution methods

- **Gauss–Seidel (GS)** update: $V_i^{(k+1)} = (1/Y_{ii})[(P_i - jQ_i)/V_i^{*(k)} - \Sigma_{j \neq i} Y_{ij} V_j]$. Each newly updated value is used straight away, which is why GS is faster than Gauss (Jacobi).
- **Acceleration factor** α: $V_{acc} = V_{old} + \alpha(V_{new} - V_{old})$, with **α ≈ 1.4–1.6** (1.6 is commonly quoted).
- **Newton–Raphson (NR)**: $[\Delta P; \Delta Q] = J\,[\Delta\delta; \Delta|V|]$. The **Jacobian** $J = [J_1\ J_2; J_3\ J_4]$ $(\partial P/\partial\delta, \partial P/\partial V; \partial Q/\partial\delta, \partial Q/\partial V)$ is **re-computed and factorised every iteration**. It is sparse and has the same sparsity pattern as Y_{bus}.
- **Fast decoupled load flow (FDLF)** (Stott & Alsaç, 1974) is based on **P–δ and Q–V decoupling**: it drops J_2 $(\partial P/\partial V)$ and J_3 $(\partial Q/\partial\delta)$.
  - Assumptions: $\cos\delta_{ij} \approx 1$, $G_{ij} \sin\delta_{ij} \ll B_{ij}$, and $Q_i \ll B_{ii}V_i^2$ (X/R high).
  - This gives $\Delta P/V = B'\Delta\delta$ and $\Delta Q/V = B''\Delta V$, where **B′ and B″ are constant** matrices that are **factorised only once**.
- **DC load flow**:
  - Assumptions: **lossless lines** (R ≈ 0), **all |V| = 1 pu**, **small angles** (sin δ ≈ δ), and Q is ignored.
  - The result is **P = B′θ**, a **linear, non-iterative** solution.
  - It gives only active power flows and angles, with no V or Q information.
  - Used for **contingency screening**, planning and market (LMP) studies.

Feature	Gauss–Seidel	Newton–Raphson	Fast Decoupled
Convergence	**Linear**, slow	**Quadratic**, fast	Linear (geometric) but fast
No. of iterations	**Large**, and it **rises with system size**	**≈ 3–5, almost independent of size**	More than NR (≈ 2–5× NR), roughly independent of size
Time per iteration	**Least**	**Most** (Jacobian built and factorised each time)	Small (constant B′, B″)
Memory	**Least**	**Most**	Less than NR (about 60%)
Total solution time (large system)	Highest	Low	**Lowest**, which suits online and contingency studies
Sensitivity	Affected by the choice of slack bus. Weak with **negative reactances** (series capacitors) and ill-conditioned systems	Reliable. Needs a reasonable initial guess	May fail when R/X is high (distribution systems)
Accuracy	Good	**Most accurate**	Slightly less accurate per iteration, but the final answer is exact (the mismatch test is exact)
Best for	Small systems	**Large systems** (the standard method)	Repeated / **online** studies

KEY FORMULAE

- $Y_{ii} = \Sigma\, y$ (all connected to i, incl. shunt) ; $Y_{ij} = -y_{ij}$
- $S_i^* = P_i - jQ_i = V_i^* \Sigma_j Y_{ij} V_j$
- $P_i = \Sigma\, |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$; $Q_i = -\Sigma\, |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$
- Order of NR Jacobian (polar) = 2(n − 1) − m
- DC load flow: $P_{ij} = (\theta_i - \theta_j)/x_{ij}$

MEMORY TRICK

Bus types: **"Slack fixes the Voltage phasor (|V|, δ); a Generator fixes what it controls (P, |V|); a Load fixes what it draws (P, Q)."** Methods: **GS = Gentle and Slow (linear); NR = Newton Races (quadratic, 3–5 iterations); FDLF = Fastest, Decoupled, Fixed matrices.**

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Quantities specified at a slack bus? → **|V| and δ**.
- Quantities specified at a PV bus? → **P and |V|**.
- Off-diagonal element Y_{ij} of Y_{bus}? → **Negative of the admittance between buses i and j**.
- Y_{bus} becomes unsymmetrical in the presence of? → **Phase-shifting transformers**.
- Convergence characteristic of the NR method? → **Quadratic**.
- Load flow method best for repeated online contingency analysis? → **Fast decoupled (or DC load flow for screening)**.

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7. Power Systems II – Faults, Protection & Stability

NOTE

Faults, protection and stability together give about 5–8 MCQs in PSU/AE papers. Most-asked items: the operator **a**, sequence networks for LG/LL/LLG faults, fault MVA, IDMT relay time (PSM/TMS), relay characteristics on the R–X plane, which CB suits which voltage, SF_6/vacuum properties, RRRV and current chopping, Buchholz relay, Petersen coil, the swing equation and the equal-area criterion. Learn the tables. The numerical questions are short one-step substitutions.

7.1 Symmetrical Components ★★★

- **Fortescue's theorem (1918)**: any set of **n unbalanced phasors** can be resolved into **n balanced sets**. A 3-phase unbalanced set gives **3 sets**: positive, negative and zero sequence.
- **Positive sequence**: 3 equal phasors 120° apart, same phase sequence as the supply (**a-b-c**).
- **Negative sequence**: 3 equal phasors 120° apart, **opposite phase sequence (a-c-b)**.
- **Zero sequence**: 3 equal phasors that are **in phase** (zero displacement).
- Operator **$a = 1\angle 120° = -0.5 + j0.866$**; **$a^2 = 1\angle 240° = -0.5 - j0.866$**; **$a^3 = 1$**; $a^4 = a$.
- **$1 + a + a^2 = 0$**; $a - a^2 = \mathbf{j\sqrt{3}}$; $1 - a^2 = \sqrt{3}\angle 30°$; $1 - a = \sqrt{3}\angle -30°$; $a^2 - a = -j\sqrt{3}$.
- Multiplying by **a** rotates a phasor **120° anticlockwise** and leaves its magnitude unchanged. Multiplying by j rotates it by 90°.
- **Zero-sequence current needs a return path** (neutral wire / ground). Neutral current **$I_n = 3I_{a0}$**.
- There is **no zero-sequence current** in a **3-wire system, a delta connection or an ungrounded star**. In a delta, zero-sequence current can **circulate inside the delta** but cannot leave it.
- **Line voltages never contain zero-sequence components**, because $V_{ab} + V_{bc} + V_{ca} = 0$ always.
- A balanced 3-phase system has **only positive-sequence** components. Negative and zero sequence are both zero.
- An impedance Z_n in the neutral appears as **$3Z_n$** in the **zero-sequence network only**. It does not appear in the positive or negative networks.
- The **positive-sequence network** is the only one that contains **sources (emfs)**. Negative and zero networks are passive, because generators produce only balanced positive-sequence emf.
- Unbalance factor = $|I_2|/|I_1|$ (negative-sequence current is what heats rotors).

KEY FORMULAE

- $V_a = V_{a0} + V_{a1} + V_{a2}$; $V_b = V_{a0} + a^2V_{a1} + aV_{a2}$; $V_c = V_{a0} + aV_{a1} + a^2V_{a2}$
- $[V_{abc}] = [A][V_{012}]$, with **$A = [1\ 1\ 1;\ 1\ a^2\ a;\ 1\ a\ a^2]$**; **$A^{-1} = (1/3)[1\ 1\ 1;\ 1\ a\ a^2;\ 1\ a^2\ a]$**
- $V_{a0} = (V_a + V_b + V_c)/3$
- $V_{a1} = (V_a + aV_b + a^2V_c)/3$
- $V_{a2} = (V_a + a^2V_b + aV_c)/3$
- Complex power: **$S = V_aI_a^* + V_bI_b^* + V_cI_c^* = 3(V_{a0}I_{a0}^* + V_{a1}I_{a1}^* + V_{a2}I_{a2}^*)$**. The factor 3 appears because A is not unitary ($A^TA^* = 3I$).
- There is **no cross-product term**, so power is the sum of the powers of the individual sequences.

Element	Sequence impedance relation	Remarks
Transmission line (static, symmetrical)	**$Z_1 = Z_2 < Z_0$**	$Z_0 \approx$ **2–3.5 × Z_1** (≈3 typical). Zero-sequence current returns through earth/ground wire, so the path is longer and the mutual flux is in phase.
Transformer	**$Z_1 = Z_2 = Z_0$** (= leakage impedance)	Z_0 is drawn only if the winding connection allows zero-sequence current (see next table). In a 3-limb core-type unit Z_0 is slightly lower.
Synchronous generator	**$Z_0 < Z_2 < Z_1$**	Z_0 is the smallest because it is only slot and end-winding leakage. $X_2 \approx (X_d'' + X_q'')/2$. $Z_1 = X_d''$, X_d' or X_d depending on the period studied.
Synchronous / induction motor	$Z_2 < Z_1$	Induction motor $Z_2 \approx$ standstill (blocked-rotor) impedance, which is low. This is why unbalanced voltage overheats motors.
Static loads, reactors, capacitors	$Z_1 = Z_2$	Any balanced static element has $Z_1 = Z_2$.

Transformer connection (P–S)	Zero-sequence equivalent
Y_g – Y_g (both neutrals grounded)	Z_0 is in series between P and S, so zero-sequence current **flows through both sides**.
Y_g – Y (one side ungrounded)	**Open** on both sides. No zero-sequence path.
Y_g – Δ	On the Y_g side, Z_0 connects to the **reference bus**. The Δ side is **open**. Current circulates in the Δ.
Y – Δ (Y ungrounded)	Open on both sides.
Δ – Δ	Open on both sides. Zero-sequence current circulates in the Δ windings only.
Y grounded through Z_n	Add **$3Z_n$** in series in the zero-sequence path.

- A Y–Δ transformer shifts **positive-sequence quantities by +30°** and **negative-sequence quantities by −30°** (ANSI convention, stepping up from LV to HV side: HV positive sequence leads LV by 30°). Zero sequence does not pass through it.
- A **delta tertiary** in Y–Y transformers provides a path for **3rd-harmonic and zero-sequence currents** and lowers Z_0.
- A **zigzag (interconnected star) grounding transformer** provides a neutral for an ungrounded/Δ system. It offers **low Z_0** and high Z_1.

MEMORY TRICK

"**Delta Drinks Zero**": a delta absorbs zero-sequence current and lets none of it out. For the zero-sequence network of a transformer, draw a **closed switch on a grounded-Y side** and an **open switch on an ungrounded-Y side**. A **Δ side shorts the winding to reference** but is open towards the line.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Value of $1 + a + a^2$? → **0**.
- Zero-sequence currents cannot flow in? → **Delta-connected lines / ungrounded star** (no return path).
- Neutral impedance Z_n appears in the zero-sequence network as? → **$3Z_n$**.
- For a transmission line, which is larger, Z_0 or Z_1? → **Z_0** ($Z_1 = Z_2 < Z_0$).
- Smallest sequence impedance of a synchronous generator? → **Zero sequence**.
- If $I_a = 10$, $I_b = I_c = 0$, then I_{a0}? → **10/3 A** (and $I_{a1} = I_{a2} = 10/3$ A).

7.2 Symmetrical Fault Analysis ★★★

- **Symmetrical fault** = 3-phase (LLL or LLLG) fault. The system stays balanced, so only the **positive-sequence network** is needed and per-phase analysis applies.
- A 3-phase fault is the **most severe in most cases** (it sets the CB breaking capacity) but the **least frequent**. An **LG fault is the most frequent** and the least severe in general.
- Exception: at the terminals of a **solidly grounded generator**, the LG fault current can exceed the 3-ph fault current because $Z_0 < Z_1$ (see 7.3).

Fault type	Frequency of occurrence	Severity
LG (single line-to-ground)	**≈ 70–80%** (Wadhwa quotes up to 85%)	Least severe (generally)
LL (line-to-line)	**≈ 10–15%** (≈ 8% in some texts)	—
LLG (double line-to-ground)	**≈ 10%** (≈ 5% in some texts)	—
LLL / LLLG (symmetrical)	**≈ 5%** (≈ 2% in some texts)	**Most severe** (generally)

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- **Overhead lines** have the most faults (≈ 50% of all faults), then cables, switchgear, transformers and generators.
- The **short-circuit current of an alternator** is limited by the **reactance**, because R is negligible. After a sudden SC, the AC envelope decays through three periods.
- **DC offset**: the fault current has a decaying DC component whose size depends on the **instant of fault** (voltage angle) and the X/R ratio. Offset is **maximum when the fault occurs at voltage zero** (purely inductive circuit). The first peak can then be ≈ **2×** the symmetrical peak. It is zero when the fault occurs at voltage maximum.
- The DC component decays with time constant $\mathbf{T_a = L/R}$ (armature time constant, ~0.1–0.3 s).
- The asymmetrical rms current is $\mathbf{\sqrt{(I_{ac}^2 + I_{dc}^2)}}$. Its theoretical maximum is $\sqrt{3} \times I_{sym}$. The **momentary (first-cycle) current ≈ 1.6 × symmetrical rms** is used in practice.
- **Doubling effect** = the maximum instantaneous current reaches ≈ twice the peak of the symmetrical current.

Period	Reactance	Typical value (turbo-alternator, pu)	Time constant	Used for
Subtransient (first 2–3 cycles)	X_d'' (damper winding effect)	0.1–0.25	$T_d'' \approx$ 0.03–0.05 s	**CB making / momentary rating**, relay setting
Transient (next ~1–2 s)	X_d' (field winding effect)	0.15–0.35	$T_d' \approx$ 1–2 s	**Breaking current**, transient stability
Steady state	X_d (synchronous)	1.0–2.0	—	Steady-state SC current, steady-state stability

- Order: $\mathbf{X_d'' < X_d' < X_d}$, and so $\mathbf{I'' > I' > I}$. A salient-pole machine without dampers has $X_d'' \approx X_d'$.
- The SC current of an induction motor dies out in a few cycles because it has no field excitation. It contributes only to the momentary (subtransient) current.
- **Per-unit system**: $Z_{pu} = Z(\Omega) \times MVA_{base}/(kV_{base})^2$. To change base: $\mathbf{Z_{pu,new} = Z_{pu,old} \times (MVA_{new}/MVA_{old}) \times (kV_{old}/kV_{new})^2}$.
- A transformer's per-unit impedance is the **same referred to either side**, which is one advantage of the pu system. Typical X: 5–10% (distribution) up to 12–15% (large power transformers).
- **Thevenin (fault-point) method**: $I_f = V_{th}/(Z_{th} + Z_f)$. V_{th} = pre-fault voltage at the fault point (usually 1 pu, with load current neglected). Z_{th} = impedance seen from the fault point with sources shorted.
- With a **bus impedance matrix Z_{bus}**, a fault at bus k gives $I_f = V_k^0/(Z_{kk} + Z_f)$. Post-fault voltage at bus i is $V_i = V_i^0 - Z_{ik}I_f$. The **diagonal element Z_{kk} = Thevenin impedance** at bus k.
- **Short-circuit level / fault level** of a bus = $\sqrt{3} \times V_L$ (pre-fault) × I_f. A **high fault level** means a **strong (stiff) bus**, i.e. low source impedance.
- An **infinite bus** has constant V and f and **zero internal impedance**, so its fault level is infinite.

KEY FORMULAE

- **Fault MVA = Base MVA / Z_{pu}** = (Base MVA × 100)/%Z
- $\mathbf{I_{sc} = I_{base}/Z_{pu}} = I_{full\text{-}load} \times 100/\%Z$
- $I_{base} = MVA_{base}/(\sqrt{3}\ kV_{base})$ kA
- Example: a 10 MVA transformer with 10% reactance can feed at most 10 × 100/10 = **100 MVA** into a fault (infinite source).
- %X needed in a reactor to limit fault MVA to S_f: $\mathbf{\%X_{total} = Base\ MVA \times 100 / S_f}$, so $\%X_{reactor} = \%X_{total} - \%X_{existing}$.

Current-limiting reactor	Location	Advantage	Disadvantage
Generator reactor	In series with each generator	Protects the generator	Constant voltage drop and loss. Rarely needed in modern machines, which have high leakage X. If a bus-bar fault occurs, the whole bus is lost.
Feeder reactor	In series with each outgoing feeder	A fault on one feeder does not drop bus voltage much, so other feeders are unaffected	Constant voltage drop and I^2R loss in normal operation. **No protection against bus-bar faults.**
Bus-bar reactor – ring system	Between bus sections; each generator feeds one section	Normally carries little current, so **low loss and drop**	—
Bus-bar reactor – tie-bar system	Each section connected to a common tie-bar through a reactor	Effectively 2 reactors in series between sections. Sections can be added without changing the reactors.	Needs an extra (tie) bus-bar

- Reactors are **air-cored** (dry type) or oil-immersed without an iron core, so that the **reactance stays constant** (iron would saturate at fault current).
- A reactor is chosen so that the **reactance is high and resistance is very low**, which keeps losses small.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Most frequent fault in power systems? → **Single line-to-ground (LG)**.
- Reactance used to find the CB making (momentary) current? → **Subtransient X_d''**.
- Fault MVA at a bus with Z = 0.05 pu on 100 MVA base? → **2000 MVA**.
- Why are current-limiting reactors air-cored? → **To avoid saturation (constant X at high current)**.
- Maximum DC offset in fault current occurs when the fault happens at? → **Voltage zero**.
- Feeder reactors give no protection against? → **Bus-bar faults**.

7.3 Unsymmetrical Faults ★★★

- Unsymmetrical (shunt) faults: **LG, LL, LLG**. Series (open-conductor) faults: one or two conductors open.
- Analysis connects the three **sequence networks** at the fault point. E = pre-fault phase voltage (positive-sequence Thevenin emf), Z_f = fault impedance.

Fault	Fault conditions	Connection of sequence networks	Fault current
LG (phase a – ground)	$V_a = Z_f I_a$; $I_b = I_c = 0$	**All three in series** (+ $3Z_f$). $I_{a1} = I_{a2} = I_{a0}$	$\mathbf{I_f = 3E/(Z_1 + Z_2 + Z_0 + 3Z_f)}$
LL (b–c)	$I_a = 0$; $I_b = -I_c$; $V_b - V_c = Z_f I_b$	**Positive and negative in parallel (opposition); no zero sequence**. $I_{a1} = -I_{a2}$	$I_{a1} = E/(Z_1 + Z_2 + Z_f)$; $\mathbf{\lvert I_f \rvert = \sqrt{3}E/(Z_1 + Z_2 + Z_f)}$
LLG (b–c–g)	$I_a = 0$; $V_b = V_c = Z_f(I_b + I_c)$	**All three in parallel**: Z_1 in series with $[Z_2 \parallel (Z_0 + 3Z_f)]$	$I_{a1} = E/[Z_1 + Z_2(Z_0+3Z_f)/(Z_2+Z_0+3Z_f)]$; $I_n = 3I_{a0}$
LLL	Balanced	Positive sequence only	$I_f = E/(Z_1 + Z_f)$

KEY FORMULAE

- LLG current division: $I_{a2} = -I_{a1}(Z_0+3Z_f)/(Z_2+Z_0+3Z_f)$; $I_{a0} = -I_{a1}Z_2/(Z_2+Z_0+3Z_f)$
- LL fault: $I_b = -I_c = \mathbf{-j\sqrt{3}\ I_{a1}}$
- Ratio (bolted, $Z_1 = Z_2$): $\mathbf{I_{LG}/I_{3\varphi} = 3Z_1/(2Z_1 + Z_0)}$, which is **> 1 if $Z_0 < Z_1$**
- $\mathbf{I_{LL}/I_{3\varphi} = \sqrt{3}/2 \approx 0.866}$ (when $Z_1 = Z_2$)
- Neutral reactance that makes LG fault current = 3-ph fault current: $\mathbf{X_n = (X_1 - X_0)/3}$
- Example: E = 1 pu, $X_1 = X_2 = 0.2$, $X_0 = 0.05$ (solidly grounded) → $I_{LG} = 3/0.45 =$ **6.67 pu**, $I_{3\varphi} = 1/0.2 =$ **5 pu**, $I_{LL} = \sqrt{3}/0.4 =$ **4.33 pu**

- For a **solidly grounded generator**, the usual severity order is **LG > LLL > LL** (Z_0 small). Standards require the generator neutral to be grounded through impedance so that $\mathbf{I_{LG} \le I_{3\varphi}}$.
- For a fault far out on a line (where $Z_0 > Z_1$), **LLL is the most severe**.
- If the neutral is **ungrounded**, $Z_0 = \infty$, so **LG fault current = 0** (apart from capacitive current) and an LLG fault behaves like an LL fault.
- In an LG fault on an ungrounded system, the **healthy-phase voltages rise to $\sqrt{3}$ × phase voltage** (line voltage). The neutral shifts to the faulted-phase potential.

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Neutral grounding

- Advantages of grounding: **arcing grounds are eliminated**, healthy phases stay near phase voltage, earth-fault relaying works, insulation can be graded (reduced BIL), and the voltage stresses from lightning and switching are lower.
- **Arcing ground** (ungrounded system): an intermittent arc caused by charging current. It produces overvoltages up to **5–6 × normal** and is cured by grounding the neutral.
- Ground at **each voltage level**, at the **source (generator/transformer) neutral**, not at the load end. Each generator neutral is usually grounded separately through its own impedance, with one grounded at a time in a parallel group.

Method	Typical application	Features
Solid (effective) grounding	**LV (< 600 V / 415 V)** and **EHV (≥ 66–132 kV and above)**	Highest fault current. Allows reduced insulation (BIL) and cheaper arresters (80% rating).
Resistance grounding	**3.3–33 kV**, generators	Limits fault current and damps arcing grounds. Improves stability because the resistor absorbs power during the fault. Fault current is in phase with voltage.
Reactance grounding	Where capacitance current is large (cables), synchronous motors and condensers	Fault current lags by ≈ 90°. X_0/X_1 is kept low (≤ 10) to limit transient overvoltages.
Resonant grounding – Petersen coil (arc suppression coil / earth-fault neutraliser)	Overhead lines (~33 kV class) with frequent **transient** earth faults	Inductive current I_L cancels capacitive current I_C, so the arc self-extinguishes and the system can run with a sustained LG fault for a while
Grounding transformer (zigzag or Y–Δ)	Systems without an accessible neutral (Δ)	Low Z_0; carries current only during earth faults
Voltage-transformer grounding	Generator in unit scheme (via distribution transformer + resistor)	Very high effective resistance. Used for stator earth-fault detection.

KEY FORMULAE

- Capacitive fault current in an ungrounded system (LG): $\mathbf{I_C = 3\,V_{ph}\,\omega C}$ = 3 × normal per-phase charging current (C = phase-to-ground capacitance)
- Petersen coil tuning, $I_L = I_C$: $V_{ph}/(\omega L) = 3V_{ph}\omega C$, so $\mathbf{L = 1/(3\omega^2 C)}$
- kVA rating of Petersen coil = $V_{ph} \times I_L$. It is **short-time rated** (e.g. 30 s), or continuously rated if the system may run faulted for a long time.
- **Effective grounding** (IEC/IEEE): $\mathbf{X_0/X_1 \le 3}$ **and** $\mathbf{R_0/X_1 \le 1}$. The healthy-phase voltage during an LG fault then stays ≤ **80% of line voltage** (coefficient of earthing ≤ 80%, earth-fault factor ≤ 1.4).

MEMORY TRICK

"**LG – Line in Series; LL – Lose zero, Link 1 & 2; LLG – aLL in parallel**". The numerator of the LG current is **3E** (3 phases' worth), the LL current carries **√3E**, and the Petersen coil has **3 in the denominator** (L = 1/3ω²C).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Sequence networks for an LG fault are connected in? → **Series**.
- Which sequence component is absent in an LL fault? → **Zero sequence**.
- Inductance of a Petersen coil for a system with phase-to-earth capacitance C? → $\mathbf{L = 1/(3\omega^2 C)}$.
- Healthy-phase voltage during an LG fault on an isolated-neutral system? → $\mathbf{\sqrt{3}\,V_{ph}}$ **(line voltage)**.
- Arcing ground is eliminated by? → **Neutral grounding** (Petersen coil in particular).
- Ratio of LL to 3-phase fault current ($Z_1 = Z_2$)? → **0.866**.

7.4 Protective Relays – Principles ★★★

- **Protective relay** = a device that detects an abnormal condition and **closes the trip circuit of a CB**. The CB actually interrupts the current. The relay only senses and commands.
- Basic chain: **CT/PT → relay → trip coil → CB**. The trip circuit runs on a **DC station battery** (110 V / 220 V DC), so it does not depend on the faulted AC supply.
- CT secondary rating is **1 A or 5 A**. PT secondary is **110 V** (line). **Never open-circuit a CT secondary** on load: there is no back-mmf, so the core saturates and dangerous high voltage appears.
- Protection CTs (e.g. class **5P10, 5P20**, PS class) must not saturate at fault current. Metering CTs (class 0.2, 0.5, 1) **should saturate** to protect the instruments.
- **Zones of protection**: each zone covers one element (generator, transformer, bus, line). **Adjacent zones overlap** around the CB so that **no part is left unprotected**. A fault in the overlap trips CBs of both zones.
- **Primary protection** = first line of defence, operating fast within its own zone.
- **Back-up protection** operates with a time delay if the primary fails. It may be **local** (in the same station, e.g. breaker-failure protection) or **remote** (at the adjacent station). Back-up should be **independent** of the primary (separate CT/PT, relay, ideally separate trip supply).
- **Pick-up value** = minimum current at which the relay operates. **Reset (drop-off) value** = current below which it resets. **Reset ratio** = drop-off/pick-up. It is ≈ 1 for static/numerical relays and lower for electromagnetic ones.
- **Burden** = VA consumed by the relay at rated current. Electromagnetic relays take a few VA. Static and numerical relays take **< 0.5 VA**.
- Relay operating time = a few cycles. Total fault clearing time = relay time + CB time (modern EHV = **2–3 cycles CB**, total ≈ 80–100 ms).

Requirement	Meaning
Selectivity (discrimination)	Isolate **only the faulty section**, with minimum disconnection
Speed	Clear fault fast to reduce damage and keep **stability**
Sensitivity	Operate on **minimum fault current** (low VA burden to operate)
Reliability (dependability + security)	Operates when it must and does not maloperate
Stability	Remains **inoperative for through (external) faults**, power swings and load
Simplicity and economy	Minimum equipment and cost consistent with the above

Relay type	Principle / features	Use
Electromagnetic attraction (plunger, hinged armature, balanced beam)	Force ∝ I^2. Works on **AC and DC**. **Instantaneous**. AC types need a shading ring to stop chatter (force pulsates at 2f).	Instantaneous OC, auxiliary relays
Induction disc / cup	Torque ∝ $\varphi_1\varphi_2 \sin\theta$ (two fluxes displaced in time and space). **AC only**. Shaded-pole or watt-hour (wattmetric) structure. Induction cup is fastest, with low inertia and high torque/inertia ratio.	Disc: **IDMT OC/EF**. Cup: directional, distance.
Thermal	Bimetal strip heated by I^2R	Motor overload
Static (solid-state)	Transistors/ICs, **no moving parts**, low burden, fast. Sensitive to **temperature and voltage spikes**, and needs an auxiliary supply.	1970s–90s
Numerical (microprocessor)	Samples and digitises signals (ADC, DSP algorithms). **Multi-function**, self-monitoring, event/fault recording, communication (IEC 61850). Settings in software.	Present standard

Over-current relays

- **Instantaneous OC relay** (ANSI 50): no intentional delay (< 0.1 s). Discriminates by **current magnitude**, so it works only where fault current varies a lot with location.
- **Definite time OC (DTOC)**: operates after a set time whatever the current magnitude above pick-up.
- **Inverse time OC** (ANSI 51): the higher the current, the shorter the time. **IDMT** = Inverse Definite Minimum Time. It is inverse at low multiples and

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tends to a **definite minimum time at high PSM (≈ 20)** because of core saturation.

- **Plug setting (current setting)**: OC relay **50–200%** of rated (steps of 25%). Earth-fault relay **10–70% or 20–80%**.
- **Time multiplier setting (TMS)**: **0.05 (or 0.1) to 1.0**. It scales the operating time and changes the starting position of the disc (backstop).
- **Plug setting multiplier (PSM)** = fault current in relay coil ÷ relay pick-up current.
- Earth-fault relays are fed from the **residual connection** of 3 CTs ($I_{res} = 3I_0$) or from a **core-balance CT (CBCT)**.

KEY FORMULAE

- **PSM = $I_{fault\ (primary)}$ / (CT ratio × relay current setting)** = I_f /(CT ratio × rated sec. current × PS)
- Actual operating time = (time from curve at that PSM, for TMS = 1) × **TMS**
- IEC 60255 / BS 142 curves: **$t = TMS \times k/(PSM^{\alpha} - 1)$**
- Example: CT 400/5, PS 125% (6.25 A), fault 5000 A → relay current = 62.5 A, PSM = **10**; normal-inverse t (TMS 1) = $0.14/(10^{0.02}-1) \approx$ **2.97 s**; with TMS 0.2 → **0.59 s**

IEC curve	Formula (TMS = 1)	t at PSM 10	Use
Normal (standard) inverse (3/10 s curve)	**$0.14/(PSM^{0.02} - 1)$**	**≈ 3 s**	General; fault level varies widely with generation
Very inverse	**$13.5/(PSM - 1)$**	**1.5 s**	Fault current drops sharply with distance from source
Extremely inverse	**$80/(PSM^2 - 1)$**	**≈ 0.81 s**	Grading with **fuses**, cold-load pick-up, distribution feeders (≈ I^2t)
Long-time inverse	$120/(PSM - 1)$	13.3 s	Overload / earth-fault back-up

Directional relays and grading

- **Directional relay** (ANSI 67): operates only for power/current flow in a chosen direction. It needs a **voltage (polarising) quantity** as reference, and the current coil works with the voltage coil (induction cup / wattmetric).
- Torque $T = K\,V\,I\cos(\theta - \tau)$, where τ = **maximum torque angle (MTA)**. Common connection: **90° (quadrature) connection**, e.g. I_a with V_{bc}, with MTA 30° or 45°.
- **Directional OC relays are needed** on **parallel feeders** (at the receiving end) and on **ring mains**, where fault current can flow either way.
- **Voltage collapse** during a close-in 3-ph fault can stop a directional relay operating. Memory action (in static/numerical relays) solves this.

Grading method	Basis	Drawback
Time grading (definite time)	Relay nearest the fault has the shortest time. Each upstream relay adds ≈ **0.3–0.5 s** (grading margin ≈ 0.4 s).	The fault **nearest the source**, which has the highest current, is cleared in the **longest time**
Current grading (instantaneous)	Fault current falls with distance; settings are made accordingly	Poor discrimination where the line impedance between relays is small. Fault level varies with generation.
Time + current (IDMT)	Inverse characteristic: high current gives faster tripping	Most widely used for radial feeders

- Grading margin covers CB interrupting time (≈ 0.1 s) + relay overshoot (≈ 0.05 s) + errors + safety margin, giving ≈ **0.4 s** for electromechanical relays and ≈ **0.25–0.3 s** for numerical relays.
- In radial feeders, grading starts from the **farthest (load-end) relay**, which gets the lowest TMS.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Operating time of a standard IDMT relay at PSM 10, TMS 1? → **≈ 3 s**.
- Relay suitable for grading with fuses? → **Extremely inverse OC**.
- Induction-type relays work on? → **AC only**. Attracted-armature relays work on **both AC and DC**.
- Why do protective zones overlap? → **So that no part (especially the CB) is left unprotected**.
- Property that keeps a relay from tripping for through faults? → **Stability**.
- Directional OC relays are essential for? → **Parallel feeders and ring mains**.

7.5 Differential & Distance Protection ★★★

Differential protection

- **Principle**: compares current entering and leaving a zone (Kirchhoff). In healthy or through-fault conditions the difference is ≈ 0. For an internal fault the difference current flows in the operating coil. It is **unit protection**: it responds only to faults **inside its zone** and is inherently selective and fast.
- **Merz–Price circulating current scheme**: identical CTs at both ends, secondaries connected so that current **circulates** through the pilots. The relay sits at the **equipotential (mid) point** of the pilots. Used for generators, transformers and bus-bars.
- **Balanced (opposed) voltage scheme**: CT secondaries in **series opposition**; no current flows under normal conditions. Used for feeders (e.g. **Translay**). It needs **air-cored CTs** (transactors), because CT secondaries are effectively open-circuited.
- Problems with simple differential: **CT mismatch / ratio error**, unequal **CT saturation** on heavy through faults, **pilot-cable capacitance/impedance**, **tap-changer** ratio change, and **magnetising inrush** in transformers.
- **Percentage (biased) differential relay**: operating coil carries $(I_1 - I_2)$. **Restraining (bias) coil** carries $(I_1 + I_2)/2$. It trips when $|I_1 - I_2| > K \times |I_1 + I_2|/2$. Bias (slope) K ≈ **10–40%**, often 20–30% for transformers with OLTC. Pick-up rises with through current, so the relay **stays stable on heavy through faults**.

Transformer protection

- **CT connection is opposite to the winding connection**. On the **Y (star) side, CTs are in Δ**. On the **Δ side, CTs are in Y**. This (a) corrects the **30° phase shift** and (b) **blocks zero-sequence current** on the grounded-Y side for external earth faults. Numerical relays do this in software.
- Δ-connected CTs deliver √3 × their secondary current, so their secondary rating is made **1/√3** of the other side's (e.g. 5/√3 A, i.e. CT ratio × √3) to match magnitudes. Example: 33/11 kV Δ/Y, with 11 kV CTs connected in Δ.
- **Magnetising inrush** on energisation: up to **6–10 × rated current**, worst when switched at **voltage zero** with residual flux (flux can reach $2\varphi_m + \varphi_r$). It flows in only one winding, so it looks like an internal fault to the differential relay.
- Inrush is rich in **2nd harmonic** (typically > 15–20% of the fundamental), so **2nd-harmonic restraint (blocking)** is used. **Over-excitation (V/f)** produces **5th harmonic**, which gives 5th-harmonic restraint.
- **Restricted earth fault (REF)** protection covers earth faults in the star winding, including faults near the neutral that the overall differential cannot detect.
- Earth faults **close to the star point** produce little fault current. About **10–20% of the winding near the neutral** remains unprotected with resistance earthing.
- Other transformer protection: OC/EF back-up, **overfluxing (V/f) relay**, **winding and oil temperature indicators (WTI/OTI)**, pressure relief device (PRD), oil-level indicator, and for OLTC a separate oil-surge relay.

Buchholz relay – feature	Fact
Type	**Gas-actuated** relay for **oil-immersed transformers with a conservator**
Location	In the **pipe between main tank and conservator**. The pipe rises slightly towards the conservator.
Detects	**Incipient (slow-developing) faults**: insulation failure, core heating, inter-turn faults, loss of oil. Severe internal faults cause an oil surge.
Upper float	Gas collects slowly, giving an **alarm**

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Lower float / flap	Sudden **oil surge**, giving a **trip**
Used for	Transformers rated **above ~500 kVA** (economic limit; some texts say 750 kVA)
Limitation	Relatively slow. Does not detect faults **outside the tank** (bushings, cables). Cannot be used on sealed/conservatorless units.
Gas analysis	H_2 / C_2H_2 (acetylene) → **arcing**. CO/CO_2 → **paper/cellulose** overheating. CH_4, C_2H_4 → oil overheating (DGA – IS 10593/IEC 60599).

Generator (alternator) protection

Abnormality	Protection	Note
Stator phase-to-phase fault	**Merz–Price (biased) differential** (87G)	Fast, unit protection
Stator earth fault	Neutral grounded through **distribution transformer + resistor**, with OV/OC relay in secondary	Covers ~**95%** of winding. **100%** stator EF uses **3rd-harmonic** voltage or low-frequency injection.
Rotor (field) earth fault	DC injection or potentiometer method	Field is **unearthed**, so the **first earth fault is harmless**. A **second fault** shorts part of the field and causes **unbalanced magnetic pull and severe vibration**.
Loss of excitation (field failure)	**Offset mho relay** (40) in the 3rd/4th quadrant of the R–X plane	Machine runs as an **induction generator**, drawing reactive power. Rotor overheats from slip-frequency currents.
Reverse power (motoring)	Reverse power relay (32)	Protects the **prime mover**, not the generator. Steam turbine blades overheat. Setting ≈ 0.5–3% (steam), much higher for diesel (≈ 5–25%).
Unbalanced loading	**Negative-sequence relay** (46)	I_2 induces **double-frequency (100 Hz) currents in the rotor**, causing rotor heating. Characteristic **I_2^2t = K** (small K for large turbo-alternators).
Others	Over-speed, over-voltage, over-fluxing, inter-turn (split-phase / transverse), pole slipping, under-frequency, bearing temperature	Inter-turn faults are not seen by the differential relay

Bus-bar protection

- Bus faults are rare but very severe, and a whole station may be lost. Most bus faults are **phase-to-earth**.
- **High-impedance circulating-current differential**: the most common method. It is stable against CT saturation on through faults, with a **stabilising resistor** and a **Metrosil (non-linear resistor)** to limit voltage.
- **Frame-leakage (fault-bus) protection** is for metal-clad switchgear: the frame is insulated from earth and a CT is placed in the earth connection.
- **Low-impedance biased differential** (numerical) suits complex multi-section bus-bars. Breaker-failure protection (LBB) is integrated with bus protection.

Distance protection

- **Distance (impedance) relay** (ANSI 21) measures **Z = V/I** at the relay point. Z is proportional to the distance to the fault. It operates when the measured Z < set Z (the **reach**). It is used on **transmission lines** (non-unit protection).
- It is **independent of source impedance/fault level**, which is its advantage over OC relays.
- **Under-reach**: the relay sees the fault as farther away (arc resistance, infeed). **Over-reach**: it sees the fault as closer (DC offset transient, outfeed, CVT transient).
- **Power swing**: the apparent impedance moves through the characteristic during stable swings and can cause false trips. **Power-swing blocking (PSB)** uses a blinder or an outer zone timer.

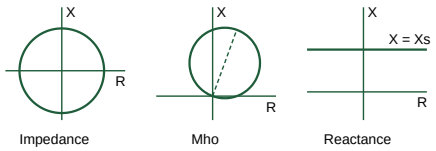

Fig. 7.1 Operating characteristics on the R–X plane. The relay trips for impedances inside the circle, or below the line for the reactance relay. The impedance circle is centred at the origin, the mho circle passes through the origin, and the reactance characteristic is a straight line parallel to the R-axis.

Relay	R–X characteristic	Directional?	Arc resistance	Power swing	Best for
Impedance (plain)	Circle **centred at origin**	**No** (needs a separate directional unit)	Moderately affected	Moderately affected	**Medium** lines
Reactance	**Straight line parallel to R-axis** (X = const)	No (needs mho starter)	**Unaffected** (measures X only)	**Most affected**	**Short lines** (arc R is large compared with line Z)
Mho (admittance / angle-impedance)	Circle **passing through origin**, diameter along line angle	**Inherently directional**	**Most affected**	**Least affected** (occupies the least R–X area)	**Long EHV lines**; also the **starter** for reactance relays
Quadrilateral (numerical)	Polygon; reach set independently on R and X	Yes	Good resistive coverage	Blinders limit it	Short lines, earth faults
Offset mho	Mho circle shifted to include origin	Partly	—	—	Zone 3, **loss of excitation**

Zone (stepped distance)	Reach	Time
Zone 1	**80–90%** of protected line (not 100%, to avoid over-reach from CT/PT/relay errors)	**Instantaneous**
Zone 2	100% of own line + **20–50%** of the next (shortest) line, i.e. **≈ 120–150%**	**≈ 0.3–0.5 s**
Zone 3	Own line + entire next line + 20–25% of the third. Remote back-up, ≈ 200%+.	**≈ 0.6–1.0 s**
Reverse zone (some schemes)	Behind the relay	Back-up for bus-bar

Pilot / carrier protection

- **Pilot** = communication channel between the two ends of a line, so faults on **100% of the line** are cleared instantly at both ends. Channels: **pilot wire** (short lines, < ~15–25 km), **power-line carrier (PLC)**, microwave, **optical fibre (OPGW)**.
- **PLCC frequency: 50–500 kHz**. Equipment: **wave (line) trap** (a parallel LC tuned to the carrier frequency, giving high Z to carrier and low Z to 50 Hz), **coupling capacitor** (passes carrier, blocks 50 Hz), and line-matching unit.
- Schemes: **phase comparison** (compares phase of currents at the two ends), **directional comparison**, **carrier-aided distance** (permissive under-reach / over-reach transfer trip – PUTT/POTT, and blocking schemes), **carrier intertripping**.

MEMORY TRICK

"**Short – Reactance; Medium – Impedance; Long – Mho**" (SRI-ML). **Mho** is "**direction-wise and swing-proof**" but "**arc-shy**". Reactance is "**arc-proof but swing-shy**". Zones run **80 → 120 → 200** with times **0 → 0.3 → 1 s**.

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- Inrush current of a transformer is rich in? → **2nd harmonic**.
- CTs on the star side of a Y–Δ transformer under differential protection are connected in? → **Delta**.
- Distance relay least affected by power swings and inherently directional? → **Mho relay**.
- Distance relay best suited for short lines? → **Reactance relay**.
- Buchholz relay protects against? → **Incipient internal faults in oil-immersed transformers**.
- Negative-sequence relay on an alternator protects against? → **Rotor overheating from unbalanced loading**.

7.6 Circuit Breakers & Fuses ★★★

Arc phenomenon

- When the contacts separate, an **arc** forms. It is ionised gas kept conducting by **thermal ionisation** and **field (high-voltage-gradient) emission**. The arc keeps the circuit closed until it is extinguished.
- Factors that maintain the arc: **potential difference between contacts** and **ionised particles between contacts**.
- **Arc voltage** is in phase with current and nearly constant (resistive). The arc is **easier to extinguish in AC** because current passes through **natural zero 100 times/s** at 50 Hz.
- Deionisation is by **lengthening, cooling, splitting the arc, and blasting** fresh dielectric through it.

Arc interruption method	Principle	Used in
High-resistance method	Increase arc resistance with time (lengthen, cool, split the arc, arc chutes) until current falls too low to sustain it. **Large energy** dissipated in the arc.	**DC CBs**, low-capacity AC CBs (air-break)
Low-resistance / current-zero method	Keep arc resistance low until **natural current zero**, then rapidly build up dielectric strength to prevent restrike	**All HV AC CBs** (oil, ABCB, SF_6, vacuum)

- Theories of current-zero interruption: **Recovery rate theory (Slepian)**: the arc is extinguished if the **rate of dielectric strength recovery > rate of rise of restriking voltage** (a "race"). **Energy balance theory (Cassie)**: the arc is extinguished if the heat lost exceeds the heat generated after current zero. (Mayr's model applies near current zero.)
- **Restriking voltage (TRV)** = the **transient** high-frequency voltage across the contacts **immediately after arc extinction**. Its natural frequency is $f_n = 1/(2\pi\sqrt{(LC)})$.
- **Recovery voltage** = the **power-frequency (50 Hz) rms voltage** across the contacts after the transient has died out.
- **RRRV (rate of rise of restriking voltage)**, in **kV/µs**, is the key factor deciding whether the arc restrikes. A high RRRV means harder interruption.
- Restriking voltage is **highest at zero power factor** (fault current lags by 90°), because current zero then coincides with **voltage maximum**. That is why SC (inductive) currents are hard to break.
- **First pole to clear factor** = **1.5** for an ungrounded 3-ph fault and 1.3 for an effectively grounded system.

KEY FORMULAE

- Restriking voltage (no damping): $v = V_m(1 - \cos \omega_n t)$, with $\omega_n = 1/\sqrt{(LC)}$. **Peak = $2V_m$** at $t = \pi\sqrt{(LC)}$.
- **Maximum RRRV = $V_m/\sqrt{(LC)} = \omega_n V_m$** (at $t = \pi/2\omega_n$). **Average RRRV** = $2V_m/(\pi\sqrt{(LC)})$.
- Current chopping voltage: $\frac{1}{2}Li^2 = \frac{1}{2}Cv^2$, so **$v = i\sqrt{(L/C)}$** (i = chopped current)
- Resistance switching, critical damping: **$R = 0.5\sqrt{(L/C)}$**. For R < this, the TRV is oscillation-free.
- Breaking capacity (sym.) = **$\sqrt{3} \times V_L \times I_{sym,rms}$** MVA
- Asymmetrical breaking current = **$\sqrt{(I_{ac}^2 + I_{dc}^2)}$**
- **Making current (peak) = 2.55 × symmetrical breaking current (rms)** = √2 × 1.8 (1.8 allows for doubling with decay)

- **Current chopping** = forced interruption of **small inductive currents** before natural zero (e.g. **unloaded transformer** magnetising current, reactors). It happens in CBs with strong de-ionising power that is independent of current: **ABCB** (and vacuum, slightly). The energy stored in L transfers to C, which produces **high overvoltage**. Remedy: **resistance switching** or non-linear resistors across the contacts.
- **Capacitive current switching** (unloaded lines, capacitor banks): after interruption the line stays charged at V_m. Half a cycle later the contact voltage is $2V_m$, and a restrike can raise the line to **$3V_m$**, then $5V_m$ and so on.
- **Resistance switching** = a shunt resistor across the contacts/arc. It reduces RRRV, damps the TRV, limits chopping overvoltages, and shares voltage between multi-break contacts. Used in ABCBs.
- Multi-break CBs use **grading capacitors** across the breaks to **equalise the voltage** between breaks.

CB type	Arc medium / principle	Voltage range	Key facts
Air-break (ACB)	Atmospheric air. **High-resistance** method using **arc runners / arc chutes** and magnetic blow-out.	**LV (≤ 1 kV)**, older up to 11 kV	Indoor LV switchboards; DC CBs
Bulk oil (BOCB)	Transformer oil. Arc decomposes oil into gas, mostly **hydrogen (~70%)**, which has high thermal conductivity and cools the arc.	Up to **33 kV** (older to 132 kV)	Large oil volume, so **fire/explosion risk**; oil also insulates from earth. Plain-break and controlled (explosion pot) types.
Minimum oil (MOCB)	Small oil volume, used **only for arc extinction**. Insulation to earth by **porcelain/solid**.	**3.3–132 kV** (up to 220 kV earlier)	Less oil, less fire risk, smaller size. Oil deteriorates fast and needs frequent maintenance.
Air-blast (ABCB)	Compressed air at **20–30 kg/cm²** (axial or cross blast)	**132–400 kV+**	**Very fast** (2–3 cycles), suits frequent operation, no fire risk. Downsides: **current chopping**, noisy, compressor plant needed.
SF_6	**Sulphur hexafluoride**, puffer type at ≈ 5–6 bar	**33 kV to 765 kV+** (also 11 kV RMUs), GIS	See SF_6 facts below. **Most used for EHV**.
Vacuum (VCB)	Vacuum **10^{-5}–10^{-7} torr**. Arc is a **metal-vapour** arc from the contacts, which condenses at current zero.	**3.3–33 kV** (36 kV); 66–132 kV rare	**Fastest dielectric recovery**, long life, maintenance-free, compact, silent, no fire risk. Contacts **Cu–Cr**. **Best for distribution (11/33 kV)**.

- **SF_6 properties**: colourless, odourless, non-toxic, non-inflammable, chemically inert. **Electronegative**: it captures free electrons to form heavy negative ions, which makes the arc deionise fast.
- SF_6 dielectric strength is ≈ **2.5 (2–3) × air** at atmospheric pressure. At ≈ 3 bar it is comparable to transformer oil.
- SF_6 is **≈ 5 times heavier than air** (molar mass 146) and has excellent thermal conductivity / heat transfer. Arc time constant is very short, so RRRV capability is high.
- Arcing produces **metal fluorides** (white powder) and lower fluorides (SF_4, S_2F_{10}), which are **toxic** and attack moisture-sensitive parts. Gas must be kept dry (moisture filters).
- SF_6 liquefies at high pressure / low temperature (heaters needed in cold climates). It is a potent **greenhouse gas (GWP ≈ 23,500)**, so leakage is controlled.
- Vacuum vs SF_6: vacuum is preferred at MV (≤ 33 kV) and SF_6 at HV/EHV. In vacuum the dielectric strength recovers to ≈ kV/µs within microseconds after current zero.

Rating (IS/IEC 62271-100)	Definition
Symmetrical breaking capacity	rms value of AC component of current at contact separation; MVA = √3 V I
Asymmetrical breaking current	rms of total current (AC + DC) at contact separation
Making capacity	Peak of the first major loop of current when closing onto a fault = **2.55 × sym. breaking (rms)** (2.5 × at 50 Hz in IEC 62271 for τ = 45 ms)

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Short-time rating	rms current the CB carries in closed position for a short time without damage: **1 s or 3 s**; limited by thermal and electromagnetic stresses
Rated operating sequence (duty cycle)	**O – t – CO – t′ – CO**; t = 3 min, or **0.3 s** for CBs for rapid auto-reclosing; t′ = 3 min
Rated voltage / frequency / normal current	e.g. 12 kV, 36 kV, 145 kV, 245 kV, 420 kV (highest system voltage)

- **Auto-reclosing**: about **80–90% of faults on overhead lines are transient** (lightning flashover, birds, branches), so the CB recloses automatically after a **dead time**, which must exceed the arc de-ionisation time (≈ **0.3–0.5 s** on EHV).
- EHV lines use **single-shot** auto-reclose, often **single-pole**. Distribution feeders use multi-shot (up to 3–4) reclosers. **Not used on cables or transformers**, because their faults are permanent.
- **Isolator (disconnector)** = an **off-load** device for isolation and visible break. Sequence: **open CB first, then isolator**; for closing, isolator first, then CB. An **earth switch** is interlocked with the isolator.

LV device	Function / key value
MCB	Thermal (bimetal → overload) + magnetic (solenoid → short circuit) trip. Rated up to **100 A (125 A)**, breaking capacity typically **3–10 kA** (IS/IEC 60898). Curves: **B (3–5 I_n)** lighting; **C (5–10 I_n)** general/motors; **D (10–20 I_n)** high inrush.
MCCB	Moulded case CB, higher currents (**~up to 1600–2500 A**), adjustable trip, breaking capacity up to ~100 kA (IEC 60947-2)
ELCB (voltage-operated)	Older type; senses voltage on the earth wire. Now obsolete.
RCCB / RCD (current-operated)	Core-balance CT senses the **residual current ($I_{phase} - I_{neutral}$). 30 mA** for personal protection (shock); **100 mA / 300 mA** for equipment/fire protection. Trips within **0.3 s** at $I_{\Delta n}$ (≈ 40 ms at $5I_{\Delta n}$). **Does not protect against overload/SC** (RCBO = RCCB + MCB).

Fuses

- The fuse is the **cheapest** protective device. It acts both as the **detector and the interrupter** but must be **replaced** after operation. It has an **inverse time–current** characteristic.
- **Fusing factor = minimum fusing current / current rating**, always **> 1**. Rewirable (semi-enclosed) ≈ **2**; HRC ≈ **1.1–1.25**, the lowest, i.e. closest protection.
- **Minimum fusing current** = the current at which the element melts in ≈ infinite time (4 h test).
- Element materials: **silver** (HRC – no oxidation, high conductivity), **copper**, **tin–lead alloy (63:37)** for small currents. Desirable: **low melting point, high conductivity**, low cost, freedom from oxidation.
- **HRC (High Rupturing Capacity) cartridge fuse**: ceramic/steatite body filled with **quartz sand (silica)**. Silica + silver vapour forms a high-resistance glassy substance that quenches the arc. Breaking capacity is very high (≈ **80–120 kA**). It is **current-limiting**: it interrupts before the first peak.
- **Cut-off current** = the instantaneous current at which the element melts (lower than the prospective peak). Total operating time = **pre-arcing (melting) time + arcing time**.
- **I^2t** (pre-arcing and total) measures the let-through energy and is used for **discrimination**: a minor (downstream) fuse's total I^2t must be < the major (upstream) fuse's pre-arcing I^2t. A size ratio of ≈ **2:1** usually gives discrimination.
- Other types: **expulsion / drop-out (DO) fuse** on DT poles (11 kV), **semiconductor (ultra-fast)** fuses for thyristors (I^2t matched), and striker-pin HRC fuses in RMUs.

Lightning & surge protection, insulation coordination

- **Overvoltages**: external (**lightning**, the most severe in magnitude) and internal (**switching**, arcing ground, resonance, Ferranti effect). In EHV/UHV (≥ 400 kV) systems, **switching surges** decide the insulation design. Below that, lightning decides it.
- **Standard lightning impulse 1.2/50 µs** (front/tail). Switching impulse **250/2500 µs**.
- **Lightning arrester (surge arrester)** = non-linear resistor connected **phase-to-earth** as close as possible to the equipment, mainly the **transformer**. It offers very high resistance at normal voltage and low resistance at surge voltage, so it diverts surge current to earth.
- Types: **rod gap, horn gap, expulsion type, valve type (SiC + series gaps), metal oxide (ZnO) gapless**.
- **Metal oxide (ZnO) arrester**: highly non-linear (V–I exponent α ≈ **25–50**, compared with 2–6 for SiC). Negligible leakage at normal voltage, so **no series spark gaps** are needed. Faster response, better protective level and energy absorption. It is the **present standard**.
- Arrester rating: ≈ **80% of highest line voltage** for effectively earthed systems ("80% arrester"), **100%** for isolated/non-effectively earthed systems. Examples for 11 kV: 9 kV (80%) or 12 kV (100%).
- **Surge absorber** (e.g. Ferranti absorber, series inductor + parallel R/C) **reduces the steepness** (wave-front) of a surge and protects machine windings. **Surge capacitors** are used at rotating machines.
- **Ground (earth / shield) wire**: on the tower top, it shields phase conductors from direct strokes. **Shielding (protective) angle ≈ 30°** (20–45° in practice; ~20° or less for EHV and two earth wires). **Tower footing resistance** should be low, **< 10 Ω**, to avoid **back-flashover**. Counterpoise wires are used where soil resistivity is high.
- **BIL (Basic Impulse Insulation Level)** = crest value of standard lightning impulse that the insulation withstands.
- **Insulation coordination**: arrange equipment insulation so that the **protective level of the LA is below the BIL** of equipment by a safety margin ≈ **20–25%**. Insulation in the station is graded: **line insulation > bus / switchgear > transformer**, and the LA is set lowest so that any flashover occurs at a cheap, self-restoring place.
- **Volt–time curve**: insulation breakdown voltage against time to breakdown. The LA curve must lie **below** the equipment curve at all times.

System voltage (kV rms)	11	33	66	132	220	400
Typical BIL (kV peak)	75	170	325	550 / 650	950 / 1050	1425 / 1550

MEMORY TRICK

CB by voltage, "**Air – Vacuum – Oil – SF_6 – Blast**": LV **Air** → MV **Vacuum** (≤ 33 kV) → **Oil** (MOCB ≤ 132 kV) → **SF_6** (33–765 kV) → **ABCB** (EHV, older). For making current, "**2.55 = √2 × 1.8**". **Chopping** means a **small inductive current** in an **ABCB**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Making capacity of a CB with symmetrical breaking current 20 kA? → **2.55 × 20 = 51 kA (peak)**.
- Current chopping is most pronounced in? → **Air-blast CB**, when interrupting **small inductive currents**.
- SF_6 is a good arc quencher because it is? → **Electronegative**.
- CB best suited for 11/33 kV distribution? → **Vacuum CB**.
- Restriking voltage is the? → **Transient voltage across the contacts just after arc extinction**; RRRV unit = **kV/µs**.
- Fusing factor of an HRC fuse? → **≈ 1.1** (always > 1).

7.7 Power System Stability ★★★

- **Stability** = the ability of a power system to stay in **synchronism** (rotor angle stability) and return to a steady operating state after a disturbance.
- **Steady-state stability**: response to **small, gradual** load changes. **Steady-state stability limit** = maximum power that can flow without losing synchronism under gradual change, at **δ = 90°** (round rotor, R = 0).
- **Transient stability**: response to **large, sudden** disturbances (faults, line switching, loss of generation). It is judged over the **first swing (≈ 1 s)**. The transient stability limit is **below** the steady-state limit.
- **Dynamic (small-signal) stability**: stability over a longer period (several seconds), **including the effect of controllers** (AVR, governor). Fast AVRs can introduce **negative damping**, which leads to low-frequency oscillations (0.2–2 Hz). A **Power System Stabiliser (PSS)** adds damping.
- Other classes (IEEE/CIGRE): **voltage stability** (reactive power deficit, voltage collapse) and **frequency stability**.
- Assumptions in classical transient stability analysis: **R neglected**, **mechanical input P_m constant** (governor action slow), machine represented by **constant**

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E′ behind X_d', rotor angle δ coincides with E′, loads as constant impedances, and damping neglected.

KEY FORMULAE

- **Power–angle equation**: $P_e = (EV/X) \sin \delta$; $\mathbf{P_{max} = EV/X}$ (steady-state stability limit). $Q = (EV \cos\delta - V^2)/X$.
- **Swing equation**: $\mathbf{M\, d^2\delta/dt^2 = P_m - P_e = P_a}$ (accelerating power; damping neglected)
- In pu: $\mathbf{(2H/\omega_s)\, d^2\delta/dt^2 = P_m - P_e}$, or $(H/\pi f)\, d^2\delta/dt^2 = P_m - P_e$ (δ in electrical rad)
- **Inertia constant H = stored kinetic energy at synchronous speed (MJ) / machine rating (MVA)** = $\frac{1}{2} J\omega_{sm}^2/S$, unit **MJ/MVA (seconds)**
- **M = GH/(πf)** MJ-s/elec-rad, or **M = GH/(180f)** MJ-s/elec-degree (G = MVA rating). M is the angular momentum at synchronous speed.
- Change of base: $\mathbf{H_{new} = H_{old} \times (S_{machine}/S_{base})}$. Coherent machines in parallel: $H_{eq} = \Sigma\, H_i S_i / S_{base}$.
- **Synchronising power coefficient** $P_s = dP/d\delta = (EV/X) \cos\delta$. $P_s > 0$ means stable; it is **zero at δ = 90°**.
- Natural frequency of small oscillations: $\mathbf{\omega_n = \sqrt{(\omega_s P_s/2H)}}$ rad/s (P_s in pu)
- Steady-state stability margin = $(P_{max} - P_0)/P_{max}$

Machine	Typical H (MJ/MVA)
Turbo-alternator (thermal, 2-pole/4-pole)	**≈ 4–9** (high speed, so high KE)
Hydro-generator (water-wheel)	**≈ 2–4**
Synchronous condenser	≈ 1–1.25 (large: ~2)
Synchronous motor	≈ 2

- H varies over a **narrow range** across machine sizes (unlike J), which is why it is used in studies.
- A machine with a **larger H** (larger M) accelerates more slowly during a fault, so it has a **longer critical clearing time** and better transient stability.

Equal area criterion (single machine – infinite bus)

- It is applicable to a **single machine connected to an infinite bus** (or two machines). It needs **no solution of the swing equation**. For multi-machine systems, numerical step-by-step / Runge-Kutta methods are used.
- Statement: the system is stable if the **accelerating area A_1 ($P_m > P_e$) = decelerating area A_2 ($P_e > P_m$)** before δ reaches δ_{max}, where the post-fault curve crosses P_m again.
- The rotor speed deviation is **zero at δ_{max}**, where the swing reverses.
- **Critical clearing angle δ_c** = the largest angle at which the fault can be cleared and the system still be stable ($A_1 = A_{2,max}$). The **critical clearing time t_c** is the corresponding time.
- Sudden increase of input from 0 for a lossless machine on an infinite bus: the maximum shaft power that can be applied suddenly without losing synchronism is less than P_{max} (≈ **0.725 P_{max}**, new operating angle $\delta_1 \approx 46.4°$, $\delta_{max} \approx 133.6°$).

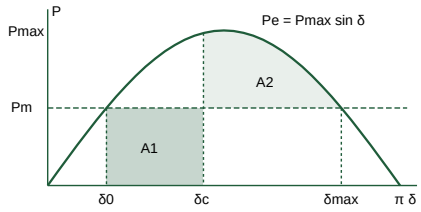

Fig. 7.2 Equal-area criterion for a 3-phase fault at the generator terminals ($P_e = 0$ during the fault), cleared at δ_c with the system restored to its pre-fault state. Drawn for $\delta_0 = 30°$: $A_1 = A_2$ gives $\delta_c \approx 79.6°$ and $\delta_{max} \approx 150°$.

KEY FORMULAE

- 3-ph fault at generator terminals ($P_e = 0$ during fault), post-fault network = pre-fault, $\delta_{max} = \pi - \delta_0$:
  $\cos\delta_c = (\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0$ (δ in radians)
- Critical clearing time for this case (δ grows as $\frac{1}{2}(P_m/M)t^2$): $\mathbf{t_c = \sqrt{[2M(\delta_c - \delta_0)/P_m]} = \sqrt{[4H(\delta_c - \delta_0)/(\omega_s P_m)]}}$
- General case (during-fault $P_{max} = r_1 P_{max}$, post-fault $= r_2 P_{max}$):
  $\cos\delta_c = [(P_m/P_{max})(\delta_{max} - \delta_0) + r_2 \cos\delta_{max} - r_1 \cos\delta_0]/(r_2 - r_1)$, with $\delta_{max} = \pi - \sin^{-1}(P_m/r_2 P_{max})$
- Check: $\delta_0 = 30° \rightarrow \cos\delta_c = (2\pi/3)(0.5) - 0.866 = 0.181$, so **$\delta_c \approx 79.6°$**

Method to improve stability	How it helps
High-speed CBs and fast relays	Fault cleared before δ_c, so A_1 is smaller. This is the **most effective** method for transient stability.
Auto-reclosing (high-speed)	Restores the transfer path quickly after transient faults
Single-pole switching / reclosing	Only the faulted phase opens; the 2 healthy phases keep transferring power during the dead time
Reduce series reactance X: parallel lines, **bundled conductors**, **series capacitor compensation**, low-X transformers	$P_{max} = EV/X$ rises
Higher transmission voltage	$P_{max} \propto V^2$, so a higher margin
Fast-acting excitation (high-response AVR) + PSS	Raises E during a fault; PSS damps oscillations
Braking resistors (dynamic braking)	Shunt resistors switched in during the fault absorb the excess generator power (reduce accelerating power)
Fast valving / turbine bypass, generator tripping	P_m reduced quickly
High-inertia machines (larger H)	Slower acceleration
HVDC links	Asynchronous tie, with fast power modulation. AC systems at both ends need not be in synchronism.
Neutral grounding through resistance; shunt compensation (SVC, STATCOM)	Resistor absorbs power during earth faults; SVC/STATCOM hold voltage

- **Series compensation** raises the stability limit. **Shunt reactors** do not raise it; they control the Ferranti rise on light load. Series compensation ≈ **25–70%** of line X. Risk: **sub-synchronous resonance (SSR)** with turbine shafts.
- For long lines the practical loading is limited by **stability**, not thermal rating. The operating δ is kept ≈ **30–45°** to leave a margin.
- **Effect of fault type on transient stability** (most to least severe): **LLL > LLG > LL > LG**, because the 3-ph fault blocks almost all power transfer. A fault at the **sending end (near generator)** is worse than one at mid-line.

MEMORY TRICK

"**Steady – Small – 90°; Transient – Big – first swing; Dynamic – with controllers**". For the swing equation remember "**M = GH/πf**" ("**G**ood **H**abits over **π f**"). For the critical angle: "**$\cos\delta_c = (\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0$**": **π minus twice**, times sine, minus cosine.

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- Unit of the inertia constant H? → **MJ/MVA (seconds)**.
- Steady-state stability limit occurs at δ = ? → **90°**; P_{max} = **EV/X**.
- Equal-area criterion is applicable to? → **A single machine connected to an infinite bus (or two-machine system)**.
- Most effective way to improve transient stability? → **High-speed fault clearing (fast CBs) and auto-reclosing**.
- Series capacitors improve stability because they? → **Reduce net line reactance, raising P_{max}**.
- A 100 MVA machine has H = 5 MJ/MVA. Its H on a 500 MVA base? → **1 MJ/MVA**.

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8. Control Systems

NOTE

Control Systems typically gives **4–6 MCQs** in PSU/AE papers. They are mostly formula-based and quick to score: the second-order time response (M_p, t_p, t_s), the steady-state error table, the Routh range of K, root locus asymptotes and centroid, gain and phase margin, lead/lag compensators, PID effects and controllability. Learn every boxed formula and the standard numericals ($K/[s(s+1)(s+2)] \rightarrow K_{max} = 6$).

8.1 Modelling & Transfer Function ★★★

- **Control system** = an arrangement of components that regulates or commands a system to get a desired output.
- **Open-loop system**: the control action does **not depend on the output**. It has no feedback. It is simple, cheap and stable, but **inaccurate**, and it cannot correct disturbances. It needs calibration.
- **Closed-loop (feedback) system**: the control action **depends on the output**, which is measured and compared with the reference to produce an **error signal**. It is accurate and less sensitive to disturbances and parameter changes, but it is complex and costly and **can become unstable** (it may oscillate).

Open-loop examples	Closed-loop examples
Traffic signal with **fixed timer**	Traffic signal with vehicle-sensing (adaptive)
Washing machine (timer based), toaster, electric hand dryer	Air-conditioner/refrigerator with **thermostat**, automatic electric iron
Immersion rod, electric bulb, fan regulator	**Automatic voltage regulator (AVR)**, servo voltage stabiliser
Field-controlled DC motor	Armature-controlled DC motor, missile/radar tracking, autopilot
Stepper motor drive (open loop)	Human body temperature control, toilet flush tank (float), water-level controller

Effects of (negative) feedback

Parameter	Effect of negative feedback
Overall gain	**Reduced** by factor (1 + GH)
Sensitivity to parameter variation (of G)	**Reduced**: S^T_G = **1/(1 + GH)**
Sensitivity to feedback element H	$S^T_H = -GH/(1 + GH) \approx$ **−1** (H must be accurate)
Bandwidth	**Increased**
Speed of response / time constant	Faster; τ reduced to τ/(1 + K)
Effect of noise/disturbance	**Reduced** (in forward path)
Non-linear distortion	Reduced
Stability	**Affected**. It may improve or worsen; high loop gain can cause instability.
Accuracy (steady-state error)	Improved

- Positive feedback gives T = G/(1 − GH). It raises the gain and is used in **oscillators** and regenerative circuits, but it is not used for control.
- Example: for G = K/(1 + sτ) with unity feedback, T = [K/(1+K)]/[1 + sτ/(1+K)]. The DC gain falls to K/(1+K), the time constant falls to **τ/(1+K)** and the bandwidth rises by (1+K).

Transfer function, poles, zeros, type, order

- **Transfer function (TF)** = Laplace transform of the output ÷ Laplace transform of the input, with **all initial conditions zero**. It applies only to **LTI** (linear time-invariant) systems.
- TF = Laplace transform of the **impulse response**. The TF is independent of the input and does not describe the internal structure (state model does).
- **Poles** = values of s that make the TF infinite (roots of the denominator). **Zeros** = values of s that make the TF zero (roots of the numerator). Pole: ×, zero: ○ on the s-plane.
- **Characteristic equation** = denominator of the closed-loop TF set to zero: **1 + G(s)H(s) = 0**. Its roots are the closed-loop poles.
- **Order** = highest power of s in the denominator (number of poles). **Type** = number of **open-loop poles at the origin** of G(s)H(s), i.e. the number of pure integrators.
- Example: $G(s)H(s) = 10(s+2)/[s^2(s+1)(s+5)] \rightarrow$ **Type 2, Order 4**.
- **Proper TF**: degree of numerator ≤ degree of denominator. Strictly proper: numerator degree < denominator degree. Physical systems are proper.

Mechanical–electrical analogies

Mechanical (translational)	Rotational	Force–Voltage (F–V, loop)	Force–Current (F–I, node)
Force F	Torque T	**Voltage V**	**Current I**
Mass M	Inertia J	**Inductance L**	**Capacitance C**
Damping (friction) B	B	**Resistance R**	**Conductance 1/R**
Spring stiffness K	K	**1/C** (elastance)	**1/L**
Displacement x	θ	Charge q	Flux linkage ψ
Velocity v	ω	Current i	Voltage v

- The F–I analogy is also called the **Mobility analogy**. It keeps the topology: masses become node-to-ground capacitors and parallel elements stay parallel. The F–V analogy is the **direct analogy**, and in it series elements become parallel and vice versa.
- Mass–spring–damper: $F = M\,d^2x/dt^2 + B\,dx/dt + Kx$, so $X(s)/F(s) = 1/(Ms^2 + Bs + K)$. Then $\omega_n = \sqrt{(K/M)}$ and $\zeta = B/(2\sqrt{(KM)})$.

Control components

Component	Key facts
Armature-controlled DC servomotor	Field current constant, control on armature voltage. Back emf gives inherent feedback, so it behaves as a **closed-loop** system. $\theta(s)/V_a(s) = K_T/[s[(R_a+sL_a)(Js+B) + K_TK_b]]$. Used for **large** power; L_a is small, so it gives faster response.
Field-controlled DC servomotor	Armature current constant, control on field. **Open-loop** (no back-emf feedback). $\theta(s)/V_f(s) = K/[s(R_f+sL_f)(Js+B)]$. Used for **small** power; L_f is large, so it is slower.
AC servomotor	**Two-phase induction motor** (reference + control winding at 90°). Rotor has **high R/X ratio**, which gives a linear torque–speed curve with negative slope (stable). It uses a drag-cup or squirrel-cage rotor with a small diameter and long length, for low inertia.
Synchro (selsyn)	A rotor with a single-phase AC winding and a 3-phase **star-connected stator** (windings 120° apart). **Synchro transmitter + control transformer = error detector**. Output ∝ $\sin(\theta_1-\theta_2) \approx$ error for small angles. In the null (electrical zero) position the CT rotor is **90° (perpendicular)** to the transmitter rotor. The output is an amplitude-modulated AC signal.
Tachogenerator	Output voltage ∝ **speed**. TF $V(s)/\theta(s) = \mathbf{K_t s}$ (it acts as a differentiator); $V(s)/\omega(s) = K_t$. Used for **velocity (derivative) feedback**, which increases damping. AC tacho: output frequency = excitation frequency.
Potentiometer	Error detector for position. A pair of pots gives $V = K_p(\theta_r - \theta_c)$. Used for DC systems.
Stepper motor	Digital actuator. Step angle = $360°/(m \cdot N_r)$, where m = number of stator phases. For the VR type this equals $(N_s - N_r)\times 360°/(N_sN_r)$. Used in **open-loop** position control (printers, CNC).

MEMORY TRICK

"Armature has Arms (feedback arms) → Closed"; "Field is Free → Open". F–V analogy: "**MBK → L R 1/C**" (Mass is Lazy L, B is Resistive). In F–I you swap: M→C, B→1/R, K→1/L.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- A transfer function is defined under which initial conditions? → **All zero**.
- In the force–current analogy, mass is analogous to? → **Capacitance**.
- Which DC servomotor behaves as a closed-loop system? → **Armature controlled** (due to back emf).
- Negative feedback, the effect on bandwidth? → **Increases**, while gain decreases.
- Which pair forms the error detector in an AC position-control system? → **Synchro transmitter + synchro control transformer**.
- The TF of a tachogenerator (output voltage vs shaft position)? → $\mathbf{K_t s}$.

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8.2 Block Diagrams & Signal Flow Graphs ★★★

- **Block diagram**: blocks (TFs), summing points, take-off (branch) points and signal arrows. It applies to **LTI** systems only.
- The canonical feedback loop has forward path G and feedback H.

KEY FORMULAE

- Closed-loop TF: C/R = **G/(1 + GH)** for negative feedback and **G/(1 − GH)** for positive feedback.
- Unity feedback (H = 1): C/R = G/(1 + G).
- Error E/R = 1/(1 + GH). Feedback signal B/R = GH/(1 + GH).
- Loop gain (open-loop TF) = **G(s)H(s)**.

Block diagram reduction rules

Operation	Result / compensation
Blocks in cascade (series)	G_1G_2 (only if there is no loading)
Blocks in parallel	$G_1 \pm G_2$
Eliminating a feedback loop	**G/(1 ± GH)**
Moving a summing point **ahead (before)** a block G	Insert **1/G** in the added signal path
Moving a summing point **beyond (after)** a block G	Insert **G** in the added signal path
Moving a take-off point **ahead (before)** a block G	Insert **G** in the take-off branch
Moving a take-off point **beyond (after)** a block G	Insert **1/G** in the take-off branch
Interchanging adjacent summing points	Allowed (no change)
Non-unity feedback → unity feedback	Equivalent forward G′ = G/[1 + G(H − 1)]

- A summing point and a take-off point next to each other **cannot simply be interchanged**.
- When there are multiple inputs (reference + disturbance), use **superposition**: set the other inputs to zero and add the outputs.
- Disturbance D entering between G_1 and G_2: C/D = $G_2/(1 + G_1G_2H)$. A high G_1 suppresses the disturbance.

Signal flow graph (SFG): S.J. Mason, 1953

Term	Meaning
Node	Represents a **variable (signal)**. Its value = sum of all incoming signals.
Branch	A directed line segment joining two nodes. Its **gain (transmittance)** = the TF.
Source (input) node	Has **only outgoing** branches
Sink (output) node	Has **only incoming** branches
Mixed (chain) node	Has both incoming and outgoing branches
Path	A traversal of connected branches in the arrow direction, with **no node visited more than once**
Forward path	A path from the input node to the output node
Loop	A closed path that starts and ends at the same node, without passing any node twice
Self-loop	A loop of a single branch (one node)
Non-touching loops	Loops with **no common node**
Path gain / loop gain	Product of the branch gains along the path or loop

KEY FORMULAE: MASON'S GAIN FORMULA

- $\mathbf{T = (1/\Delta)\,\Sigma\, P_k\Delta_k}$
- P_k = gain of the k-th forward path.
- $\mathbf{\Delta = 1 - \Sigma L_1 + \Sigma L_2 - \Sigma L_3 + \ldots}$, where ΣL_1 = sum of all individual loop gains, ΣL_2 = sum of gain products of all combinations of **two non-touching loops**, ΣL_3 = three non-touching loops, and so on.
- $\mathbf{\Delta_k}$ = Δ for the part of the graph **not touching** the k-th forward path. If every loop touches path k, $\Delta_k = 1$.

- Δ = 1 + (loop-gain terms) is the **graph determinant**. Setting Δ = 0 gives the **characteristic equation**.
- SFGs apply only to **linear** systems. The SFG of a system is **not unique**, but its overall gain is.
- **Conversion from block diagram to SFG**: each summing point and take-off point becomes a node, and each block becomes a branch. A negative feedback is shown as branch gain −H.
- Example: a simple loop G with feedback H has one forward path $P_1 = G$ and one loop $L_1 = -GH$. Then $\Delta = 1 + GH$ and $\Delta_1 = 1$, so $T = G/(1 + GH)$ ✓.
- Two loops L_1, L_2 that do not touch each other: $\Delta = 1 - (L_1 + L_2) + L_1L_2$.

MEMORY TRICK

Mason: **"Paths times leftovers, over Delta"**. In Δ the signs alternate **− + − +**: subtract singles, add pairs, subtract triples.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- A node in an SFG represents? → **A variable (signal)**; a branch represents gain.
- A node having only outgoing branches is? → **Source node**.
- Moving a take-off point beyond a block G requires the branch to include? → **1/G**.
- Δ_k in Mason's formula is? → **The value of Δ for the part of the graph not touching the k-th forward path**.
- The closed-loop TF with positive feedback? → **G/(1 − GH)**.

8.3 Time Response ★★★

- **Time response = transient response + steady-state response**. The transient part dies out (for a stable system), and the steady-state part is what remains as t → ∞.

Standard test signals

Signal	r(t)	R(s)	Use
Impulse	δ(t)	**1**	Gives the TF directly; sudden shock
Step	A·u(t)	**A/s**	Sudden change; most common, for transient specs
Ramp (velocity)	At	**A/s^2**	Constant-velocity input
Parabolic (acceleration)	$At^2/2$	**A/s^3**	Constant acceleration
Sinusoidal	A sin ωt	$A\omega/(s^2+\omega^2)$	Frequency response

- Impulse = d/dt (step), step = d/dt (ramp). So the **impulse response = derivative of the step response**, and the step response = integral of the impulse response.

First-order system: C/R = 1/(1 + sT)

- Unit-step response: $c(t) = \mathbf{1 - e^{-t/T}}$. The pole is at s = −1/T, and T = **time constant**.
- At t = T, c = **63.2%**. At 2T 86.5%, 3T **95%**, 4T **98.2%**, 5T 99.3%.
- Settling time t_s = **4T** (2% band) or **3T** (5% band). Rise time (10–90%) = **2.2T**. Delay time (50%) = 0.693T.
- Initial slope of the step response = **1/T**. There is no overshoot; it is monotonic.
- Steady-state error to a step = 0 (for this closed-loop form). Error to a unit ramp = **T**. Impulse response = $(1/T)e^{-t/T}$.
- Smaller T gives a faster response; the pole lies farther from the jω-axis.

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Second-order system

KEY FORMULAE: STANDARD SECOND-ORDER SYSTEM

- $C(s)/R(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. Unity-feedback form $G(s) = \omega_n^2/[s(s + 2\zeta\omega_n)]$.
- Poles: $s = -\zeta\omega_n \pm j\omega_n\sqrt{(1-\zeta^2)}$. $|s| = \omega_n$; **cos θ = ζ** (θ measured from the negative real axis).
- Damped frequency $\omega_d = \omega_n\sqrt{(1-\zeta^2)}$. Attenuation (decay rate) $\sigma = \zeta\omega_n = 1/\tau$.
- Rise time (0–100%) $t_r = (\pi - \theta)/\omega_d$, with $\theta = \cos^{-1}\zeta$ in radians.
- Peak time $t_p = \pi/\omega_d$.
- Max. overshoot $M_p = e^{-\pi\zeta/\sqrt{(1-\zeta^2)}}$ (×100 for %). It depends on **ζ only**.
- Settling time $t_s = 4/(\zeta\omega_n)$ (2% band) or $3/(\zeta\omega_n)$ (5% band).
- Delay time $t_d \approx (1 + 0.7\zeta)/\omega_n$.
- Number of oscillations before settling = $t_s/T_d = t_s\omega_d/2\pi$, which for the 2% band = $2\sqrt{(1-\zeta^2)}/(\pi\zeta)$.
- Damped period $T_d = 2\pi/\omega_d$. Time of the n-th peak = $n\pi/\omega_d$.
- Logarithmic decrement $\delta = \ln(x_n/x_{n+1}) = 2\pi\zeta/\sqrt{(1-\zeta^2)}$.

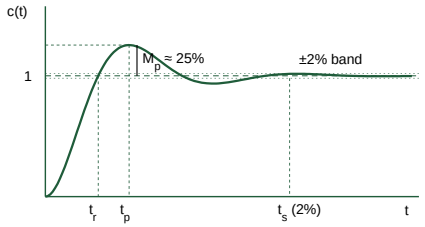

Fig. 8.1 Unit-step response of an underdamped second-order system (ζ = 0.4): t_r, $t_p = \pi/\omega_d$, M_p = 25.4%, $t_s = 4/\zeta\omega_n$

Damping ratio ζ	Case	Closed-loop poles	Step response
ζ = 0	**Undamped**	Purely imaginary $\pm j\omega_n$	**Sustained oscillations** at ω_n (marginally stable)
0 < ζ < 1	**Underdamped**	Complex conjugate, LHP	Damped oscillations at ω_d, with overshoot
ζ = 1	**Critically damped**	Real, equal: $-\omega_n$, $-\omega_n$	No overshoot; **fastest without overshoot**
ζ > 1	**Overdamped**	Real, distinct, LHP	Sluggish, no overshoot
ζ < 0	Negatively damped	RHP	Growing oscillations, **unstable**

ζ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
% M_p	72.9	52.7	37.2	25.4	**16.3**	**9.5**	**4.6**	1.5

- Practical systems are designed with **ζ ≈ 0.4–0.8**. ζ = 0.707 is often called **optimum**: $M_p \approx 4.3\%$ and no resonant peak.
- When ζ increases: $M_p \downarrow$, $t_r \uparrow$, $t_p \uparrow$. When ω_n increases at constant ζ: t_r, t_p and t_s all $\downarrow$ and M_p is unchanged.
- On the s-plane, **radial lines = constant ζ** (constant M_p), **circles about the origin = constant ω_n**, **vertical lines = constant ζω_n** (constant t_s), and **horizontal lines = constant ω_d** (constant t_p).
- **Worked example**: G = 25/[s(s+6)] with unity feedback gives $s^2 + 6s + 25$. So ω_n = **5 rad/s**, $2\zeta\omega_n = 6 \rightarrow \zeta$ = **0.6**, ω_d = **4 rad/s**, θ = 53.13° (0.927 rad). This gives t_r = (3.142 − 0.927)/4 = **0.554 s**, $t_p = \pi/4$ = **0.785 s**, M_p = **9.5%**, t_s(2%) = 4/3 = **1.33 s**.
- Adding **velocity (tacho) feedback** $K_t s$ to $G = \omega_n^2/[s(s+2\zeta\omega_n)]$ increases the effective damping to $\zeta' = \zeta + K_t\omega_n/2$ while ω_n is unchanged. It reduces M_p but increases the ramp error.

Effect of adding poles/zeros; dominant poles

Addition	Effect
Zero added to the closed-loop TF (LHP)	Faster: $t_r \downarrow$, but **overshoot ↑** (adds a derivative effect)
Pole added to the closed-loop TF	Sluggish: $t_r \uparrow$, overshoot $\downarrow$
Pole added to the open-loop G(s)H(s)	Root locus shifts **right**, so it is **less stable** and overshoot rises
Zero added to the open-loop G(s)H(s)	Root locus shifts **left**, so it is **more stable** and damping improves
RHP zero	**Non-minimum phase**; gives initial **undershoot** (inverse response)

- **Dominant poles** = closed-loop poles **nearest the jω-axis** (slowest decay). They govern the transient. Other poles may be neglected if their real parts are **≥ 5 times** (some books say 5–10 times) farther from the jω-axis.
- A pole and a nearby zero (a **dipole**) nearly cancel each other.
- While neglecting a far pole, keep the DC gain unchanged: 10/[(s+1)(s+10)] ≈ **1/(s+1)**.

MEMORY TRICK

M_p for ζ = 0.5, 0.6, 0.7 = **"16, 9.5, 4.6"**. **"Peak = Pi over ω_d; Settle = Four over σ."** The first-order checkpoints are **63–86–95–98** at T, 2T, 3T, 4T.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- First-order system: % of final value reached in one time constant? → **63.2%**.
- Maximum overshoot depends on? → **Damping ratio ζ only**.
- Closed-loop poles on the imaginary axis (ζ = 0) give? → **Sustained (undamped) oscillations**.
- $s^2 + 4s + 16$: ω_n and ζ? → **4 rad/s, 0.5** (M_p = 16.3%).
- Settling time (2% criterion) of a second-order system? → **$4/\zeta\omega_n$**.
- The response that is fastest without overshoot? → **Critically damped (ζ = 1)**.

8.4 Steady-State Error ★★★

- **Steady-state error** $e_{ss} = \lim_{t\to\infty} e(t)$. It measures **accuracy**. It depends on the **type** of the system and on the **input**.
- The final value theorem applies only if the system is **stable**, i.e. sE(s) has no poles on the jω-axis or in the RHP.

KEY FORMULAE

- Unity feedback: E(s) = R(s)/[1 + G(s)], so $\mathbf{e_{ss} = \lim_{s\to 0} sR(s)/[1 + G(s)]}$. For non-unity feedback, use G(s)H(s) in the constants. This gives the actuating-signal error; the true output error needs H(0) = 1.
- Position error constant $\mathbf{K_p = \lim_{s\to 0} G(s)H(s)}$.
- Velocity error constant $\mathbf{K_v = \lim_{s\to 0} sG(s)H(s)}$.
- Acceleration error constant $\mathbf{K_a = \lim_{s\to 0} s^2G(s)H(s)}$.
- Unit step: $e_{ss} = \mathbf{1/(1 + K_p)}$. Unit ramp: $\mathbf{1/K_v}$. Unit parabola ($t^2/2$): $\mathbf{1/K_a}$. For magnitude A, multiply by A.

Type	K_p	K_v	K_a	Step e_{ss}	Ramp e_{ss}	Parabolic e_{ss}
0	K	0	0	**1/(1+K)**	**∞**	**∞**
1	∞	K	0	**0**	**1/K**	**∞**
2	∞	∞	K	**0**	**0**	**1/K**
3	∞	∞	∞	0	0	0

- K here is the **Bode (time-constant-form) gain**, i.e. G(s)H(s) written as $K(1+sT_a).../[s^N(1+sT_1)...]$.
- **Higher type → better steady-state accuracy**, but stability gets worse. Type ≥ 3 is rarely used because it is hard to stabilise.
- Type 0 tracks a step with a finite error. Type 1 tracks a step with zero error and a ramp with a finite error. Type 2 tracks a ramp with zero error and a parabola with a finite error.
- Example: G = 10/[s(s+2)] gives $K_v = 10/2 =$ **5**. Unit ramp e_{ss} = **0.2**, step $e_{ss} = 0$, parabola $e_{ss} = \infty$.
- Example: G = 20/[(s+1)(s+4)] gives $K_p = 20/4 = 5$. Step error = 1/6 = **0.167**.
- Increasing the gain K reduces a finite e_{ss} but reduces relative stability. The **accuracy vs stability** trade-off is solved by compensation (lag/PI).
- **Static error coefficients** (K_p, K_v, K_a) give only a finite/zero/infinite value and work only for standard inputs. **Generalised (dynamic) error coefficients** C_0, C_1, C_2...: $E(s)/R(s) = 1/(1+GH) = C_0 + C_1 s + C_2 s^2/2! + ...$, giving $e_{ss}(t) = C_0 r(t) + C_1 \dot{r}(t) + C_2 \ddot{r}(t)/2! + ...$ They give error as a **function of time** for any input.

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Sensitivity

- Sensitivity of T to parameter K: $S^T_K = (\partial T/T)/(\partial K/K)$ = % change in T ÷ % change in K.
- Open-loop: S^T_G = **1** (fully sensitive). Closed-loop: S^T_G = **1/(1 + GH)**, much smaller when the loop gain is high. $S^T_H = -GH/(1+GH) \approx -1$.
- Example: if the loop gain GH = 99, a 10% change in G changes T by only 10/100 = **0.1%**.

MEMORY TRICK

The table is a **staircase**. Each step down in type moves the "finite" cell one input to the right: zeros to the left of it, ∞ to the right. "**Type = number of inputs it can beat (zero error) completely**."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Steady-state error of a type-1 system for a unit-step input? → **Zero**.
- Type-0 system with a ramp input, e_{ss}? → **Infinite**.
- K_v for G = K/[s(s+a)]? → **K/a**.
- A type-2 system has finite e_{ss} for which input? → **Parabolic (acceleration)**.
- Closed-loop sensitivity to forward-path variations? → **1/(1 + GH)**.

8.5 Stability – Routh-Hurwitz & Root Locus ★★★

- **BIBO stability**: every bounded input gives a bounded output. This holds if the **impulse response is absolutely integrable** ($\int |h(t)|dt < \infty$), i.e. all poles are in the LHP.
- **Absolute stability** asks only whether the system is stable or not. **Relative stability** asks how stable it is, measured by gain margin, phase margin, ζ, or distance of the poles from the jω-axis.
- An LTI system is stable only if **all closed-loop poles lie in the left half of the s-plane** (negative real parts).

Closed-loop pole location	Impulse response	Stability
All in the LHP	Decays to zero	**Stable**
Any pole in the RHP	Grows without bound	**Unstable**
Simple (non-repeated) poles on the jω-axis, rest in the LHP	Constant/sustained oscillation	**Marginally (limitedly) stable**
Repeated poles on the jω-axis (e.g. $1/s^2$, $1/(s^2+\omega^2)^2$)	Grows as t or t sin ωt	**Unstable**
Single pole at origin	Constant (a step; does not decay)	Marginally stable, but not BIBO stable (a step input gives a ramp output)

- Zeros do **not** decide stability. They only shape the response.

Routh-Hurwitz criterion

- **Necessary (not sufficient)** condition: all coefficients of the characteristic equation are **present (non-zero)** and of the **same sign**. For orders 1 and 2 this condition is also sufficient.
- **Routh array**: the first two rows hold alternate coefficients. Each later entry = −(1/a)·det of the 2×2 formed from the two rows above.
- **Number of sign changes in the first column = number of roots in the RHP**. Stable only if there are **no sign changes** (all first-column entries positive).
- Third-order $a_0s^3 + a_1s^2 + a_2s + a_3 = 0$ (all $a > 0$) is stable if $\mathbf{a_1a_2 > a_0a_3}$. If $a_1a_2 = a_0a_3$ it is marginally stable.
- The Routh test gives **absolute stability** and the number of RHP roots, but **not the exact root locations**. Relative stability: shift $s = z - \sigma$ and test again.

Special case	Remedy	Interpretation
Zero in the first column (rest of that row non-zero)	Replace 0 by a small positive **ε** and continue (or substitute s = 1/z)	Check the signs as $\varepsilon \to 0^+$
Entire row of zeros	Form the **auxiliary equation A(s)** from the row above, and use the coefficients of **dA/ds**	Roots **symmetric about the origin** (±a, ±jω, or quadrants). Roots of A(s) are also roots of the characteristic equation. The auxiliary equation is always of **even** order. Without sign changes it means jω-axis roots → **marginal stability**.

STANDARD K-RANGE NUMERICALS

- G(s) = K/[s(s+1)(s+2)], unity feedback → $s^3 + 3s^2 + 2s + K = 0$. The s^1 row is (6 − K)/3, so stable for **0 < K < 6**. $K_{mar} = 6$, and the auxiliary equation $3s^2 + 6 = 0$ gives ω = **√2 rad/s**.
- G(s) = K/[s(s+2)(s+4)] → $s^3 + 6s^2 + 8s + K$: **0 < K < 48**, ω = √8 = **2√2 rad/s**.
- G(s) = K/[s(s+1)(s+3)] → $s^3 + 4s^2 + 3s + K$: **0 < K < 12**, ω = √3 rad/s.
- G(s) = K/[s(1+sT₁)(1+sT₂)] → stable for **0 < K < $(T_1+T_2)/(T_1T_2)$** = $1/T_1 + 1/T_2$.
- For $s^3 + a\,s^2 + b\,s + K$ (a, b > 0): K_{max} = **ab** and ω at that point = **√b**.

Root locus (W.R. Evans, 1948)

- **Root locus** = the path of the **closed-loop poles** as the gain K varies from 0 to ∞, drawn using the open-loop poles and zeros.
- **Angle condition** (decides whether a point is on the locus): ∠G(s)H(s) = **± (2q+1)180°**. **Magnitude condition** (gives K at that point): |G(s)H(s)| = **1**, so K = Π(vector lengths from poles)/Π(vector lengths from zeros).

Rule	Statement
1. Symmetry	**Symmetric about the real axis**
2. Number of branches	= number of OL poles P (if P ≥ Z) = **order** of the characteristic equation
3. Start and end	**Starts at OL poles (K = 0)** and **ends at OL zeros (K = ∞)**. P − Z branches go to infinity (zeros at ∞).
4. Real-axis segments	A real-axis point lies on the locus if the total number of real OL poles + zeros **to its right is odd**
5. Asymptote angles	θ_q = **(2q+1)180°/(P − Z)**, q = 0, 1, …, P−Z−1
6. Centroid	σ_A = **(Σ real parts of poles − Σ real parts of zeros)/(P − Z)**. It is always on the real axis.
7. Breakaway / break-in points	Write K as a function of s and solve **dK/ds = 0**; keep only roots that lie on the locus. Branches break away at **±90°** (two branches).
8. jω-axis crossing	Use **Routh**: K_{mar} from the zero row, ω from the auxiliary equation
9. Angle of departure (from a complex pole)	φ_d = **180° − Σ∠(other poles) + Σ∠(zeros)**
10. Angle of arrival (at a complex zero)	φ_a = **180° − Σ∠(other zeros) + Σ∠(poles)**

P − Z	Asymptote angles
1	180°
2	**±90°**
3	**±60°, 180°**
4	**±45°, ±135°**

- **Worked example**: G = K/[s(s+1)(s+2)]. Branches = 3. Real-axis locus on 0 to −1 and on −2 to −∞. Centroid = (0 − 1 − 2)/3 = **−1**. Asymptotes at ±60°, 180°. K = −($s^3 + 3s^2 + 2s$), so dK/ds = 0 gives $3s^2 + 6s + 2 = 0$ → s = −0.423 (valid) or −1.577 (invalid, not on the locus). Breakaway point = **−0.423**. jω crossing at **±j√2** with K = 6.
- For G = K/[s(s+a)]: breakaway at **−a/2**, and the locus is a vertical line through −a/2. The system is **never unstable** for any K > 0.
- For G = K(s+z)/[s(s+p)] with z > p: the complex part of the locus is a **circle centred at the zero** (−z).
- **Complementary root locus** covers −∞ < K < 0 (real-axis segments where the count to the right is **even**; asymptotes at q·360°/(P−Z)). The **root contour** is used when a parameter other than K varies.
- **Adding an OL pole** pulls the locus **towards the right** (towards the RHP). Stability ↓, settling slower.
- **Adding an OL zero** pulls the locus **towards the left**. Stability ↑, transient improves. This is the basis of **PD/lead** compensation.

MEMORY TRICK

"**Poles are Parents, Zeros are Zones**": branches are born at poles and die at zeros. For the real axis, count **to the right** and an **odd** count is on the locus ("Odd Right"). Centroid = "**poles minus zeros over P minus Z**".

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- Number of sign changes in the first column of the Routh array equals? → **Number of roots in the RHP**.
- A row of zeros in the Routh array indicates? → **Roots symmetric about the origin** (possible jω-axis roots).
- $s^3 + 3s^2 + 2s + K$: marginal K and frequency of oscillation? → **K = 6, ω = √2 rad/s**.
- Root locus branches start at? → **Open-loop poles (K = 0)**.
- Angles of asymptotes for P − Z = 3? → **60°, 180°, 300°**.
- Adding a zero to the open-loop TF makes the root locus shift? → **Left (more stable)**.

8.6 Frequency Response – Bode, Polar & Nyquist ★★★

- **Frequency response** = the steady-state response to a sinusoidal input. The output has the **same frequency**, but its magnitude and phase change: $|G(j\omega)|$ and $\angle G(j\omega)$. Obtained by putting **s = jω**.
- Advantages: it can be measured experimentally, it handles dead time (e^{-sT}), and noise is easy to judge. It is mainly used for stability (GM, PM).

Frequency-domain specifications

KEY FORMULAE: SECOND-ORDER SYSTEM

- Resonant peak $\mathbf{M_r = 1/(2\zeta\sqrt{(1-\zeta^2)})}$, valid for **ζ < 0.707**. For ζ ≥ 0.707 there is no peak ($M_r = 1$).
- Resonant frequency $\mathbf{\omega_r = \omega_n\sqrt{1 - 2\zeta^2}}$. Note $\omega_r < \omega_d < \omega_n$.
- Bandwidth $\mathbf{\omega_b = \omega_n[(1 - 2\zeta^2) + \sqrt{(4\zeta^4 - 4\zeta^2 + 2)}]^{1/2}}$. For ζ = 0.707, $\omega_b = \omega_n$.
- Bandwidth = range of frequency over which |M| ≥ 0.707 of its zero-frequency value (the **−3 dB** point).
- Phase margin (approx.): **PM ≈ 100ζ** (in degrees, for ζ ≤ 0.6–0.7). Example: ζ = 0.5 → PM ≈ 52° (exact), ≈ 50° (approx.).
- Rise time × bandwidth ≈ constant, so **large bandwidth → fast response**, but also more noise.

- M_r and M_p both depend only on ζ; a large M_r means a large M_p. Desirable M_r = **1.1 to 1.5**. At ζ = 0.5, $M_r = 1/(2\times0.5\times0.866) =$ **1.155**.
- **Cut-off rate** = slope of the log-magnitude curve near the cut-off frequency. It shows the ability to separate signal from noise.

Bode plot (H.W. Bode)

- Two plots against **log ω**: magnitude in **dB (20 log|G|)** and phase in degrees. Multiplicative factors become additive, which is why asymptotic straight lines are used.
- 1 decade = 10:1 frequency ratio; 1 octave = 2:1. **20 dB/decade = 6 dB/octave**.

Factor	Magnitude slope	Phase
Constant K	Horizontal line at 20 log K	0° (K > 0); −180° if K < 0
Pole at origin $1/s^N$	**−20N dB/dec**, passes 0 dB at ω = 1	**−90N°** (constant)
Zero at origin s^N	+20N dB/dec	+90N°
Simple pole 1/(1 + sT)	0 up to the **corner $\omega_c = 1/T$**, then **−20 dB/dec**	0° → −90°; **−45° at the corner**
Simple zero (1 + sT)	+20 dB/dec after 1/T	0° → +90°; +45° at the corner
Quadratic pole $1/[1 + 2\zeta(s/\omega_n) + (s/\omega_n)^2]$	**−40 dB/dec** after ω_n	0° → −180°; **−90° at ω_n**
Dead time e^{-sT}	0 dB (|G| = 1)	**−ωT rad** (lag grows without limit)

- Max. error of the asymptotic magnitude for a simple pole = **3 dB at the corner frequency** (1 dB one octave either side). For a quadratic factor the error at $\omega_n = -20\log(2\zeta)$, which is large for small ζ.
- **Initial slope reveals the type**: 0 dB/dec → type 0; −20 → type 1; −40 → type 2.
- Type 0: the low-frequency level 20 log K_p gives K_p. Type 1: the initial line (or its extension) cuts 0 dB at **ω = K_v**. Type 2: it cuts 0 dB at **ω = $\sqrt{K_a}$**.
- Final slope = **−20(P − Z) dB/dec**. Final phase = −90°(P − Z) for minimum-phase systems.

- **Minimum-phase system**: no poles or zeros in the RHP (and no dead time). Magnitude and phase are uniquely related (Bode's gain–phase relation), so the TF can be found from the magnitude plot alone.
- **Non-minimum-phase**: has RHP zeros (or dead time). For the same magnitude it has a larger phase lag. All-pass systems (1 − sT)/(1 + sT) have |G| = 1 and phase $-2\tan^{-1}\omega T$.

Gain margin & phase margin

KEY FORMULAE

- **Phase crossover frequency ω_{pc}**: where $\angle G(j\omega)H(j\omega) = -180°$.
- **Gain crossover frequency ω_{gc}**: where $|G(j\omega)H(j\omega)| = 1$ **(0 dB)**.
- **Gain margin** GM = $1/|G(j\omega_{pc})H(j\omega_{pc})|$; in dB, GM = $-20\log|GH(j\omega_{pc})|$.
- **Phase margin** PM = $180° + \angle G(j\omega_{gc})H(j\omega_{gc})$.

- GM = the factor by which the gain can be **increased** before the system becomes unstable. PM = the additional **phase lag** that can be added at ω_{gc} before instability.
- **Minimum-phase systems**: stable if **GM > 0 dB and PM > 0°**, i.e. $\omega_{gc} < \omega_{pc}$. Marginally stable if GM = 0 dB and PM = 0° ($\omega_{gc} = \omega_{pc}$). Unstable if both are negative.
- Good design: **GM ≈ 6–10 dB**, **PM ≈ 30°–60°** (commonly 45°).
- For first- and second-order systems (no ω_{pc}, since the phase never reaches −180°), **GM = ∞**.
- Example: if |GH| = 0.5 at ω_{pc}, GM = 2 = **6.02 dB**. If $\angle GH = -150°$ at ω_{gc}, PM = **30°**.
- Pure dead time reduces PM without changing the magnitude plot, so it is destabilising.

Polar plot

- **Polar plot** = locus of G(jω) in the complex plane (magnitude and phase) as ω varies from **0 to ∞**.
- Starting point (ω → 0) depends on the **type**. The end point (ω → ∞) for a strictly proper system is the **origin**, approached at angle **−90°×(P − Z)**.

Type	Start (ω = 0)	Example shape
Type 0	Finite point K on the **positive real axis**	K/(1+sT): a **semicircle** in the 4th quadrant, from K to 0
Type 1	∞ along **−90°** (asymptotic to a line parallel to the −j axis)	K/[s(1+sT)]: comes from −j∞ with asymptote at real part −KT, ends at the origin at −180°
Type 2	∞ along **−180°**	K/[s^2(1+sT)]: ends at the origin at −270°

Order (type-0, all poles)	Ends at the origin tangent to	Quadrants crossed
1	−90° (−j axis)	IV
2	−180° (negative real axis)	IV, III
3	−270° (+j axis)	IV, III, II, so it crosses the negative real axis (ω_{pc} exists)
4	−360°	IV, III, II, I

Nyquist stability criterion (H. Nyquist, 1932)

- Based on Cauchy's **principle of the argument** (conformal mapping). The **Nyquist contour** encloses the entire RHP, with small indentations around poles on the jω-axis.
- The plot of G(s)H(s) is taken along that contour, and its encirclements of the **critical point (−1 + j0)** are counted.

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NYQUIST CRITERION

- **N = P − Z** (Nagrath & Gopal convention). N = number of **anticlockwise (counter-clockwise)** encirclements of (−1, j0), P = number of OL poles of G(s)H(s) in the RHP, Z = number of closed-loop poles (zeros of 1 + GH) in the RHP.
- Equivalent form (Ogata/Kuo): **Z = N + P**, with N counted as **clockwise** encirclements.
- For stability **Z = 0**, so the number of anticlockwise encirclements of −1 + j0 must equal P.
- If the OL system is stable (P = 0), the closed loop is stable if the Nyquist plot **does not encircle −1 + j0**.

- The Nyquist criterion gives both absolute and relative stability. It also works with dead time and with **open-loop unstable** systems, which is its advantage over the Bode-based test.
- Polar plot crosses the negative real axis at −a: then GM = 1/a. Passing through −1 means marginal stability.
- **Inverse polar plot** = plot of 1/G(jω).

M-circles, N-circles, Nichols chart

Locus	Centre	Radius
Constant-M circle (closed-loop magnitude M = |G/(1+G)|)	$(-M^2/(M^2-1), 0)$	$\|M/(M^2-1)\|$
M = 1	A **straight line** x = −1/2 (perpendicular bisector of 0 and −1)	
Constant-N circle (N = tan α, α = closed-loop phase)	**(−1/2, 1/(2N))**	**$\sqrt{(N^2+1)}/(2N)$**

- M > 1 circles lie to the **left** of x = −1/2; M < 1 circles lie to the right. As M → ∞ the circle shrinks to the point (−1, 0). Every N-circle passes through **(0, 0) and (−1, 0)**.
- The M-circle that the G(jω) plot just **touches** gives M_r, and the frequency at the point of tangency gives ω_r.
- **Nichols chart** = M and N loci drawn on a plot of **open-loop magnitude in dB (y-axis) vs open-loop phase in degrees (x-axis)** (gain–phase plot). From the OL plot it reads closed-loop M_r, ω_r and bandwidth. A change in gain K simply shifts the curve **vertically**. The critical point is (−180°, 0 dB).

MEMORY TRICK

"**GM at Phase crossover, PM at Gain crossover**" (they cross over). For stability, "**Gain crosses first**": $\omega_{gc} < \omega_{pc}$. Nyquist: "**N = P − Z, Anti-clockwise is Positive.**"

PREVIOUSLY ASKED / LIKELY QUESTIONS

- The slope of the Bode magnitude plot for a double pole at the origin? → **−40 dB/decade**.
- Maximum error between the asymptotic and actual Bode plot for a simple pole? → **3 dB at the corner frequency**.
- Phase margin is measured at? → **Gain crossover frequency** (|GH| = 1).
- Resonant peak exists only when? → **ζ < 0.707**.
- The M = 1 locus is? → **A straight line x = −1/2**.
- A system with a zero in the RHP is called? → **Non-minimum phase**.

8.7 Compensators & Controllers ★★★

- **Compensation** = adding a device to the system to meet performance specifications that gain adjustment alone cannot meet.
- **Series (cascade) compensation** places the device in the forward path. **Feedback (parallel) compensation** places it in a minor feedback loop. **Load** compensation is also used.

Lead compensator

KEY FORMULAE: LEAD (A < 1)

- $G_c(s) = (s + 1/T)/(s + 1/(\alpha T)) = \alpha(1 + sT)/(1 + s\alpha T)$, with **0 < α < 1**.
- Zero at −1/T, pole at −1/(αT). The **zero is nearer the origin** than the pole.
- Max phase lead **$\sin\varphi_m = (1-\alpha)/(1+\alpha)$**, so $\alpha = (1 - \sin\varphi_m)/(1 + \sin\varphi_m)$.
- It occurs at **$\omega_m = 1/(T\sqrt{\alpha})$** = **geometric mean** of the two corner frequencies 1/T and 1/(αT).
- At ω_m, the magnitude of $(1 + sT)/(1 + s\alpha T)$ = **$1/\sqrt{\alpha}$**, i.e. a gain of **$-10 \log \alpha$ dB**.
- RC lead network: $R_1 \parallel C_1$ in series, R_2 shunt to output. $T = R_1C_1$, $\alpha = R_2/(R_1 + R_2)$.

- The lead compensator is a **high-pass filter** and approximates **PD**. It adds phase lead near ω_{gc}, which increases the PM.
- Effects: **improves the transient response** and relative stability, **increases bandwidth** (faster, $t_r \downarrow$, $t_s \downarrow$), reduces overshoot, and shifts the root locus **left**. Its effect on e_{ss} is small. Drawbacks: noise is amplified (high-frequency gain), and an extra amplifier gain of $1/\alpha$ is needed.
- Max practical lead per stage ≈ **60°** (α ≈ 0.07). As α → 0, $\varphi_m \to 90°$. For more lead, cascade two stages. Example: α = 1/3 gives $\sin\varphi_m = 0.5 \to \varphi_m = 30°$; α = 0.172 gives 45°.

Lag compensator

- $G_c(s) = (1 + sT)/(1 + s\beta T)$, with **β > 1**. The **pole is nearer the origin** than the zero. Max phase lag $\sin\varphi_m = (\beta - 1)/(\beta + 1)$ at $\omega_m = 1/(T\sqrt{\beta})$.
- The lag compensator is a **low-pass filter** and approximates **PI**. Its corner frequencies are placed well below ω_{gc}.
- Effects: it **improves steady-state accuracy** (raises low-frequency gain, so K_v rises by about β), **reduces bandwidth** (slower response, t_r and $t_s \uparrow$), and attenuates high-frequency noise. It uses the attenuation at high frequency (not the phase lag) to raise the PM.
- **Lead–lag**: combines both, improving transient and steady state together. The lag section comes at low frequency and the lead section at high frequency. $G_c = [(1 + sT_1)/(1 + s\alpha T_1)]\cdot[(1 + sT_2)/(1 + s\beta T_2)]$, α < 1 < β.

Feature	Lead	Lag
Pole–zero	Zero nearer origin	Pole nearer origin
Filter type	**High-pass**	**Low-pass**
Analogous controller	**PD**	**PI**
Main improvement	**Transient response**, PM	**Steady-state error**
Bandwidth	**Increases**	**Decreases**
Speed of response	Faster	Slower
Noise	Amplified	Suppressed
Root locus	Shifts left	Small change near the origin (pole–zero dipole)

Controllers: P, I, D and combinations

- **P controller**: $u = K_p e$. Increasing K_p **reduces** (but does not eliminate) e_{ss}, speeds up the response, and increases overshoot. Too high a value causes instability. It leaves an **offset**.
- **I controller**: $u = K_i \int e\,dt$, i.e. K_i/s. It adds a pole at the origin, so the **type increases by 1** and **e_{ss} is eliminated** for a step. It reduces relative stability (adds −90° phase) and makes the response more oscillatory. **Integral wind-up** occurs under actuator saturation.
- **D controller**: $u = K_d\,de/dt$, i.e. $K_d s$. It anticipates the error, **increases damping** and stability, reduces overshoot, **amplifies high-frequency noise**, and has **no effect on e_{ss}**. It is **never used alone** because it gives no output for a constant error.
- **PI** = $K_p(1 + 1/(T_i s))$: adds a pole at the origin and a zero at $-1/T_i$. It is **lag-like**: improves e_{ss} but reduces stability and bandwidth.
- **PD** = $K_p(1 + T_d s)$: adds a zero at $-1/T_d$. It is **lead-like**: improves damping and transient, increases bandwidth, and leaves the type unchanged.
- **PID** = $K_p[1 + 1/(T_i s) + T_d s] = (K_d s^2 + K_p s + K_i)/s$. It adds one pole at the origin and **two zeros**. It is the most widely used (≈ 90% of industrial loops) and acts like a lead–lag.

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Increase of	Rise time	Overshoot	Settling time	Steady-state error	Stability
K_p	Decreases	Increases	Small change	**Decreases**	Degrades
K_i	Decreases	Increases	Increases	**Eliminated**	Degrades
K_d	Small change	**Decreases**	Decreases	**No effect**	Improves (if K_d small)

Ziegler–Nichols tuning (1942)

Method	Controller	K_p	T_i	T_d
Ultimate gain (closed-loop, 2nd method): raise K_p with I and D off until sustained oscillation at K_{cr}, period P_{cr}	P	$0.5K_{cr}$	∞	0
	PI	$0.45K_{cr}$	$P_{cr}/1.2$	0
	PID	$\mathbf{0.6K_{cr}}$	$\mathbf{0.5P_{cr}}$	$\mathbf{0.125P_{cr}}$
Process reaction curve (open-loop, 1st method): S-shaped step response with delay L and time constant T	P	T/L	∞	0
	PI	0.9T/L	L/0.3	0
	PID	1.2T/L	2L	0.5L

- Z–N tuning aims at a **quarter-amplitude decay ratio**, which gives roughly 25% overshoot; fine tuning is needed afterwards.
- Op-amp realisation: P = inverting amplifier, I = integrator (C in feedback), D = differentiator (C at input).

MEMORY TRICK

"**LEAD to Lead the Transient; LAG to Lag-behind the error.**" **PD ≈ Lead (zero), PI ≈ Lag (pole at origin)**. Integral = "**I kill the error**"; Derivative = "**D Damps**". Z–N PID: "**0.6 – 0.5 – 0.125**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- A lead compensator is essentially a? → **High-pass filter** (improves transient response).
- Max phase lead from a lead network with α = 1/3? → **30°**.
- The controller that eliminates steady-state error but may cause oscillations? → **Integral (I)**.
- The derivative controller's effect on steady-state error? → **None**; it improves damping.
- Lag compensation affects bandwidth how? → **Reduces bandwidth**.
- Frequency of maximum phase lead? → $\omega_m = 1/(T\sqrt{\alpha})$.

8.8 State Space Analysis ★★★

- **State** = the smallest set of variables (**state variables**) whose values at t_0, together with the input for $t \geq t_0$, fully determine the future behaviour.
- Number of state variables = **order** of the system = number of **independent energy-storage elements** (inductor currents and capacitor voltages are natural choices).
- State-space applies to **MIMO, non-linear and time-varying** systems and uses **non-zero initial conditions**. It works in the time domain. The TF approach handles only SISO LTI systems with zero ICs.
- The choice of state variables is **not unique**. The TF is unique, and the eigenvalues are invariant under a similarity transformation.

KEY FORMULAE

- State equation $\dot{\mathbf{x}} = \mathbf{Ax + Bu}$; output equation $\mathbf{y = Cx + Du}$.
- A (n×n) = system matrix. B (n×m) = input matrix. C (p×n) = output matrix. D (p×m) = direct transmission (feedforward) matrix. n = number of states, m = inputs, p = outputs.
- Transfer function: $\mathbf{G(s) = C(sI - A)^{-1}B + D}$.
- Characteristic equation $\mathbf{|sI - A| = 0}$. **Eigenvalues of A = poles** of the system.
- Resolvent matrix $\Phi(s) = (sI - A)^{-1}$.
- State transition matrix (STM) $\boldsymbol{\varphi(t) = e^{At} = L^{-1}[(sI - A)^{-1}]} = I + At + A^2t^2/2! + \ldots$
- Solution: $x(t) = \varphi(t)x(0) + \int_0^t \varphi(t-\tau)Bu(\tau)d\tau$ (zero-input response + zero-state response).
- Trace(A) = sum of eigenvalues; det(A) = product of eigenvalues.

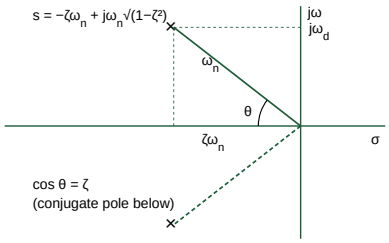

Fig. 8.2 Location of the underdamped second-order poles (eigenvalues): distance from origin = ω_n, real part = $-\zeta\omega_n$, imaginary part = ω_d

Properties of the state transition matrix

Property	Statement
Initial value	$\boldsymbol{\varphi(0) = I}$ (identity)
Inverse	$\boldsymbol{\varphi^{-1}(t) = \varphi(-t)}$
Transition (semigroup)	$\boldsymbol{\varphi(t_2 - t_1)\varphi(t_1 - t_0) = \varphi(t_2 - t_0)}$
Addition	$\varphi(t_1 + t_2) = \varphi(t_1)\varphi(t_2)$
Power	$[\varphi(t)]^k = \varphi(kt)$
Derivative	$d\varphi/dt = A\varphi(t) = \varphi(t)A$
Meaning	Gives the **zero-input (free) response**: $x(t) = \varphi(t)x(0)$

- **Worked example**: $A = [[0, 1], [-2, -3]]$. $|sI - A| = s^2 + 3s + 2 = (s + 1)(s + 2)$, so the eigenvalues are **−1, −2**. Then $(sI - A)^{-1} = (1/[(s+1)(s+2)])\cdot[[s+3, 1], [-2, s]]$, which gives $\varphi(t) = [[2e^{-t} - e^{-2t}, e^{-t} - e^{-2t}], [-2e^{-t} + 2e^{-2t}, -e^{-t} + 2e^{-2t}]]$. Check: $\varphi(0) = I$ ✓.
- For a diagonal $A = \text{diag}(\lambda_1, \lambda_2)$, $\varphi(t) = \text{diag}(e^{\lambda_1 t}, e^{\lambda_2 t})$.
- Other ways to find e^{At}: the series, Cayley–Hamilton (every matrix satisfies its own characteristic equation), the Sylvester formula, and diagonalisation, where $e^{At} = Pe^{\Lambda t}P^{-1}$ with P = **modal matrix** (columns = eigenvectors).

Controllability & observability (R.E. Kalman, 1960)

- **Controllable**: an input can transfer every state from any initial value to any final value in finite time.
- **Observable**: every initial state $x(t_0)$ can be found from the measured output over a finite time.

Test	Controllability	Observability
Kalman test	$Q_c = \mathbf{[B\ AB\ A^2B \ldots\ A^{n-1}B]}$ has **rank n** (for SISO, $\lvert Q_c \rvert \neq 0$)	$Q_o = \mathbf{[C^T\ A^TC^T \ldots\ (A^T)^{n-1}C^T]}$ (or [C; CA; …; CA^{n-1}]) has **rank n**
Gilbert test (diagonal canonical form, distinct eigenvalues)	**No row of $\tilde{B}$ is zero**	**No column of $\tilde{C}$ is zero**
Relation	**Duality**: (A, B) is controllable ⇔ (A^T, B^T) is observable	

- **Pole–zero cancellation** in the TF means the system is **either uncontrollable or unobservable** (or both). The TF shows only the controllable **and** observable part.
- Controllability is needed for **pole placement** by state feedback (u = −Kx). Observability is needed to design a **state observer** (estimator).
- Example: $A = [[-1, 0], [0, -2]]$, $B = [1, 0]^T$. The second row of B is zero, so state x_2 is **uncontrollable**. $Q_c = [[1, -1], [0, 0]]$ has rank 1 < 2.
- **Output controllability** rank of [CB CAB … CA^{n-1}B D] = p (number of outputs).

Canonical forms

For $G(s) = (b_2s^2 + b_1s + b_0)/(s^3 + a_2s^2 + a_1s + a_0)$:

Form	Structure of A	Special feature
Controllable canonical (phase-variable / companion)	1's on the super-diagonal; **last row = $[-a_0\ -a_1\ -a_2]$**	**B = [0 0 1]^T**, C = $[b_0\ b_1\ b_2]$. Always **controllable**. Phase variables are successive derivatives ($x_2 = \dot{x}_1$ …).
Observable canonical	**Transpose** of the controllable form: last column = $[-a_0\ -a_1\ -a_2]^T$	**C = [0 0 1]**, B = $[b_0\ b_1\ b_2]^T$. Always **observable**.
Diagonal canonical	A = diag($\lambda_1, \lambda_2, \lambda_3$), from **partial fractions**	Requires **distinct** poles. States are **decoupled** (modes). Used for the Gilbert test.

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Jordan canonical	Jordan blocks: repeated λ on the diagonal, **1's on the super-diagonal** of each block	Used when poles are **repeated**

- The coefficients a_i appear with a **negative sign** in the companion matrix. Example: G = 1/(s² + 3s + 2) gives A = [[0, 1], [−2, −3]], B = $[0, 1]^T$, C = [1, 0].
- Converting a TF to state space is called **realisation**. A **minimal realisation** (least number of states) is both controllable and observable.
- State-variable approaches for discrete systems: x(k+1) = Gx(k) + Hu(k), with STM G^k.

MEMORY TRICK

"**$C(sI-A)^{-1}B + D$**: read it as **"C See, A is Inverted, then B and D"**." For STM properties, remember "**Zero gives I, minus gives inverse**". Kalman: "**B, AB, A^2B... for Control; C, CA, CA^2... for Observation**."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- The TF from a state model is? → **$C(sI - A)^{-1}B + D$**.
- Poles of a system in state-space form are? → **Eigenvalues of A**.
- State transition matrix φ(t) equals? → **$e^{At} = L^{-1}[(sI - A)^{-1}]$**; φ(0) = **I**.
- Kalman's controllability matrix for an n-th order system? → **$[B\ AB\ \dots\ A^{n-1}B]$** with rank n.
- Pole–zero cancellation in a TF indicates? → **Loss of controllability or observability**.
- The minimum number of state variables for an RLC series circuit? → **2** (i_L, v_C).

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9. Electrical & Electronic Measurements

NOTE

Measurements give 5–8 easy marks in most AE/AM papers. Questions cluster on: which instrument reads AC/DC/rms/average, shunt & multiplier numericals, % limiting error, two-wattmeter method, energy-meter creeping and meter constant, which bridge measures what (Hay vs Maxwell vs Schering), CT open-secondary danger, dual-slope DVM, 3½-digit resolution and Lissajous patterns. Learn the tables in this chapter thoroughly.

9.1 Measurement Basics & Error Analysis ★★★

- **Measurement** = comparison of an unknown quantity with a **standard**. Methods: **direct** (compare with standard) and **indirect** (via calibrated instrument).
- **Absolute instruments** give the value in terms of instrument constants and deflection (e.g. **tangent galvanometer, Rayleigh current balance**) – used only in standards labs. **Secondary instruments** are calibrated against absolute ones (all ordinary meters).
- Secondary instruments: **indicating** (ammeter, voltmeter, wattmeter), **recording** (chart recorder), **integrating** (energy meter, ampere-hour meter).
- **Deflection type** (PMMC ammeter) – fast but less accurate. **Null type** (potentiometer, bridges) – **more accurate & sensitive**, draw no current at balance.

Static characteristic	Meaning
Accuracy	Closeness to the **true value**
Precision	Closeness of repeated readings to **each other** (reproducibility/repeatability); precise ≠ accurate
Sensitivity	**Δoutput / Δinput** (slope of calibration curve); reciprocal = **deflection factor / inverse sensitivity**
Resolution (discrimination)	**Smallest change** of input that can be detected
Threshold	Smallest input (from **zero**) that gives a detectable output
Dead zone	Largest input change to which there is **no response** (due to friction, backlash)
Drift	Slow change of reading with time – **zero drift, span drift, zonal drift**
Linearity	Max deviation from the best straight line, as % of FSD
Hysteresis	Different readings for increasing and decreasing input

- **Dynamic characteristics**: **speed of response, measuring lag, fidelity** (faithful reproduction), **dynamic error**. Zero-order (potentiometer), **first-order** (thermometer, RC – time constant τ; 63.2% in 1τ), **second-order** (galvanometer, spring–mass – ω_n, ζ).
- **Loading effect**: a low-resistance voltmeter connected across a high-resistance circuit reads **low**; an ammeter adds series resistance. Remedy: voltmeter with **high Ω/V**, ammeter with **very low** resistance.

Error type	Examples	Remedy
Gross (human)	Misreading, wrong connection, wrong calculation	Care, take ≥ 3 readings
Systematic – **instrumental**	Friction, spring ageing, weak magnet, **loading**, calibration error	Calibration, correction factors
Systematic – **environmental**	Temperature, humidity, stray magnetic/electric fields	Shielding, air-conditioning, compensation
Systematic – **observational**	**Parallax**	**Mirror strip + knife-edge pointer**
Random (residual)	Unknown, small, fluctuating causes	**Statistical analysis** of many readings

KEY FORMULAE

- Static error $\delta A = A_m - A_t$; static correction $\delta C = A_t - A_m = -\delta A$
- Relative (%) error = $(A_m - A_t)/A_t \times 100$
- **Limiting (guarantee) error** is specified as % of FSD. At a reading A: **% error = (% FSD error) × FSD/A** → error **increases as reading falls**; always read in the **upper part of scale** (above ~⅔ FSD).
- $X = X_1 \pm X_2$: **absolute** limiting errors add, $\Delta X = \pm(\Delta X_1 + \Delta X_2)$ (worst for a difference of nearly equal quantities)
- $X = X_1X_2$ or X_1/X_2 : **% (relative) errors add** → $\pm(\varepsilon_1 + \varepsilon_2)$
- $X = X_1^n$: % error = $\mathbf{n \times \varepsilon_1}$ (e.g. $P = I^2R \rightarrow 2\varepsilon_I + \varepsilon_R$)
- Arithmetic mean $\bar{x} = \Sigma x/n$; deviation $d = x - \bar{x}$; $\Sigma d = 0$
- Standard deviation $\sigma = \sqrt{(\Sigma d^2/n)}$ for $n \geq 20$; $\mathbf{s = \sqrt{(\Sigma d^2/(n-1))}}$ for $n < 20$; variance = σ^2
- **Probable error** of one reading $r = 0.6745\,\sigma$; probable error of mean $r_m = r/\sqrt{(n-1)}$
- Gaussian: ±1σ → **68.27%**, ±2σ → 95.45%, ±3σ → **99.73%** of readings; ±0.6745σ → 50%

- Example: 0–150 V voltmeter, accuracy ±1% FSD → ±1.5 V anywhere on scale. At 75 V reading → **±2%**; at 30 V → ±5%.
- Example: P = VI, V with ±1%, I with ±2% → P within **±3%**. P = V^2/R with V ±1%, R ±0.5% → **±2.5%**.

Standard	Where kept / remark
International	Defined by international agreement (BIPM, Sèvres, France); not available for ordinary calibration
Primary	National labs – in India **CSIR-NPL, New Delhi**; used to calibrate secondary standards
Secondary	Industrial/calibration labs (NABL-accredited)
Working	Everyday lab use – standard cells, standard resistors (**manganin**, low temperature coefficient)

- Modern voltage standard: **Josephson junction** array; resistance standard: **quantum Hall effect**. Since 2019 SI redefinition, **h, e, k, N_A** have fixed exact values.
- Calibration hierarchy = **traceability**: working → secondary → primary → international.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Parallax error is which type? → **Observational (systematic)** error; removed by **mirror-backed scale**.
- Error that is reduced by taking many readings and averaging? → **Random error**.
- A 100 A ammeter with ±2% FSD error reads 50 A; limiting error? → **±4%**.
- Probable error in terms of σ? → **0.6745 σ**.
- % error in $P = I^2R$ if I has ±1% and R ±1%? → **±3%**.
- Closeness of repeated readings to each other? → **Precision**.

9.2 Analog Indicating Instruments ★★★

- Three torques are essential: **deflecting** (moves pointer; from magnetic, thermal, electrostatic, induction effect), **controlling** (opposes deflection, gives a definite reading, returns pointer to zero) and **damping** (brings pointer to rest quickly, prevents oscillation). Damping torque acts **only while the moving system moves**.
- **Spring control**: pair of **phosphor-bronze** hairsprings wound in **opposite directions**; $T_c = K\theta$ → with $T_d \propto I$ gives a **uniform scale**; works in any position.
- **Gravity control**: small weight; $T_c \propto \sin\theta$ → **cramped (non-uniform) scale**; instrument must be kept **vertical**; cheap, no ageing.
- Damping should be **slightly under-damped** (in practice) – critical damping is the ideal theoretical condition.

Damping method	Used in	Remark
Eddy current	**PMMC** (aluminium former), induction instruments, energy meter (braking magnet)	Most efficient; needs strong field

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Air friction	**Moving iron, electrodynamometer**, hot-wire	Weak operating field – a permanent magnet for eddy damping would distort it
Fluid friction (oil)	**Electrostatic** instruments (also raises dielectric strength)	Must be kept level; not common

KEY FORMULAE

- PMMC: T_d = **NBAI** = GI ; θ = GI/K → **θ ∝ I** (uniform scale)
- Moving iron: T_d = **½ I² dL/dθ** ; θ = (I^2/2K)(dL/dθ) → **θ ∝ I²** (square-law scale)
- Electrodynamometer: T_d = **$I_1 I_2$ cos φ dM/dθ** ; ammeter/voltmeter θ ∝ I² ; wattmeter θ ∝ VI cos φ
- Electrostatic: T_d = **½ V² dC/dθ** → θ ∝ V²
- Ammeter shunt: **$R_{sh} = R_m/(m-1)$**, m = I/I_m
- Voltmeter multiplier: **$R_s = R_m(m-1)$**, m = V/V_m
- Voltmeter sensitivity **S = 1/I_{fs} Ω/V** ; R_V = S × range

- **PMMC**: permanent magnet (Alnico/ceramic), coil on **aluminium former**, soft-iron cylindrical core (radial uniform field). **DC only** (on AC it reads the **average = zero** for pure sine). **Linear scale**, high sensitivity, low power (25–200 μW), high torque/weight, **eddy-current damping**, accuracy up to ~0.1%. Errors: magnet ageing, spring ageing, temperature (use **manganin swamping resistor**). Not damaged easily but costly.
- Variants: **fluxmeter** (**negligible controlling torque**, heavy electromagnetic damping) and **ballistic galvanometer** (measures charge; small controlling torque, very low damping, long period).
- **Moving iron (MI)**: **attraction** type (iron drawn into coil) and **repulsion** type (two similarly magnetised irons repel – **radial vane / co-axial vane**; more common). Reads **rms**, works on **AC & DC**, **non-uniform (square-law) scale** – cramped at the beginning (shaped vane makes it nearly linear above ~10% FSD). Cheap, robust, high power consumption. Damping: **air friction**.
- MI errors – DC & AC: **hysteresis**, **temperature**, **stray fields**; AC only: **frequency** (coil inductance; corrected by a **capacitor across the series (swamping) resistance** of an MI voltmeter) and **eddy currents**.
- **Electrodynamometer**: fixed coils (current) + moving coil, **air-cored** → no hysteresis/eddy error, same calibration on AC & DC → **transfer instrument** (calibrate on DC, use on AC). Weak field → low sensitivity, high power consumption; **air-friction damping**. Used mainly as a **wattmeter**.
- **Electrostatic**: only **voltmeter**; **quadrant** type (low/medium V) and **attracted-disc** (absolute, very high V) types. Draws **negligible power (no current on DC)**, reads **rms**, square-law scale, used for **high voltages** (kV range); no hysteresis/eddy/stray-field errors.
- **Induction type**: works on rotating/shifting field (shaded pole or Ferraris principle); **AC only**; large scale (~300°), used as ammeter/voltmeter/wattmeter and **energy meter**; frequency and temperature errors large.
- **Rectifier type**: PMMC + bridge rectifier. PMMC responds to **average**, scale marked in **rms for sine** = average × **form factor 1.11**. Reads wrong for non-sinusoidal waves (square wave: FF = 1 → reads **11% high**). Used in multimeters (AC ranges); high sensitivity.
- **Thermocouple instruments**: heater + thermocouple + PMMC. Respond to **heating effect → true rms** for any waveform; usable at **radio frequencies** (up to ~50–100 MHz). Square-law scale; small overload capacity (heater burns out). **Hot-wire** instrument (expansion of Pt-Ir wire) – AC/DC, rms, now obsolete.

Instrument	AC/DC	Reads	Scale	Damping
PMMC	**DC only**	Average	**Uniform**	Eddy current
Moving iron	Both	**rms**	Non-uniform (square law)	Air friction
Electrodynamometer	Both	rms	Square law (V, A); **uniform as wattmeter**	Air friction
Electrostatic	Both	rms	Square law	Fluid/air
Induction	**AC only**	rms	Uniform-ish, long (~300°)	Eddy current
Rectifier	AC (sine)	Average, marked rms (×1.11)	~Uniform	Eddy current
Thermocouple / hot-wire	Both, **RF**	**True rms**	Square law	Eddy current

- Example: PMMC 10 mA FSD, R_m = 5 Ω. Ammeter 0–1 A → m = 100, R_{sh} = 5/99 ≈ **0.0505 Ω**. Voltmeter 0–100 V → V_m = 0.05 V, m = 2000, R_s = 5 × 1999 = **9995 Ω**.
- Example: I_{fs} = 50 μA → S = **20 kΩ/V**; on 10 V range R_V = 200 kΩ. 1 mA → 1 kΩ/V.
- **Ayrton (universal) shunt** – multi-range ammeter, prevents meter from being left unshunted during range change. Shunts of **manganin**; four-terminal construction for heavy currents.
- Ohmmeter: **series type** – zero at right end (full deflection), ∞ at left; **shunt type** – for low resistance, zero at left.

MEMORY TRICK

"**P**MMC = **P**ure DC, **P**roportional scale." "MI = **M**ost **I**ndestructible, reads anything (AC/DC), cramped start." "**E**lectro**S**tatic = **E**xtra **S**mall power, high voltage." Only **air-cored dynamometer** = transfer instrument.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- PMMC connected to a 50 Hz sinusoidal supply reads? → **Zero**.
- Instrument used as a transfer instrument? → **Electrodynamometer** type.
- Instrument for measuring true rms of RF current? → **Thermocouple** type.
- Rectifier voltmeter calibrated on sine reads a square wave of 10 V peak as? → **11.1 V**.
- Type of damping in moving-iron instruments? → **Air friction**.
- Multiplier to extend a 100 Ω, 1 mA meter to 10 V? → **9.9 kΩ**.

9.3 Measurement of Power, Energy & Power Factor ★★★

- **Electrodynamometer wattmeter**: **current coil (CC)** = fixed coils, few turns, thick wire, in **series** with load; **pressure/potential coil (PC)** = moving coil with high non-inductive series resistance, in **parallel** with load. Reads **VI cos φ** on AC (average power), **uniform scale**, AC & DC.
- **Connection error**: PC on **supply side** → reading includes **CC loss (I^2R_c)** → preferred for **low-current** loads; PC on **load side** → reading includes **PC loss (V^2/R_p)** → preferred for **high-current** loads. Eliminated by a **compensating coil** (in series with PC, wound with CC, opposing).
- **PC inductance error**: PC current lags V by β = $\tan^{-1}(\omega L_p/R_p)$. For **lagging pf** the wattmeter **reads high**; error is serious at **low pf**. Compensated by a **capacitor across part of the PC series resistance**.
- Other errors: eddy currents, stray fields (astatic construction / shielding), temperature, vibration of moving system.
- **LPF (low power factor) wattmeter**: full-scale deflection at low pf (e.g. **0.1–0.2**); low-resistance PC (larger PC current), compensating coil, capacitor compensation for PC inductance, very light moving system. Used in **open-circuit test of transformer**.
- Wattmeter range is decided by **current & voltage ratings** – it may be overloaded (CC overheats) while reading below full scale at low pf. Ratings are in **A, V and W**.

KEY FORMULAE

- True power / wattmeter reading (PC inductance) – correction factor K = **cos φ / [cos β · cos(φ − β)]**
- Error ≈ **VI sin φ tan β** (lagging pf, wattmeter reads more)
- Two-wattmeter (balanced, lagging): **$W_1 = V_L I_L \cos(30° - φ)$**, **$W_2 = V_L I_L \cos(30° + φ)$**
- P = $W_1 + W_2 = \sqrt{3} V_L I_L \cos φ$; $W_1 - W_2 = V_L I_L \sin φ$
- **$\tan φ = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)$** ; **$Q = \sqrt{3}(W_1 - W_2)$**
- One-wattmeter reactive power: CC in line R, PC across Y–B → reading = $V_L I_L \sin φ$; **Q = √3 × reading**
- Energy meter: **meter constant K = revolutions per kWh** ; energy recorded = N/K
- % error = (Energy recorded − true energy)/true energy × 100

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pf (balanced, lagging)	φ	W_1 and W_2
1.0	0°	$W_1 = W_2$ (each = P/2)
0.866	30°	$W_1 = 2W_2$
0.5	60°	$W_2 = 0$, W_1 = total power
< 0.5	> 60°	W_2 **negative** – reverse its CC or PC connections, subtract reading
0	90°	$W_1 = -W_2$ (equal & opposite), P = 0

- **Blondel's theorem**: power in an **n-wire** system is measured by **(n − 1) wattmeters**, with the common PC terminal on the wire having no CC. 3-phase 3-wire → **2 wattmeters** (balanced or unbalanced, star or delta); 3-phase 4-wire → **3 wattmeters**.
- **One-wattmeter method** – only for **balanced** load (with access to neutral or artificial star); P = 3W.
- Two-wattmeter method gives total power for **balanced or unbalanced** 3-wire loads, but the **tan φ / pf formula** is valid only for **balanced sinusoidal** loads.

Induction type energy meter (single phase)

- **Driving system**: **series magnet** (current coil – few turns, thick wire) and **shunt magnet** (pressure coil – many turns, thin wire, **highly inductive**). Flux of shunt magnet must lag V by exactly **90°** – achieved by **copper shading bands/loops** on the central limb (**pf / lag adjustment**).
- **Moving system**: thin **aluminium disc** on a spindle (jewel bearings / magnetic suspension), worm gear drives the **register (counting mechanism)**. **No controlling spring** – it is an integrating instrument, disc rotates continuously.
- **Braking system**: **permanent (braking) magnet** near disc edge → eddy-current braking torque **∝ speed**. Driving torque ∝ power → **speed ∝ power**, revolutions ∝ energy. Moving the braking magnet **towards the centre** reduces braking torque → disc **runs faster** (full-load speed adjustment).
- **Creeping**: slow continuous rotation of disc when **only PC is energised (no load)**, due to over-compensation for friction, over-voltage, vibration, stray fields. Prevented by **two diametrically opposite holes** drilled in the disc (creep holes) or a small iron piece on spindle attracted by the brake magnet. Creep test: at **110% rated voltage**, no load, disc must **not complete one revolution**.
- **Friction compensation** (light-load adjustment): small **shading vane/loop** under shunt magnet producing a small forward torque; adjusted at **5–10% of full load** at upf.
- Adjustment sequence: **full load (upf) – brake magnet**; **light load – friction compensator**; **lag (0.5 pf lagging) – shading band**; then creep test.
- Other errors: phase (incorrect lag), speed, frequency, temperature, overload (self-braking by series flux) – overload compensation by **magnetic shunts**.
- **Three-phase energy meter**: two-element (3-wire, Blondel) or three-element (4-wire) on a common spindle.
- Example: meter constant 3600 rev/kWh; load 1 kW for 1 min → true energy 1/60 kWh → should make **60 rev**. If disc makes 59 rev → error = (59 – 60)/60 = **−1.67%** (meter **slow**).
- Example: 230 V, 10 A, upf for 1 h = 2.3 kWh; meter with K = 600 rev/kWh should make **1380 rev**.
- **Electronic (static) energy meters**: voltage & current sensed (potential divider; CT / shunt / Rogowski coil), digitised by ADC, multiplied in a **DSP/energy IC**; output pulses → **LED pulse constant imp/kWh** (replaces rev/kWh). No moving parts, no creep, accuracy class 1.0/0.5S/0.2S, tamper detection, **ToD (time-of-day)** tariff, MD recording.
- **Smart meters**: two-way communication (AMI), remote connect/disconnect, **prepaid** mode, net-metering for rooftop solar. India – **RDSS** scheme targets ~25 crore prepaid smart meters.
- **Maximum demand indicator (MDI)**: records max average demand over a **demand interval (typically 15 or 30 min)**; **Merz-Price** type (pusher pointer on energy meter, reset each month). Thermal type responds to heating (lagging).

Meter	Type / principle	Key fact
Power factor meter	**Electrodynamometer** (crossed moving coils, one with R, other with L) / **moving iron** (rotating field – Nalder-Lipman, Westinghouse)	**No controlling torque**; pointer stays at last position when de-energised
Frequency meter	**Vibrating reed** (Frahm) – steel reeds tuned in steps of **0.5 Hz**; the reed in resonance vibrates with maximum amplitude (with an unpolarised magnet reeds respond to **2f**, scale marked in f); **Weston** (two coils at 90°, one in R, one in L – ratio of currents varies with f); ferrodynamic; electrical resonance	Reed type: robust, discrete reading
Synchroscope	Electrodynamometer / moving iron / Weston (lamp-dial) type; or lamp methods (dark lamp, bright lamp)	Pointer rotates **clockwise = incoming machine fast**; close breaker at **12 o'clock (in phase)**
Phase sequence indicator	**Rotating type** (small induction motor, disc rotation direction) or **static** (two lamps + capacitor/inductor)	Indicates RYB or RBY
Ohmmeter / megger	Cross-coil ratiometer	No control spring

MEMORY TRICK

Two-wattmeter: "**Equal → Unity, One Zero → 0.5, Negative → below 0.5, Equal & Opposite → Zero pf**." Energy-meter creep: "**Holes stop Creep**."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In two-wattmeter method one wattmeter reads zero; load pf? → **0.5**.
- Number of wattmeters for a 3-phase 4-wire unbalanced load? → **3** (Blondel).
- Creeping in energy meter is prevented by? → **Two holes drilled diametrically opposite** in the disc.
- Braking torque in an energy meter is produced by? → **Permanent magnet (eddy currents)**; it is ∝ **speed**.
- Flux of shunt magnet is made to lag voltage by 90° using? → **Copper shading bands**.
- Instrument without controlling torque? → **PF meter / megger / fluxmeter / energy meter**.

9.4 Bridges & Potentiometers ★★★

Resistance class	Range	Method
Low	**< 1 Ω** (μΩ to 1 Ω)	**Kelvin double bridge**, potentiometer, ammeter–voltmeter, Ducter (micro-ohmmeter)
Medium	**1 Ω – 100 kΩ**	**Wheatstone bridge**, substitution, ammeter–voltmeter, ohmmeter, Carey-Foster (comparing two nearly equal R)
High	**> 100 kΩ** (MΩ–TΩ)	**Megger**, **loss of charge**, direct deflection, megohm bridge; use **guard terminal** to bypass surface leakage

- **Wheatstone bridge**: balance $R_1R_4 = R_2R_3$ (products of opposite arms equal), i.e. P/Q = R/S → $R_x = (P/Q)S$. Galvanometer as detector. At low R, **lead and contact resistances** cause error; at high R, leakage and low galvanometer sensitivity.
- **Kelvin double bridge**: second set of ratio arms (p/q = P/Q) eliminates effect of the **link/lead resistance** r → R = (P/Q)S. Four-terminal resistors (current and potential terminals).
- **Loss of charge method**: capacitor C discharges through R: **R = 0.4343 t / [C $\log_{10}$(V/v)]** = t/[C ln(V/v)]; for very high R (insulation of cables).

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KEY FORMULAE – AC BRIDGES

- General balance: $\mathbf{Z_1Z_4 = Z_2Z_3}$ → both **magnitude** ($|Z_1||Z_4| = |Z_2||Z_3|$) and **angle** ($\theta_1 + \theta_4 = \theta_2 + \theta_3$) conditions
- **Maxwell L–C (Maxwell–Wien)** ($C_4 \parallel R_4$ opposite L_1): $\mathbf{L_1 = R_2R_3C_4}$, $R_1 = R_2R_3/R_4$, $\mathbf{Q = \omega C_4R_4}$
- **Hay's** (C_4 in series with R_4): $\mathbf{L_1 = R_2R_3C_4/(1 + \omega^2C_4^2R_4^2)}$, $\mathbf{Q = 1/(\omega C_4R_4)}$; for Q > 10, $L_1 \approx R_2R_3C_4$
- **Schering**: $\mathbf{C_1 = C_2R_4/R_3}$, $r_1 = R_3C_4/C_2$, $\mathbf{D = \tan\delta = \omega C_4R_4}$ ($= \omega C_1r_1$)
- **De Sauty**: $\mathbf{C_1/C_2 = R_4/R_3}$ (lossless capacitors only)
- **Wien** (R_1C_1 series arm, $R_2 \| C_2$ arm): $\mathbf{f = 1/(2\pi\sqrt{(R_1R_2C_1C_2)})}$; $R_3/R_4 = R_1/R_2 + C_2/C_1$; if $R_1 = R_2 = R$, $C_1 = C_2 = C$ → $\mathbf{f = 1/(2\pi RC)}$ and $\mathbf{R_3 = 2R_4}$

AC bridge	Measures	Key point
Maxwell inductance (L-L)	L by comparison with standard variable L	Needs standard inductor
Maxwell L–C	L of **medium-Q coils (1 < Q < 10)**	Balance **independent of frequency**; unsuitable for Q > 10 (huge R_4) and Q < 1 (balance convergence poor)
Hay's	L of **high-Q coils (Q > 10)**	Balance **frequency dependent**
Anderson	L over a wide range; **very low-Q** coils, precise; also self-inductance in terms of C	Modified Maxwell; 5 arms (more complex) but easy convergence
Owen	L over wide range, incremental inductance of iron-cored coils	Uses two standard capacitors; independent of frequency
Schering	**Capacitance & dissipation factor (tan δ)**; insulation of cables, bushings, insulators at **high voltage**	HV arms high impedance; low-voltage arms earthed for safety
De Sauty	C of **lossless (perfect)** capacitors	Simplest C bridge; modified De Sauty for lossy C
Wien	**Frequency** (audio), C; basis of Wien-bridge oscillator	Used in harmonic distortion analysers
Heaviside-Campbell, Campbell, Carey-Foster	**Mutual inductance**	Carey-Foster (Heydweiller) – M in terms of C
Resonance bridge	L, C, frequency	Series LC in one arm

- **Detectors**: **headphones/telephones** – audio range **250 Hz – 4 kHz** (most sensitive ~1 kHz); **vibration galvanometer** – **power & low audio frequencies (5 Hz – ~1000 Hz)**, tuned mechanically to supply frequency; **tunable amplifier / electronic null detector** – wide range; **CRO**.
- Sources: power-frequency supply or **audio oscillator (~1 kHz)**.
- **Wagner earth device**: auxiliary arm that brings the detector junction to **earth potential** → eliminates errors due to **stray capacitance to earth** (used with Schering, high-frequency bridges). Shielding also used.
- **Sliding balance** (Maxwell at low Q): adjusting one element disturbs the other balance – poor convergence.

Potentiometers

- Potentiometer = **null (balance) instrument** comparing unknown emf with a known voltage drop; at balance **draws no current** from source → measures true **emf**, **no loading**, effectively **infinite input impedance**; accuracy independent of galvanometer calibration.
- **Standardisation**: working current set using a **standard cell** – **Weston cadmium (saturated) cell, 1.0186 V at 20 °C** (unsaturated cell ≈ 1.0190 V). Standard cell must not supply more than **~100 µA**; internal resistance ~500–1000 Ω.
- **Crompton (laboratory) DC potentiometer**: 15 dial resistors + slide wire (circular), range ~1.6 V; resolution ~0.1 mV.
- Uses: calibration of **voltmeter** (direct, with volt-ratio box for higher V), **ammeter** (drop across **standard resistance**), wattmeter, measurement of resistance (comparison with standard R).
- **Volt-ratio box**: potential divider (e.g. 1.5 V to 1500 V) to extend potentiometer range.
- **AC potentiometers**: standardised with a **transfer instrument** (electrodynamometer/thermal ammeter) since no AC standard cell exists.
  - **Polar type** – reads magnitude & phase angle directly; e.g. **Drysdale-Tinsley** (phase-shifting transformer).
  - **Coordinate type** – reads in-phase & quadrature components; e.g. **Gall-Tinsley**, Campbell-Larsen.

MEMORY TRICK

"**Hay** is **High** (Q > 10); **Max** is **Medium** (1–10); **Anderson** for the **Anaemic** (low Q)." "**Sch**ering = **Sch**ool for insulators (tan δ, HV)." "**Wien** = **W**hich frequency?"

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Bridge for low resistance (< 1 Ω)? → **Kelvin double bridge**.
- Bridge to measure inductance of a coil with Q = 25? → **Hay's bridge**.
- Dissipation factor of a cable insulation is measured by? → **Schering bridge**.
- Bridge used for frequency measurement? → **Wien bridge**.
- Emf of Weston standard cell? → **1.0186 V** (at 20 °C).
- Effect of stray earth capacitance in AC bridges is eliminated by? → **Wagner earth device**.

9.5 Instrument Transformers ★★★

- Purpose: extend range of AC meters, **isolate** meters/relays from HV, **standardise** meter ratings (1 A/5 A, 110 V), feed protective relays.
- **Current transformer (CT)**: primary in **series** with line (often a single bar – **bar-primary** or window/ring type); secondary feeds ammeter/CC/relay (**very low impedance**) → operates close to **short circuit**. Primary current set by the **load**, not by CT burden.
- Standard secondary current: **1 A or 5 A** (1 A preferred for long leads – lower lead burden I^2R).
- **Never open the CT secondary with primary carrying current** – no secondary mmf to oppose primary, the whole primary current becomes magnetising current → **core saturates**, very **high peaky secondary voltage** (danger to life and insulation), excessive core loss/heating, **residual magnetism** spoils accuracy. Always **short the secondary** (shorting link) before removing a meter.
- **Potential (voltage) transformer (PT/VT)**: primary across line, secondary **110 V** (line) or $110/\sqrt{3}$ V; feeds voltmeters/PCs (**high impedance**) → operates nearly at **open circuit** (like a small, very accurate power transformer). Secondary must **never be short-circuited** (fuse it).
- **CVT (capacitor voltage transformer)**: capacitor divider + intermediate EMVT + tuning reactor; economical for **EHV (≥ 132 kV)**; doubles as coupling capacitor for **PLCC**. Slower transient response than electromagnetic VT.
- One terminal of each secondary is **earthed** for safety.

KEY FORMULAE

- Nominal ratio K_n (rated I_p/I_s or V_p/V_s); actual ratio R; turns ratio $n = N_s/N_p$ (CT)
- **Ratio error (%)** $\mathbf{= (K_n - R)/R \times 100}$ ($= (K_nI_s - I_p)/I_p \times 100$)
- CT actual ratio $\mathbf{R \approx n + (I_w\cos\delta + I_m\sin\delta)/I_s}$ (I_w = core-loss component, I_m = magnetising component of exciting current, δ = secondary burden angle)
- CT phase angle $\theta \approx (180/\pi) \times \mathbf{(I_m\cos\delta - I_w\sin\delta)/(nI_s)}$ degrees
- Burden (VA) = $I_s^2 \times Z_{burden}$; e.g. 15 VA at 5 A → 0.6 Ω ; at 1 A → 15 Ω

- **Ratio error** = actual ratio differs from nominal; **phase-angle error** = secondary (reversed) phasor not exactly 180° from primary – affects **wattmeters, energy meters, pf meters** only (not ammeters/voltmeters).
- Errors caused by **exciting current** (CT) and by **winding impedance drops** (PT). Reduced by **high-permeability core (CRGO, Mumetal, nickel-iron)**, low flux density, short magnetic path, low burden.
- **Turns compensation**: CT – secondary turns reduced by **1–2 turns** below nominal so the ratio error at rated burden ≈ 0; PT – primary turns slightly reduced (or secondary increased). Turns compensation corrects **ratio error only**, not phase angle.
- CT ratio error: increases with burden; positive (+) phase angle normally. In CT, ratio and phase errors **increase at low primary current** (since I_e is a larger fraction).

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Accuracy class (IS 16227 / IEC 61869)	Use
CT **0.1, 0.2, 0.5, 1** (and 0.2S, 0.5S special), 3, 5	**Metering** – class = % ratio error at rated current; 0.2S/0.5S for revenue (tariff) meters, accurate down to 1% load
CT **5P, 10P** with ALF, e.g. **5P20**	**Protection** – composite error ≤ **5% / 10%** up to **ALF × rated current** (accuracy limit factor 5, 10, 15, 20, 30)
CT **class PS / PX**	Differential & distance protection – specified by **knee-point voltage**, magnetising current, R_{ct}
VT **0.1, 0.2, 0.5, 1, 3**	Metering (% voltage error at 80–120% of rated V)
VT **3P, 6P**	Protection (5% to V_f × rated)

- **Knee-point voltage**: point on excitation curve where a **10% increase in voltage** produces a **50% increase in exciting current**.
- **Instrument security factor (ISF / FS)** for metering CTs – usually **≤ 5 (or 10)**: metering CT should **saturate** at fault currents to protect meters. Protection CT should **not saturate** up to ALF × I_n.
- Metering CT works at low flux density; protection CT core larger.

Point	CT	PT	Power transformer
Connection	Series	Parallel (shunt)	Parallel
Operating condition	Near **short circuit**	Near **open circuit**	Varies with load
Primary quantity depends on	Line load (not burden)	Supply voltage	Secondary load
Flux density	Low; varies with line current	Nearly constant	Nearly constant, high (~1.5–1.7 T)
Secondary rating	1 A / 5 A	110 V	Any
Dangerous condition	**Open secondary**	**Short secondary**	Short circuit
Design criterion	Accuracy (ratio, phase)	Accuracy	Efficiency, regulation

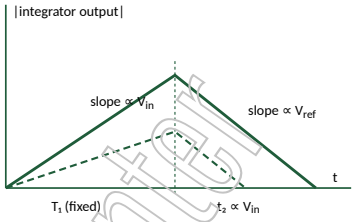
PREVIOUSLY ASKED / LIKELY QUESTIONS

- Why must CT secondary never be opened on load? → Core saturates and **dangerously high voltage** is induced in secondary.
- Standard secondary voltage of a PT? → **110 V**.
- Rated secondary current of a CT? → **1 A or 5 A**.
- Phase-angle error of CT affects readings of? → **Wattmeter / energy meter** (not ammeter).
- Meaning of 5P20? → Composite error ≤ **5%** up to **20 × rated current** (protection CT).
- Voltage transformer type economical at 400 kV? → **CVT**.

9.6 Digital Instruments ★★★

- Advantages over analog: **no parallax/observational error**, high accuracy & resolution, high input impedance (**~10 MΩ** typical DVM), auto-ranging, auto-polarity, data output (BCD, USB).
- Basic DVM = signal conditioning (attenuator, amplifier) + **ADC** + counter + display (LED/LCD/7-segment).

DVM / ADC type	Principle	Feature
Ramp (single-slope)	Time for linear ramp to go from V_{in} to 0 (or 0 to V_{in}) counted with clock	Simple; error from **ramp non-linearity, clock drift, noise**; accuracy ~±0.05%
Dual-slope integrating	Integrate V_{in} for fixed time T_1, then integrate $-V_{ref}$ till zero (time t_2): $\mathbf{V_{in} = V_{ref} \times t_2/T_1}$	**Most accurate**; independent of R, C & clock frequency; excellent **noise (normal-mode) rejection** when T_1 = multiple of mains period (**20 ms** for 50 Hz); **slow**; used in DMMs
Successive approximation (SAR)	Binary search with DAC + comparator, MSB first	**Fast**; conversion time = **n clock cycles** for n bits (fixed)
Flash (parallel)	$\mathbf{2^n - 1}$ **comparators**	**Fastest**; costly (8-bit → 255 comparators); DSOs
Integrating V-to-F	V → pulse frequency, counted	Good noise rejection
Counter / tracking ramp (staircase)	Counter + DAC; up to 2^n clocks	Slowest variable time
Sigma-delta (Σ-Δ)	Oversampling + digital filter	High resolution (16–24 bit)



Dual-slope DVM: run-up for fixed T_1, run-down at constant slope; larger V_{in} (solid) gives longer t_2 than smaller V_{in} (dashed).

KEY FORMULAE

- n full digits → resolution $\mathbf{R = 1/10^n}$; a ½ digit (leading digit 0 or 1) is not counted in n
- **3½ digit**: display 0000 to **1999** (≈ 2000 counts), R = $1/10^3$ = **0.001**; on 1 V range resolution = **1 mV**
- 4½ digit: max **19999**, R = 10^{-4}; 3¾ digit: max 3999 (4000 counts)
- Sensitivity S = (lowest full-scale range) × R ; e.g. 4½ digit, lowest range 100 mV → S = 100 mV × 10^{-4} = **10 µV**
- Accuracy spec: ±(% of reading + n counts/digits); e.g. ±(0.5% rdg + 1 digit), 3½ digit reading 1.000 V on 2 V range → ±(5 mV + 1 mV) = **±6 mV**
- Frequency counter: $\mathbf{f = N/T_{gate}}$; ±1 count error → relative error = ±1/N = $\mathbf{\pm 1/(f\,T_{gate})}$
- Bits vs resolution: n-bit ADC → 2^n levels, step (LSB) = $V_{FS}/2^n$ (some texts $V_{FS}/(2^n - 1)$); 10-bit, 10 V → ≈ **9.77 mV**

- In sensitivity numericals (Sawhney/Helfrick) the "lowest full-scale range" is used exactly as stated in the question (e.g. 1 V range, 3½ digit → 1 mV).
- **DMM**: V (AC/DC), I, R, continuity, diode, often C, f, temperature. Average-responding DMMs are rms-calibrated for sine; **true-rms DMMs** (thermal or computing) needed for distorted waves. **Crest factor** limit (e.g. 3) specified for true-rms meters.
- **Digital frequency meter**: input → Schmitt trigger (shaping) → main gate opened for precise **gating time** (1 ms, 10 ms, 0.1 s, 1 s, 10 s) derived from **crystal oscillator time base** (e.g. 1 MHz/10 MHz, often oven-controlled) → decade counters → display. Errors: **±1 count (gating) error** and **time-base error** (crystal drift, ageing).
- Example: f = 1 kHz, gate 1 s → N = 1000, ±1 count = **±0.1%**; gate 10 s → ±0.01%.
- **Period measurement**: input period opens gate, clock pulses counted → T = N × T_{clock}. Preferred at **low frequencies** (gives better resolution than frequency mode); **multiple-period averaging** improves accuracy. Crossover frequency $f_c = \sqrt{(f_{clock}/T_{gate})}$ roughly.
- **Time-interval** measurement (start/stop channels), **ratio** measurement (f_1/f_2), totalising mode. **Prescaler** (divider) extends counter to GHz.
- **Digital phase meter**: zero-crossing detectors set/reset a flip-flop; pulse width/period → φ = (t_d/T) × 360°.
- **Q-meter**: works on **series resonance**; at resonance $V_C = Q \times V_{in}$ → $\mathbf{Q = V_C/V_{in}}$ (voltmeter calibrated directly in Q). Measures Q, L, C, distributed (self) capacitance, effective resistance. Distributed capacitance by doubling frequency: $\mathbf{C_d = (C_1 - 4C_2)/3}$ (when $f_2 = 2f_1$). Errors: **insertion resistance** of the small injection resistor (~0.02 Ω), residual inductance, voltmeter loading.
- **LCR meter**: auto-balancing bridge / V-I vector ratio method at selectable test frequencies (100 Hz, 1 kHz, 10 kHz, 100 kHz); displays L, C, R, Q, D, ESR. Four-terminal (Kelvin) test leads for low impedance.
- **Data acquisition system (DAS)**: transducer → signal conditioner → **multiplexer** → **sample-and-hold** → ADC → processor/storage/display. Sampling must satisfy **Nyquist: $\mathbf{f_s \geq 2\,f_{max}}$**; **anti-aliasing low-pass filter** before ADC. SCADA/RTU in substations is a DAS application.

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MEMORY TRICK

"**Flash is Fastest, Dual-slope is Dead-accurate (and slow), SAR is in the middle.**"
3½ digit → "**1999 and one in a thousand.**"

PREVIOUSLY ASKED / LIKELY QUESTIONS

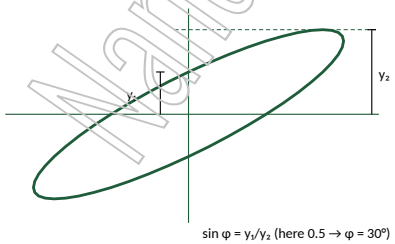
- DVM with best noise rejection? → **Dual-slope integrating** type.
- Fastest ADC? → **Flash (parallel) type**; an 8-bit flash needs **255** comparators.
- Resolution of a 3½-digit DVM on 1 V range? → **1 mV** (max display 1999).
- Error inherent in all digital counters? → **±1 count** error.
- For low frequency signals a counter should be used in? → **Period mode**.
- Q-meter works on the principle of? → **Series resonance**.

9.7 Oscilloscope & Other Measurements ★★★

- **CRO blocks**: **CRT**, vertical (Y) attenuator + amplifier, **delay line** (lets the leading edge of signal be seen after trigger), **trigger circuit**, **time-base (sweep) generator** producing a **sawtooth** applied to **horizontal (X) plates**, horizontal amplifier, HV & LV power supplies.
- **CRT**: **electron gun** – heated cathode, **control grid** (negative; controls **intensity/brightness**), pre-accelerating, **focusing** and accelerating anodes (electrostatic lens); **deflection plates** – vertical first (nearer gun, more sensitive) then horizontal; **fluorescent screen** (phosphor; **P31** green common); **aquadag** (graphite) coating collects secondary-emission electrons. CRO uses **electrostatic** deflection; TV picture tubes used **magnetic** deflection.
- Sawtooth time base: **sweep (trace)** linear ramp + rapid **flyback (retrace)** – blanked. Triggered sweep synchronises the waveform → stable display when sweep starts at the same point (level & slope) of the signal.
- **Dual-trace** CRO (one gun, electronic switch): **alternate mode** for **high** frequency, **chopped mode** for **low** frequency signals. **Dual-beam** CRO has two guns – truly simultaneous display.
- **X-Y mode**: time base off, signals to both plates → **Lissajous patterns**. Z-axis input → intensity modulation (frequency measurement by blanking).
- CRO input impedance typically **1 MΩ ‖ 15–30 pF**. Bandwidth–rise-time: $\mathbf{t_r = 0.35/BW}$ (e.g. 100 MHz → 3.5 ns). Scope rise time should be ~3–5 times faster than signal.

KEY FORMULAE

- Electrostatic deflection on screen: $\mathbf{D = L\,l\,E_d/(2\,d\,E_a)}$ (L = plate-centre to screen distance, l = plate length, d = plate spacing, E_a = accelerating voltage, E_d = deflecting voltage)
- Deflection sensitivity $\mathbf{S = D/E_d = L\,l/(2\,d\,E_a)}$ m/V ; deflection factor G = 1/S (V/m, V/div) ; $S \propto 1/E_a$
- Electron velocity $v = \sqrt{(2eE_a/m)}$
- Lissajous: $\mathbf{f_y/f_x}$ **= (no. of horizontal tangencies)/(no. of vertical tangencies)**
- Phase from ellipse: $\mathbf{\sin\varphi = y_1/y_2}$ (y-intercept ÷ max vertical deflection) = x_1/x_2
- Period on screen T = (divisions) × (time/div) ; $V_{p\text{-}p}$ = (divisions) × (V/div) × probe factor



Lissajous ellipse for two equal-frequency sine waves; y_1 = intercept on Y-axis, y_2 = maximum vertical deflection.

Phase difference (equal f, equal amplitude)	Lissajous figure
0°	**Straight line at 45°** (positive slope)
0°–90° (or 270°–360°)	Ellipse, major axis in 1st–3rd quadrant
90°	**Circle** (unequal amplitudes → ellipse with axes along X, Y)
90°–180°	Ellipse in 2nd–4th quadrant
180°	Straight line at **135°** (negative slope)

- Example: pattern touches a horizontal line at 2 points and a vertical line at 1 point → f_y/f_x = 2/1; if f_x = 50 Hz, $\mathbf{f_y = 100\ Hz}$.
- **DSO (digital storage oscilloscope)**: input → ADC (often flash) → memory → display. Specs: **sampling rate** (GS/s; Nyquist ≥ 2f, practically ≥ 5–10 × f for faithful shape, else **aliasing**), **bandwidth**, **record length (memory depth)**, vertical resolution (8 bit typical). Advantages: **stores single-shot transients**, **pre-trigger viewing** (events before the trigger), automatic measurements, FFT, PC interface. Analog storage CRO used **bistable/variable-persistence** storage tubes.
- **Sampling oscilloscope**: equivalent-time sampling of **repetitive** very high-frequency signals (GHz).
- **Probes**: **10:1 passive probe** – 9 MΩ in series with scope's 1 MΩ, with a **compensating capacitor** → reduces loading (10 MΩ input, lower C), attenuates by 10. Compensation condition $\mathbf{R_1C_1 = R_2C_2}$; adjusted with the scope's **~1 kHz square wave calibrator**: **under-compensated → rounded edges**, **over-compensated → overshoot/spikes**. Active (FET) probes, current probes (Hall effect/CT), **differential probes** for floating measurements.
- **Spectrum analyser**: displays amplitude vs **frequency** (frequency domain) – superheterodyne swept-tuned or FFT type; harmonic content, EMI. **Wave analyser** – amplitude of individual harmonics (narrow-band); **harmonic distortion analyser** – total harmonic distortion (THD) by rejecting fundamental (Wien-bridge notch).
- **Function generator**: **sine, square, triangular** (and sawtooth, pulse) from an integrator + Schmitt trigger loop; sine by diode shaping network; frequency ~0.01 Hz–MHz. **Signal generator** – RF with modulation (AM/FM).

Transducers (brief)

Transducer	Principle / key fact
RTD	Resistance rises with T (PTC); **platinum Pt100 = 100 Ω at 0 °C**, α ≈ **0.00385 /°C**; most linear & stable; −200 to ~850 °C; $R_t = R_0(1 + \alpha t)$; 3-/4-wire connection cancels lead resistance
Thermistor	Semiconductor (sintered metal oxides – Mn, Ni, Co); usually **NTC**; **very high sensitivity** (~−4%/°C), **non-linear** (exponential), small range (−60 to ~300 °C)
Thermocouple	**Seebeck effect**; self-generating (active); needs **cold (reference) junction compensation**. Type **K** (chromel–alumel, most common, up to ~1250 °C), **J** (iron–constantan), **T** (copper–constantan, low temp), **R/S** (Pt–Rh, up to ~1600 °C), B (highest)
LVDT	Passive inductive; **one primary, two secondaries in series opposition**; soft-iron core; output zero at **null**; magnitude ∝ displacement, **phase gives direction**; linear, frictionless, infinite resolution; RVDT for angle
Strain gauge	Resistance change with strain; **GF = (ΔR/R)/(ΔL/L) = 1 + 2ν + (Δρ/ρ)/ε**; metal (constantan, advance) **GF ≈ 2**; semiconductor GF ≈ **100–200** (but temperature sensitive); used in Wheatstone bridge (dummy gauge for temperature compensation); load cells
Hall effect	$V_H = BI/(n\,e\,t)$ (∝ B × I); measures **magnetic field, DC/AC current (clamp meters)**, position/speed; semiconductor (InSb, GaAs)
Piezoelectric	Quartz, PZT; active; **dynamic** force/pressure/acceleration only (cannot measure static)
Others	Capacitive (displacement, level), photo-conductive (LDR), photovoltaic (active), tacho-generator (speed)

- **Active** (self-generating): thermocouple, piezoelectric, photovoltaic cell, tacho-generator. **Passive** (need excitation): RTD, thermistor, strain gauge, LVDT, capacitive, Hall element (needs bias current).

Earth resistance & insulation resistance

- **Earth tester** (megger earth tester): hand-driven/battery generator with **current reverser** and **rotating rectifier** → AC through soil avoids **electrolytic polarisation (back emf)**, DC to the ohmmeter ratiometer; four terminals (P1, C1, P2, C2).
- **Fall-of-potential (three-point) method**: current electrode C driven far (≥ ~5× electrode size, typically 30–50 m), potential electrode P moved between;

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correct reading when P is at **~62% of the E–C distance** (flat part of the curve). R = V/I.

- Soil resistivity: **Wenner four-electrode method** – equally spaced (a) electrodes, **ρ = 2πaR**.
- Typical acceptable earth resistance (IS 3043 / utility practice): large power stations **≈ 0.5 Ω**, EHV/major substations **≈ 1 Ω**, small substations ≈ 2 Ω, distribution transformer/LT neutral ≈ 2–5 Ω (values vary by utility – read options carefully). Earth resistance is lower in **wet/salty** soil; improved by salt-charcoal, longer/parallel electrodes, chemical earthing.
- **Megger (insulation tester)**: hand-driven or electronic **DC** generator + **cross-coil (ratiometer) ohmmeter** – current (deflecting) coil and pressure (control) coil; **no controlling spring**, pointer rests anywhere when idle; open terminals → **∞**, shorted → **0**. Reading independent of generator speed (ratio). **Guard terminal** bypasses surface leakage current.

Megger test voltage (DC)	Typical application
250 V	ELV/SELV circuits, electronics
500 V	LV equipment & wiring up to ~500 V (domestic 230/415 V installations)
1 kV	LV (up to 1.1 kV) cables, motors, switchgear
2.5 kV	3.3 kV – 6.6 kV equipment
5 kV (and 10 kV)	**11 kV and above** – HV cables, transformers, generators

- LV installation acceptance (IS 732:2019, IEC 60364-6 based): test at **500 V DC** for circuits up to 500 V → IR ≥ **1 MΩ**; SELV/PELV at 250 V → ≥ 0.5 MΩ; above 500 V at 1000 V → ≥ 1 MΩ. Older rule-of-thumb: IR ≥ **50/number of outlets MΩ**. Rotating machines: ≥ (kV + 1) MΩ.
- **Polarisation index (PI) = $IR_{10\ min}/IR_{1\ min}$**: < 1 dangerous, 1–2 questionable, **≥ 2 good** (for class B/F machine insulation). **Dielectric absorption ratio (DAR) = $IR_{60\ s}/IR_{30\ s}$** (≥ 1.25–1.4 acceptable).
- IR **decreases** with rise in temperature and humidity; corrected to a reference temperature (e.g. 40 °C; IR roughly halves every 10 °C rise).

MEMORY TRICK

Lissajous: "**Y over X = Horizontal over Vertical** (touches)." Earth tester P-spike: "**62% – the magic spot**." Probe: "**Under = Rounded, Over = Overshoot**."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Time-base signal applied to horizontal plates of CRO? → **Sawtooth** wave.
- Lissajous figure for two equal-amplitude, equal-frequency signals 90° apart? → **Circle**.
- Intensity (brightness) of CRT spot is controlled by? → **Control grid** voltage.
- Feature of DSO not possible in analog CRO? → **Pre-trigger viewing / single-shot capture**.
- Transducer with a null position and phase-sensitive output? → **LVDT**.
- Gauge factor of a metallic strain gauge? → **≈ 2**.

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10. Analog Electronics

NOTE

Analog electronics usually gives 4–7 of the 60 questions. Most are one-line facts: rectifier figures (ripple factor, efficiency, PIV, TUF), the relation between α and β, CE/CB/CC properties, which biasing circuit is most stable, Barkhausen criterion and oscillator frequency/gain, op-amp gain formulae, slew rate and the 555 timer formulae. Learn the tables in this chapter thoroughly. Numerical questions are usually 1–2 step substitutions.

10.1 Semiconductor Diodes ★★★

- **Intrinsic semiconductor** = pure Si or Ge, with **$n = p = n_i$**. At **0 K** it behaves as an **insulator**. Semiconductors have a **negative temperature coefficient of resistance**: resistance falls as temperature rises.
- Si and Ge are **tetravalent** (4 valence electrons) with a **diamond-cubic** covalent lattice.
- **Energy gap** at 300 K: **Si ≈ 1.1 eV (1.12)**, **Ge ≈ 0.67–0.72 eV**, GaAs ≈ **1.43 eV**. At 0 K: Si 1.21 eV, Ge 0.785 eV. E_g decreases slightly as temperature rises.
- n_i at 300 K: **Si ≈ 1.5 × 10^{10} /cm³** (newer texts give 1 × 10^{10}), **Ge ≈ 2.5 × 10^{13} /cm³**. $n_i^2 \propto T^3 e^{-Eg/kT}$.
- **Mass-action law**: **$n \cdot p = n_i^2$**, which holds for both intrinsic and extrinsic material in thermal equilibrium.
- **n-type**: doped with **pentavalent donors** (**P, As, Sb**, Bi). Electrons are the **majority** carriers. The donor level lies just **below the conduction band** (≈ 0.01 eV in Ge, 0.05 eV in Si).
- **p-type**: doped with **trivalent acceptors** (**B, Al, Ga, In**). Holes are the majority carriers. The acceptor level lies just **above the valence band**.
- Doped material as a whole is **electrically neutral**. In n-type, $n \approx N_D$ and $p = n_i^2/N_D$.
- **Mobility of electrons > mobility of holes**, so n-type devices (npn, n-channel) are faster. Si: μ_n ≈ **1350–1500**, μ_p ≈ **450–480** cm²/V·s. Ge: μ_n ≈ 3900, μ_p ≈ 1900 cm²/V·s.
- **Conductivity** $\sigma = q(n\mu_n + p\mu_p)$. **Drift** current is due to the electric field. **Diffusion** current is due to a concentration gradient.
- **Einstein relation**: $D/\mu = \mathbf{V_T = kT/q}$.
- **Fermi level**: at the middle of the gap in an intrinsic semiconductor, moves **towards the conduction band** in n-type and **towards the valence band** in p-type.
- **PN junction**: diffusion of majority carriers leaves immobile ions behind, forming the **depletion (space-charge) region** and the **barrier potential**. Barrier potential: **Si ≈ 0.7 V**, **Ge ≈ 0.3 V**. Cut-in voltage V_γ: Si 0.5–0.7 V, Ge 0.1–0.3 V.
- **Forward bias** (p to +) **narrows** the depletion layer. **Reverse bias widens** it, and only a small reverse saturation current I_s (made of **minority** carriers) flows. I_s is of the order of nA for Si and μA for Ge.
- The depletion region extends **more into the lightly doped side**.
- **Transition (depletion) capacitance C_T** dominates in **reverse bias**. For an abrupt junction $C_T \propto 1/\sqrt{V_R}$; for a linearly graded junction $C_T \propto V_R^{-1/3}$. **Diffusion (storage) capacitance C_D** dominates in **forward bias**, $C_D \propto I$, and $C_D \gg C_T$.
- **Reverse recovery time** t_{rr} = storage time t_s + transition time t_t. It limits the switching speed of diodes.

KEY FORMULAE

- Diode (Shockley) equation: **$I = I_s(e^{V/\eta V_T} - 1)$**. η = 1 for Ge, η ≈ 2 for Si at low current (≈ 1 for Si at moderate current; texts differ).
- Thermal voltage: **$V_T = kT/q = T/11600 \approx 26$ mV at 300 K** (25.85 mV; 25 mV at room temperature in some texts).
- Dynamic (ac) resistance: **$r_d = \eta V_T/I_D \approx 26\ \text{mV}/I_D$**. Example: at 1 mA, $r_d \approx 26\ \Omega$.
- Reverse saturation current I_s **doubles for every 10 °C** rise (≈ 7 %/°C): $I_s(T_2) = I_s(T_1) \cdot 2^{(T_2 - T_1)/10}$.
- Forward voltage at constant current **falls by ≈ 2.5 mV/°C** (2–2.5 mV/°C depending on the text).
- Zener regulator: $I_S = (V_{in} - V_Z)/R_S$, **$I_Z = I_S - I_L$**. $R_{S(max)} = (V_{in(min)} - V_Z)/(I_{Z(min)} + I_{L(max)})$.

Breakdown	Zener breakdown	Avalanche breakdown
Doping	**Heavy** (thin depletion layer)	Light
Voltage	**< 5 V** (some texts < 4 V)	**> 6 V**
Mechanism	High field (~3 × 10^7 V/m) **pulls electrons out of covalent bonds** (field emission or tunnelling)	**Impact ionisation**: carrier multiplication by collision
Temperature coefficient	**Negative**	**Positive**
Knee	Soft (gradual)	Sharp

- A Zener diode of about **5–6 V** has a **near-zero temperature coefficient** because the two mechanisms cancel.
- A Zener diode is operated in **reverse bias** as a **voltage regulator / reference** and is connected in **parallel (shunt)** with the load. Without a series resistor the Zener current is not limited and the diode is destroyed.
- Zener rating: $P_Z = V_Z I_{Z(max)}$. A good regulator has a low dynamic Zener resistance r_z.

Special diode	Bias / principle	Key facts & use
LED	Forward bias; **electroluminescence** (radiative recombination)	Direct-gap materials: GaAs (IR), GaAsP (red/yellow), GaP (green), **GaN/InGaN (blue, white)**. Si/Ge are not used because they are **indirect-gap** materials. V_F ≈ 1.5–3.5 V.
Photodiode	**Reverse bias**	Reverse current ∝ light intensity. Dark current flows with no light. Used in light detection and optical communication.
Solar cell	**No external bias** (photovoltaic)	Works in the **4th quadrant** of the V–I characteristic. Fill factor ≈ 0.7–0.8.
Varactor (varicap)	**Reverse bias**	Voltage-variable capacitance (C_T). Used for **electronic tuning**, AFC, FM modulators and parametric amplifiers.
Schottky	Forward; **metal–semiconductor** junction	**Majority-carrier** device with no minority-charge storage, so it has **very fast switching** and negligible t_{rr}. V_γ ≈ **0.2–0.3 V**. Used in SMPS, RF circuits and Schottky TTL.
Tunnel (Esaki)	**Very heavily doped** (~1 in 10^3)	**Negative-resistance region** (peak and valley currents) caused by **quantum tunnelling**. Used in **microwave oscillators** and very fast switches.
PIN	Intrinsic layer between p and n	Acts as a variable resistor at RF and microwave frequencies. Used in **RF switches, attenuators** and fast photodetectors.
Laser diode	Forward bias, stimulated emission	Gives coherent, monochromatic light. Used in optical-fibre communication.
Step-recovery / IMPATT / Gunn	Microwave devices	Step-recovery diodes are used as frequency multipliers. IMPATT diodes use avalanche transit time. The Gunn device is a **transferred-electron device** in GaAs with no p-n junction.

MEMORY TRICK

Dopant groups: "**P-A-S** gives elec**trons**" (P, As, Sb are donors). "**B-A-G-I** gives holes" (B, Al, Ga, In are acceptors). The photodiode, varactor and Zener are the "reverse three". The LED and Schottky diode work forward-biased, and the solar cell needs no bias.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Which impurity makes Si n-type: boron, gallium, arsenic or indium? → **Arsenic** (pentavalent).
- Thermal voltage at 300 K? → **≈ 26 mV**.
- Dynamic resistance of a diode carrying 2 mA (η = 1)? → **13 Ω**.
- Diode having a negative-resistance region? → **Tunnel diode**.
- Temperature coefficient of avalanche breakdown? → **Positive**.
- Diode with the lowest forward drop and the fastest switching? → **Schottky diode**.

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10.2 Diode Circuits – Rectifiers, Clippers, Clampers ★★★

All values below assume ideal diodes, a resistive load and a sinusoidal input $V_m \sin \omega t$ ($I_m = V_m/R_L$).

Parameter	Half-wave	Centre-tap full-wave	Bridge full-wave
No. of diodes	1	2	4
V_{dc} (I_{dc})	$\mathbf{V_m/\pi = 0.318\ V_m}$	$\mathbf{2V_m/\pi = 0.636\ V_m}$	$\mathbf{2V_m/\pi}$
V_{rms} (I_{rms})	$\mathbf{V_m/2}$ ($I_m/2$)	$\mathbf{V_m/\sqrt{2}}$ ($I_m/\sqrt{2}$)	$V_m/\sqrt{2}$
Form factor (rms/avg)	**1.57**	**1.11**	1.11
Peak factor	2	$\sqrt{2} = 1.414$	1.414
Ripple factor γ	**1.21**	**0.482**	**0.482**
Max. rectification efficiency	**40.6 %**	**81.2 %**	**81.2 %**
PIV per diode	$\mathbf{V_m}$	$\mathbf{2V_m}$	$\mathbf{V_m}$
TUF	**0.287**	**0.693**	**0.812**
Ripple frequency	**f** (50 Hz)	**2f** (100 Hz)	**2f** (100 Hz)
Remarks	DC saturation of the transformer core	Needs a centre-tapped transformer; each half carries current for half a cycle	No centre tap; **two diode drops** in the path; most widely used

- Ripple factor $\gamma = V_{r,rms}/V_{dc} = \sqrt{(FF^2 - 1)}$. HW: $\sqrt{(1.57^2 - 1)} = 1.21$. FW: $\sqrt{(1.11^2 - 1)} = 0.482$.
- Efficiency $\eta = P_{dc}/P_{ac}$. HW: $4/\pi^2 = 40.6$ %. FW: $8/\pi^2 = 81.2$ %, assuming diode resistance $r_f = 0$.
- The transformer of a half-wave rectifier carries DC through its secondary, which causes **core saturation**. This is why its TUF is the lowest. For the centre-tap rectifier, primary TUF = 0.812, secondary TUF = 0.574 and average TUF = **0.693**.
- For the same DC output, the bridge rectifier needs diodes with **half the PIV** of a centre-tap rectifier.
- A **three-phase** half-wave rectifier has ripple frequency 3f and γ ≈ 0.17. A three-phase bridge (6-pulse) has ripple frequency **6f** and γ ≈ **0.042**.

KEY FORMULAE – FILTERS

- **Capacitor (shunt) filter, full-wave**: $\gamma = 1/(4\sqrt{3}\, f R_L C)$. Half-wave: $\gamma = 1/(2\sqrt{3}\, f R_L C)$.
- Peak-to-peak ripple: $V_{r(p\text{-}p)} \approx I_{dc}/(fC)$ for half-wave and $I_{dc}/(2fC)$ for full-wave. $V_{dc} = V_m - V_{r(p\text{-}p)}/2$.
- **Inductor (series) filter**, full-wave: $\gamma = R_L/(3\sqrt{2}\,\omega L)$. Ripple falls as the **load current increases** (R_L falls), the opposite of a capacitor filter.
- **LC (choke-input) filter**: $\gamma = \sqrt{2}/(12\omega^2 LC)$, which is **independent of the load**. Critical inductance $L_c = R_L/(3\omega)$.
- A capacitor filter suits **light loads (high R_L)**. An inductor filter suits **heavy loads**. A CLC (π) filter gives the lowest ripple but poor regulation.

- **Clipper (limiter)**: removes the part of a waveform above or below a reference level **without distorting the remaining part**.
  - **Series clipper**: the diode is in series with the load. **Shunt clipper**: the diode is in parallel with the load.
  - **Biased clipper**: a DC source V_R in series with the diode sets the clipping level at $V_R + V_\gamma$.
  - **Double-ended (slicer)** clippers use two biased diodes. Back-to-back Zener diodes give a symmetrical limiter.
- **Clamper (DC restorer / level shifter)**: uses a diode, a capacitor and a resistor to **add a DC level**. The waveform shape and its **peak-to-peak value are preserved**.
  - A **positive clamper** shifts the waveform up so that its negative peak sits at 0 V. A $\pm V_m$ input becomes 0 to $+2V_m$.
  - Condition: the time constant must be large, $\mathbf{R_L C \gg T/2}$ (typically 5RC ≥ 10 × T/2).
  - Used in TV receivers to restore the DC level of the video signal.
- **Voltage multipliers** are built from clamper and peak-rectifier stages.
  - **Half-wave doubler**: output ≈ $2V_m$, PIV of each diode = $2V_m$, ripple at f.
  - **Full-wave doubler**: ripple at 2f.
  - Tripler gives $3V_m$, quadrupler gives $4V_m$. A Cockcroft–Walton multiplier with n stages gives ≈ $2nV_m$.
  - Multipliers suit only **low-current, high-voltage** loads (CRT, X-ray, photocopier).
- **Peak detector** = a diode and a capacitor. It charges to V_m.

MEMORY TRICK

Rectifier "money table": **1.21 → 0.482** (ripple), **40.6 → 81.2** (efficiency doubles), **TUF 0.287 / 0.693 / 0.812** ("2-8-7, 6-9-3, 8-1-2"). PIV: only the centre-tap rectifier is "**doubly stressed**" ($2V_m$). A clipper **cuts**; a clamper **shifts**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Ripple factor of a full-wave rectifier without filter? → **0.482**.
- PIV of a centre-tap full-wave rectifier? → $2V_m$.
- Ripple frequency of a bridge rectifier on 50 Hz mains? → **100 Hz**.
- Highest TUF among single-phase rectifiers? → **Bridge (0.812)**.
- Circuit that changes the DC level but keeps the shape and peak-to-peak value of a signal? → **Clamper**.
- Filter whose ripple is independent of load current? → **Choke-input (LC) filter**.

10.3 BJT – Characteristics & Biasing ★★★

- **BJT** = a **bipolar** device (both electrons and holes take part) and **current-controlled**. Three regions:
  - **Emitter**: heavily doped, medium width.
  - **Base**: **very thin and lightly doped**.
  - **Collector**: moderately doped and **largest** in size because it dissipates heat.
- In an **npn** transistor the main carriers are electrons, so it is **faster than pnp**. The arrow on the emitter shows the direction of conventional current: **out** for npn ("Not Pointing iN") and **in** for pnp.
- $\mathbf{I_E = I_B + I_C}$. $I_C = \alpha I_E + I_{CBO} = \beta I_B + I_{CEO}$.
- **Early effect (base-width modulation)**: increasing V_{CB} widens the CB depletion layer and reduces the effective base width. This has four results:
  - Less recombination, so α and β rise.
  - The output characteristics slope upwards.
  - A finite output resistance $r_o = \mathbf{V_A/I_C}$ (Early voltage $V_A \approx 50$–150 V).
  - At the extreme, **punch-through (reach-through)** breakdown.
- $BV_{CEO} < BV_{CBO}$: $BV_{CEO} \approx BV_{CBO}/\beta^{1/n}$.
- High-frequency figures: $\mathbf{f_\beta}$ = frequency where β falls by 3 dB. $\mathbf{f_T}$ = frequency where $|\beta| = 1$. $\mathbf{f_T \approx \beta f_\beta}$. $f_\alpha \approx f_T$.

Region	E–B junction	C–B junction	Application
Active (forward-active)	Forward	**Reverse**	**Amplifier**
Saturation	Forward	**Forward**	Switch **ON** ($V_{CE(sat)} \approx 0.2$ V for Si)
Cut-off	Reverse	Reverse	Switch **OFF**
Reverse (inverse) active	Reverse	Forward	Rarely used (low β); TTL input transistor

KEY FORMULAE

- $\alpha = I_C/I_E$ (0.95–0.995, always < 1). $\beta = I_C/I_B$ (20–500).
- $\mathbf{\beta = \alpha/(1-\alpha)}$; $\mathbf{\alpha = \beta/(1+\beta)}$; γ (CC current gain) $= I_E/I_B = \mathbf{1 + \beta = 1/(1-\alpha)}$.
- $\mathbf{I_{CEO} = (1+\beta) I_{CBO}}$. I_{CBO} doubles every 10 °C, and V_{BE} falls about 2.5 mV/°C.
- Example: α = 0.98 gives β = 0.98/0.02 = **49**. β = 99 gives α = **0.99**.

Property	CB	CE	CC (emitter follower)
Input resistance	**Very low** (~20–50 Ω)	Medium (~1 kΩ)	**Very high** (~100s kΩ)
Output resistance	**Very high** (~MΩ)	Medium (~10–50 kΩ)	**Very low** (~tens of Ω)

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Current gain	α (**< 1**)	β (high)	**1 + β** (highest)
Voltage gain	High	High	**≈ 1 (slightly < 1)**
Power gain	Moderate	**Highest**	Low (≈ current gain)
Phase shift (V_o vs V_i)	0°	**180°**	0°
Typical use	**High-frequency** amplifiers, current buffer	**Most widely used** amplifier	**Buffer** and impedance matching

- The CC stage is called an **emitter follower** because the emitter voltage follows the base voltage. It has **100 % negative voltage-series feedback**.
- **Darlington pair** = two CC stages in cascade. $\beta_{overall} \approx \boldsymbol{\beta_1\beta_2}$, with very high input impedance. $V_{BE} \approx 1.4$ V.
- **Cascade** = CE stage followed by a CB stage. It gives a **wide bandwidth** because it minimises the Miller effect.

Biasing & stability

- **Q-point (operating point)** = (I_{CQ}, V_{CEQ}) on the DC load line. For **maximum symmetrical swing**, place it at the centre of the AC load line.
- **DC load line**: $V_{CE} = V_{CC} - I_C(R_C + R_E)$. Its end points are $I_{C(sat)} = V_{CC}/(R_C + R_E)$ and $V_{CE(off)} = V_{CC}$.
- **AC load line**: slope = $-1/r_{ac}$ with $r_{ac} = R_C \parallel R_L$. It is **steeper** than the DC load line and passes through the Q-point.
- The Q-point shifts with **β (unit-to-unit spread), I_{CBO} and V_{BE}**, all of which change with temperature.
- **Stability factor $\boldsymbol{S = \partial I_C/\partial I_{CBO}}$**. The ideal value is **S = 1**; a **lower S means a more stable** circuit. $S' = \partial I_C/\partial V_{BE}$ and $S'' = \partial I_C/\partial\beta$.
- **Thermal runaway**: I_C rises, so the junction heats, so I_{CBO} rises, so I_C rises further, until the transistor is destroyed.
  - Condition for thermal stability: $\partial P_C/\partial T_j < 1/\theta$.
  - The circuit is inherently safe if $\boldsymbol{V_{CE} < V_{CC}/2}$.
  - It is prevented by stabilised biasing (R_E) and heat sinks.
- **Compensation techniques** use diodes, thermistors or sensistors, unlike stabilisation, which uses negative feedback through resistors.

Bias circuit	Base current / key relation	Stability factor S	Remarks
Fixed bias (base resistor)	$I_B = (V_{CC} - V_{BE})/R_B$	**1 + β** (worst, e.g. 101)	Simple; the Q-point depends strongly on β; thermal runaway is possible
Collector-to-base (collector feedback)	$I_B = (V_{CC} - V_{BE})/(R_B + \beta R_C)$	$(1 + \beta)/[1 + \beta R_C/(R_C + R_B)] < 1 + \beta$	Better than fixed bias; the feedback also reduces the AC gain
Emitter bias (fixed bias + R_E)	$I_B = (V_{CC} - V_{BE})/[R_B + (1 + \beta)R_E]$	Lower	R_E gives negative feedback; it is bypassed by C_E for AC
Voltage divider (self / potential-divider bias)	$V_{Th} = V_{CC}R_2/(R_1 + R_2)$, $R_{Th} = R_1 \parallel R_2$; $I_B = (V_{Th} - V_{BE})/[R_{Th} + (1 + \beta)R_E]$	$(1 + \beta)(1 + R_{Th}/R_E)/(1 + \beta + R_{Th}/R_E) \approx 1 + R_{Th}/R_E$	**Most stable and most widely used**; I_C is nearly independent of β if $\beta R_E \geq 10R_2$

MEMORY TRICK

"**FAR**" for the active region: **F**orward E-B, **A**ctive, **R**everse C-B. For impedances, "CB is Low-in High-out, CC is High-in Low-out, CE sits in the middle". The only stage that inverts is the CE amplifier (**180°**).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- If α = 0.99, β = ? → **99**.
- Configuration with the highest input impedance and voltage gain ≈ 1? → **CC (emitter follower)**.
- Stability factor of fixed bias? → **1 + β**.
- Most stable biasing method? → **Voltage-divider bias**.
- For saturation, the junctions are biased as? → **Both forward-biased**.
- Base-width modulation is also called? → **Early effect**.

10.4 FET & MOSFET ★★★

- An **FET** is a **unipolar** device (only majority carriers carry current) and is **voltage-controlled**. Its terminals are the gate, drain and source.
- In a **JFET** the gate–channel junction is always **reverse-biased**, so the input resistance is very high (~10^8–10^{10} Ω) and $I_G \approx 0$.
- An n-channel JFET needs $V_{GS} \leq 0$. Increasing $|V_{GS}|$ widens the depletion regions and narrows the channel.
- **Pinch-off voltage V_P** = the V_{DS} (at $V_{GS} = 0$) beyond which I_D saturates at $\boldsymbol{I_{DSS}}$. $\boldsymbol{V_{GS(off)}}$ = the gate voltage that cuts off the channel. $|V_{GS(off)}| = |V_P|$.
- The JFET is a **normally-ON (depletion-type)** device: maximum current flows at $V_{GS} = 0$.
- JFET regions:
  - **Ohmic (voltage-variable resistor)**: for small V_{DS}.
  - **Saturation (pinch-off / constant-current)**: the region used for amplification.
  - **Breakdown**.
- **MOSFET (IGFET)** = gate insulated by a SiO_2 layer, giving an input resistance of **~10^{10}–10^{15} Ω**.
  - The thin oxide is easily punctured by static charge, so MOSFETs need **ESD handling precautions**.
  - There is no gate current for either polarity of V_{GS}.
- **Enhancement MOSFET (E-MOSFET)**: **no physical channel**, so it is **normally OFF**. A channel is induced (inversion layer) only when V_{GS} > **threshold voltage V_T** (typically 1–4 V). For n-channel, V_{GS} must be positive.
- **Depletion MOSFET (D-MOSFET)**: has a built-in channel, so it is **normally ON**. It can operate in both **depletion ($V_{GS} < 0$) and enhancement ($V_{GS} > 0$)** modes and follows Shockley's equation.
- MOSFET regions (n-channel E-MOSFET):
  - **Cut-off**: $V_{GS} < V_T$.
  - **Triode / ohmic / linear**: $V_{DS} < V_{GS} - V_T$. Used as a switch or a voltage-controlled resistor.
  - **Saturation (active)**: $V_{DS} \geq V_{GS} - V_T$. Used as an amplifier.

  Note: MOSFET "saturation" corresponds to the BJT *active* region.
- **Channel-length modulation** (λ) is the FET counterpart of the Early effect: $I_D = k(V_{GS} - V_T)^2(1 + \lambda V_{DS})$.
- **CMOS** = complementary pMOS (pull-up) and nMOS (pull-down) transistors. The CMOS inverter has:
  - **Near-zero static power**, because one device is always OFF. Power is drawn mainly when switching: $P = C V_{DD}^2 f$.
  - High noise margin (~45 % of V_{DD}) and a wide supply range.
  - A switching threshold of $\approx V_{DD}/2$ for a symmetrical design. pMOS is made **2–3 times wider** because $\mu_p < \mu_n$.
- The **power MOSFET** has a **positive temperature coefficient of $R_{DS(on)}$**, so devices can be **paralleled easily** and do not suffer secondary breakdown.

KEY FORMULAE

- **Shockley's equation** (JFET, D-MOSFET): $\boldsymbol{I_D = I_{DSS}(1 - V_{GS}/V_P)^2}$. At $V_{GS} = V_P/2$, $I_D = I_{DSS}/4$.
- **Transconductance** $g_m = \partial I_D/\partial V_{GS}$ (unit: siemens, mho). Typical JFET values are 1–10 mS.
  - $g_{m0} = \boldsymbol{2I_{DSS}/|V_P|}$ at $V_{GS} = 0$.
  - $g_m = g_{m0}(1 - V_{GS}/V_P) = g_{m0}\sqrt{(I_D/I_{DSS})}$.
- FET amplification factor $\boldsymbol{\mu = g_m r_d}$, where r_d = drain resistance. CS amplifier gain $A_v = -g_m(r_d \parallel R_D) \approx -g_m R_D$.
- **E-MOSFET in saturation**: $\boldsymbol{I_D = k(V_{GS} - V_T)^2}$, with $k = \frac{1}{2}\mu_n C_{ox}(W/L)$.
- Triode region: $I_D = k[2(V_{GS} - V_T)V_{DS} - V_{DS}^2]$. For small V_{DS}: $R_{DS(on)} = 1/[2k(V_{GS} - V_T)]$.
- MOSFET $g_m = 2k(V_{GS} - V_T) = \boldsymbol{2\sqrt{(k\, I_D)}} = 2I_D/(V_{GS} - V_T)$. Compare BJT $g_m = I_C/V_T$, which is larger for the same current.

Feature	BJT	FET / MOSFET
Carriers	**Bipolar** (electrons + holes)	**Unipolar** (majority carriers only)
Control	**Current**-controlled (I_B)	**Voltage**-controlled (V_{GS})

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Input impedance	Low–medium (~kΩ)	**Very high** (MΩ to 10^{15} Ω)
Thermal stability	Positive temperature coefficient of I_C, so **thermal runaway** is possible	**Negative** temperature coefficient of I_D at high current, so **no thermal runaway**; more stable
Noise	More (shot noise of junctions)	**Less noisy**, which suits RF front ends
Gain–bandwidth product	**Higher** (higher g_m)	Lower
Voltage gain	Higher	Lower
Offset voltage as a switch	$V_{CE(sat)} \approx 0.2$ V	**Zero offset**, so it makes a good analog switch/chopper
Size / IC density	Larger	**Smaller**; dominates VLSI
Radiation effects	More sensitive	Less sensitive (JFET)

PREVIOUSLY ASKED / LIKELY QUESTIONS

- I_{DSS} = 12 mA, V_P = −4 V, V_{GS} = −2 V. I_D = ? → **3 mA**.
- g_{m0} for I_{DSS} = 10 mA, $|V_P|$ = 5 V? → **4 mS**.
- Which MOSFET has no channel at V_{GS} = 0? → **Enhancement type**.
- Why does a FET not suffer thermal runaway? → **Negative temperature coefficient of drain current** (mobility falls with temperature).
- Main advantage of CMOS logic? → **Very low static power dissipation**.

10.5 Amplifiers – Small-Signal & Frequency Response ★★★

- **h-parameters (hybrid)**: $v_1 = h_i i_1 + h_r v_2$; $i_2 = h_f i_1 + h_o v_2$.
  - $\mathbf{h_{ie}}$ = input resistance (Ω), ≈ 1 kΩ.
  - $\mathbf{h_{re}}$ = reverse voltage ratio (dimensionless), ~10^{-4}.
  - $\mathbf{h_{fe}}$ = forward current gain (β), ~50–200.
  - $\mathbf{h_{oe}}$ = output admittance (S), ~25 µS.
- They are called "hybrid" because the units are mixed (Ω, S and dimensionless). h-parameters are valid at **low frequency** and for **small signals**. Each parameter is **measured** with the output **short-circuited** (h_i, h_f) or the input **open-circuited** (h_r, h_o).
- **Approximate (simplified) model**: neglect h_{re} and h_{oe} when $h_{oe}R_L < 0.1$.
- **r_e model**: r_e = 26 mV/I_E. CE input resistance $h_{ie} = \beta r_e$. CB input resistance $h_{ib} = r_e$.
- **Hybrid-π model** (high frequency):
  - $g_m = I_C/V_T$, so 40 mS at 1 mA.
  - $r_\pi = \beta/g_m$.
  - C_π ($C_{b'e}$, diffusion) and C_μ ($C_{b'c}$, transition) set the upper cut-off.
  - $f_T = g_m/[2\pi(C_\pi + C_\mu)]$.
- **Coupling methods**:
  - **RC coupling**: the most common, cheap, good audio response; it does not pass DC.
  - **Transformer coupling**: impedance matching and high efficiency, but bulky, gives poor frequency response and adds hum. Used in power output stages and tuned RF amplifiers.
  - **Direct coupling**: passes **DC and very low frequencies** (used in ICs and op-amps), but suffers **drift**.
- **Frequency response of an RC-coupled amplifier**:
  - The **lower cut-off f_L** is set by the **coupling and bypass (emitter) capacitors**. The emitter bypass capacitor usually dominates.
  - The **upper cut-off f_H** is set by the **junction (internal) capacitances** and stray or wiring capacitance, increased by the Miller effect.
  - Mid-band gain is constant.
  - Bandwidth = $f_H - f_L$. At the cut-off frequencies, gain = 0.707 × mid-band gain (**−3 dB**) and power is half.
- **Miller effect**: a capacitance C connected between the input and output of an inverting amplifier of gain $-A_v$ appears at the input as $\mathbf{C_{in} = C(1 + |A_v|)}$ and at the output as $C_{out} = C(1 + 1/|A_v|) \approx C$. This reduces f_H. The cascode stage is the remedy.
- **Gain–bandwidth product (GBW)** is constant for a given device: $A_{mid} \times BW$ = constant. Raising the gain therefore reduces the bandwidth.
- **Tuned amplifiers** use an LC tank as the load: $f_r = 1/(2\pi\sqrt{LC})$, $BW = f_r/Q$. Higher Q means better selectivity. Used in RF/IF stages.

KEY FORMULAE

- CE (h-model): $A_i = -h_{fe}/(1 + h_{oe}R_L) \approx -h_{fe}$; $Z_i \approx h_{ie}$; $\mathbf{A_v \approx -h_{fe}R_L/h_{ie}}$.
- CE (r_e model), R_E fully bypassed: $\mathbf{A_v = -R_C/r_e = -\beta R_C/(\beta r_e)}$.
  - With R_E unbypassed: $A_v = -R_C/(r_e + R_E) \approx \mathbf{-R_C/R_E}$, giving a stable gain but a lower one.
  - Input resistance rises to $\beta(r_e + R_E)$.
- Emitter follower: $A_v = R_E/(r_e + R_E) \approx 1$; $Z_i \approx \beta R_E$; $Z_o \approx r_e + R_s/\beta$.
- Cascaded stages: $\mathbf{A_{total} = A_1 \times A_2 \times \ldots \times A_n}$. In dB, the gains **add**: $G_{dB} = 20 \log A_v$ for voltage, $10 \log A_p$ for power. Loading by the next stage reduces each gain.
- Example: three stages of 20 dB each give 60 dB, i.e. a voltage gain of 1000.
- n identical non-interacting stages:
  - $\mathbf{f_{H(n)} = f_H\sqrt{(2^{1/n} - 1)}}$ and $\mathbf{f_{L(n)} = f_L/\sqrt{(2^{1/n} - 1)}}$, so the bandwidth shrinks.
  - For n = 2 the factor is **0.643**; for n = 3 it is **0.51**.
- Single-pole cut-offs: $f_L = 1/(2\pi R_{eq}C_{coupling})$ and $f_H = 1/(2\pi R_{eq}C_{in})$. Roll-off = **20 dB/decade = 6 dB/octave** per pole.

Power amplifier class	Conduction angle	Max. efficiency	Key points
Class A	**360°**	**25 %** (series-fed, R_C load); **50 %** (transformer-coupled)	Least distortion and lowest efficiency. Maximum dissipation occurs at **zero signal**. Transistor rating ≈ 2 × $P_{o(max)}$ (transformer-coupled).
Class B (push-pull)	**180°**	**78.5 %** (π/4)	Biased at cut-off and has **crossover distortion**. Push-pull cancels **even harmonics**. Max dissipation per transistor ≈ 0.2 $P_{o(max)}$, occurring at $V_p = 2V_{CC}/\pi$.
Class AB	180°–360°	Between 50 % and 78.5 %	Small forward bias (diodes or V_{BE} multiplier) **removes crossover distortion**. The usual audio output stage.
Class C	**< 180°** (typ. 90°–150°)	**> 78.5 %** (up to ~90–95 %)	Highly distorted pulses, so it needs a **tuned (LC tank) load**. Used in **RF transmitters** and frequency multipliers.
Class D	Switching (PWM)	Ideally 100 %; practically > 90 %	Transistors act as **switches**, and an LC filter recovers the audio. Used in portable audio and Class-D amplifiers.
Class E / F	Switching	Very high	Tuned switching RF amplifiers.

- **Crossover distortion** occurs in Class B because neither transistor conducts while $|v_{in}| < V_{BE} \approx 0.7$ V. It is cured by Class AB biasing.
- The **complementary-symmetry** push-pull stage uses an npn + pnp pair and needs no centre-tapped transformers. The **quasi-complementary** stage uses Darlington pairs.
- Figure of merit of a power amplifier = maximum power dissipation of the device ÷ maximum output power. For Class B it is ≈ 0.4 for both transistors together.
- **Heat sinks** lower the thermal resistance: $T_j = T_a + P_D(\theta_{jc} + \theta_{cs} + \theta_{sa})$.

MEMORY TRICK

Efficiency ladder: "**A = 25/50, B = 78.5, C = more, D = nearly all**". The angle halves going from A to B (360 → 180), and C is "less than half a cycle". Miller multiplies the capacitance by (1 + gain), which is why high gain gives low bandwidth.

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- Lower cut-off frequency of an RC-coupled amplifier is due to? → **Coupling and bypass capacitors**.
- Feedback capacitance 2 pF, gain −99. Miller input capacitance? → **200 pF**.
- Maximum efficiency of a Class B push-pull amplifier? → **78.5 %**.
- Amplifier with the highest efficiency that needs a tuned load? → **Class C**.
- Two identical stages each with f_H = 1 MHz. Overall f_H? → **≈ 0.643 MHz**.
- Coupling suitable for DC signals? → **Direct coupling**.

10.6 Feedback Amplifiers & Oscillators ★★★

- **Negative (degenerative) feedback** is used in amplifiers. The feedback signal is 180° out of phase with the input, so the gain falls.
- **Positive (regenerative) feedback** is used in **oscillators** and Schmitt triggers.
- **Loop gain** = Aβ. **Desensitivity D = 1 + Aβ**, also called the amount of feedback, expressed in dB as 20 log(1 + Aβ).
- Effects of negative feedback:
  - **Gain is reduced** by the factor (1 + Aβ) but is **stabilised** (desensitised).
  - **Bandwidth increases**: $f_H' = f_H(1 + A\beta)$ and $f_L' = f_L/(1 + A\beta)$. The gain-bandwidth product stays constant.
  - **Distortion and noise** generated inside the loop are **reduced** by (1 + Aβ). Noise present at the input is not reduced.
  - Input and output impedances change according to the topology (table below).
  - Operation becomes more linear.
- If Aβ ≫ 1, then **$A_f \approx 1/\beta$**: the gain depends only on the passive feedback network. This is the principle of op-amp circuits.

KEY FORMULAE

- Closed-loop gain: **$A_f = A/(1 + A\beta)$**. For positive feedback, $A_f = A/(1 - A\beta)$, which becomes infinite (oscillation) when Aβ = 1.
- Sensitivity: **$dA_f/A_f = [1/(1 + A\beta)] \times dA/A$**.
- Example: A = 1000, β = 0.01 gives 1 + Aβ = 11 and $A_f \approx 90.9$. A 10 % change in A gives only ≈ 0.91 % change in A_f.
- Barkhausen criterion: **$|A\beta| = 1$** and **$\angle A\beta = 0°$ (or 360°)**. In practice $|A\beta|$ is kept slightly > 1 so that oscillations start.

Topology (sampling–mixing)	Alternative name	Stabilised gain / amplifier type	Z_{in}	Z_{out}
Voltage-series	Series–shunt	Voltage gain (voltage amplifier)	**Increases ×** (1 + Aβ)	**Decreases ÷** (1 + Aβ)
Voltage-shunt	Shunt–shunt	Transresistance (I → V)	**Decreases**	**Decreases**
Current-series	Series–series	Transconductance (V → I)	**Increases**	**Increases**
Current-shunt	Shunt–series	Current gain (current amplifier)	**Decreases**	**Increases**

- The rules: **series mixing raises Z_{in}** and shunt mixing lowers it; **voltage sampling lowers Z_{out}** and current sampling raises it.
- Examples:
  - Emitter follower and op-amp non-inverting amplifier = **voltage-series**.
  - Op-amp inverting amplifier and collector-to-base bias = **voltage-shunt**.
  - CE stage with an unbypassed R_E = **current-series**.
- An **oscillator** is an amplifier with positive feedback and **no input signal**. It converts DC power into AC.
  - It starts from noise, with loop gain slightly > 1.
  - The amplitude is stabilised by non-linearity, a thermistor, or a lamp/diode AGC.
  - RC oscillators suit audio frequencies (Hz – ~1 MHz); LC oscillators suit RF (~100 kHz – 500 MHz).
- **Crystal oscillator** (quartz, **piezoelectric effect**):
  - Very high Q (**10^4–10^6**), giving **excellent frequency stability** (~ppm).
  - Equivalent circuit: a series R-L-C branch in parallel with the mounting capacitance C_M.
  - It has a **series resonance f_s** (minimum impedance, resistive) and a **parallel (anti-) resonance f_p** (maximum impedance), with f_p slightly > f_s.
  - **Between f_s and f_p the crystal is inductive**, and it is used as the inductor in Pierce/Colpitts crystal oscillators.
  - Used in clocks, watches and microprocessors. Frequency ∝ 1/thickness.
- **Multivibrators** are relaxation oscillators that give square-wave outputs.
  - Astable: no stable state (free-running square-wave generator).
  - Monostable: one stable state (one-shot, pulse stretcher).
  - Bistable: two stable states (flip-flop, memory).
- **UJT relaxation oscillator**: $T \approx RC \ln[1/(1 - \eta)]$, where η = intrinsic stand-off ratio (0.5–0.8). The UJT has a negative-resistance region and is used to trigger SCRs.

Oscillator	Feedback network	Frequency	Condition / remarks
RC phase shift	3 RC sections, each giving 60° (total 180°); the amplifier gives 180°	**$f = 1/(2\pi RC\sqrt{6})$**	β = 1/29, so **$|A| \geq 29$**. Audio range; fixed frequency.
Wien bridge	Series RC + parallel RC (lead-lag), **0° phase at f_0**; the amplifier is non-inverting	**$f = 1/(2\pi RC)$**	β = 1/3, so **$A \geq 3$** ($R_f = 2R_1$). Easily tunable with a ganged capacitor; low distortion; audio signal generators.
Hartley	**Tapped inductor** (L_1, L_2) + C	$f = 1/[2\pi\sqrt{(L_{eq}C)}]$, $L_{eq} = L_1 + L_2 + 2M$	RF; radio receiver local oscillators. Gain ≥ L_1/L_2 (op-amp form).
Colpitts	**Tapped capacitor** (C_1, C_2) + L	$f = 1/[2\pi\sqrt{(LC_{eq})}]$, $C_{eq} = C_1C_2/(C_1 + C_2)$	RF, better at high frequencies than Hartley. Gain ≥ C_2/C_1 (texts differ on subscripts).
Clapp	Colpitts + **small C_3 in series with L**	$f \approx 1/[2\pi\sqrt{(LC_3)}]$ when $C_3 \ll C_1, C_2$	**Better frequency stability** than Colpitts, because transistor and stray capacitances have less effect.
Crystal	Quartz crystal (as L or series element)	$f_s = 1/[2\pi\sqrt{(LC)}]$; $f_p = 1/[2\pi\sqrt{(L{\cdot}CC_M/(C + C_M))}]$	**Highest frequency stability**.
Tuned collector / Armstrong	Transformer (mutual) feedback	$f = 1/[2\pi\sqrt{(LC)}]$	RF.

MEMORY TRICK

"**Hartley has Henries, Colpitts has Capacitors**." The magic numbers are "**√6 and 29**" for the phase-shift oscillator and "**3 and 1/3**" for the Wien bridge. Clapp = Colpitts with a series "**C-lamp**" that clamps the frequency.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Minimum gain for an RC phase-shift oscillator? → **29**.
- Negative feedback increases Z_{in} and decreases Z_{out} for which topology? → **Voltage-series**.
- Wien bridge frequency for R = 10 kΩ, C = 0.01 µF? → **≈ 1.59 kHz**.
- Oscillator using a tapped inductor? → **Hartley**.
- Bandwidth with negative feedback becomes? → **BW × (1 + Aβ)**.
- Most stable frequency source? → **Crystal oscillator**.

10.7 Operational Amplifiers & Applications ★★★

- An **op-amp** is a direct-coupled, high-gain **differential amplifier**. The stages are:
  - Input stage: differential amplifier, which gives high CMRR.
  - Intermediate stage: further gain.
  - **Level shifter**.
  - Output stage: push-pull (Class AB) with low Z_{out}.
- **Virtual short / virtual ground**: with negative feedback and infinite A, **$V_+ \approx V_-$** and no current enters the input terminals. In the inverting amplifier the –

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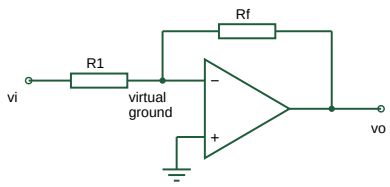
terminal is a **virtual ground**: it is at 0 V but no current flows to ground through it.

- Open-loop (no feedback) the op-amp saturates at $\pm V_{sat} \approx \pm(V_{CC} - 1 \text{ to } 2\ V)$, e.g. ±13 V to ±14 V on a ±15 V supply.
- **741 pin-out** (8-pin DIP):
  - Pin **2** = inverting input; pin **3** = non-inverting input; pin **6** = output.
  - Pin **7** = $+V_{CC}$; pin **4** = $-V_{EE}$.
  - Pins 1 and 5 = offset null; pin 8 = NC.
- The 741 has an internal **30 pF compensation capacitor**, which gives a dominant pole at ≈ 5 Hz (open-loop BW ≈ 5–10 Hz).

Parameter	Ideal	Typical µA741
Open-loop voltage gain A_{OL}	**∞**	**≈ 2×10^5 (≈ 106 dB)**
Input resistance R_{in}	**∞**	**2 MΩ**
Output resistance R_o	**0**	**75 Ω**
Bandwidth	∞	Unity-gain BW (GBW) = **1 MHz**
CMRR	**∞**	**90 dB**
Slew rate	∞	**0.5 V/µs**
Input offset voltage V_{io}	0	1 mV typ. (6 mV max)
Input offset current I_{io}	0	20 nA (200 nA max)
Input bias current I_B	0	80 nA (500 nA max)
PSRR (SVRR)	0 µV/V (∞ dB)	30 µV/V (max 150 µV/V)
Supply	–	±15 V typical (±18 V max)
Drift, noise	0	Small, non-zero

- **Input bias current** $I_B = (I_{B1} + I_{B2})/2$. **Input offset current** $I_{io} = |I_{B1} - I_{B2}|$.
- **Input offset voltage** = the differential voltage that must be applied to make the output zero.
- The bias-current effect is minimised by a **compensating resistor** $R_{comp} = R_1 \parallel R_f$ on the non-inverting input.
- **CMRR** = A_d/A_{cm}. In dB: $20 \log(A_d/A_{cm})$. A higher CMRR means better rejection of common-mode noise such as hum.
- **Slew rate** SR = $(dV_o/dt)_{max}$, in V/µs. It is caused by the compensation capacitor's limited charging current: SR = I_{max}/C_c.
- **Full-power bandwidth** f_{max} = SR/($2\pi V_m$). Above f_{max} a sine output distorts into a triangular wave.
- Op-amp frequency compensation: dominant-pole, pole-zero or lag compensation keeps the phase margin adequate (typ. ≥ 45°).



Inverting amplifier: vo = −(Rf/R1)·vi

Fig. 10.1 Inverting amplifier. The − input is at virtual ground, so i = vi/R1 flows through Rf. Input resistance = R1.

KEY FORMULAE – LINEAR APPLICATIONS

- **Inverting amplifier**: $A_v = -R_f/R_1$; $R_{in} = R_1$.
- **Non-inverting amplifier**: $A_v = 1 + R_f/R_1$ (always ≥ 1); R_{in} very high.
- **Voltage follower** (R_f = 0 or $R_1 = \infty$): $A_v = 1$, highest Z_{in} and lowest Z_{out}. Used as a **buffer**. It has 100 % voltage-series feedback (β = 1), so its bandwidth equals the GBW.
- **Inverting summer**: $V_o = -R_f(V_1/R_1 + V_2/R_2 + \ldots)$. With all resistors equal it gives $-(V_1 + V_2 + \ldots)$. With $R_f = R/n$ it is an averager.
- **Difference (subtractor) amplifier**: $V_o = (R_f/R_1)(V_2 - V_1)$ when $R_f/R_1 = R_3/R_2$ (matched ratios are needed for high CMRR).
- **Instrumentation amplifier** (3 op-amps): $A_v = (1 + 2R/R_G)(R_3/R_2)$.
  - The gain is set by a **single resistor R_G**.
  - It has very high Z_{in} and high CMRR.
  - Used for **transducer and bridge signals** (strain gauge, thermocouple). Example IC: AD620.
- **Integrator**: $V_o = -(1/RC)\int v_i\, dt$.
  - A square-wave input gives a **triangular** output. Behaves as a low-pass filter (−20 dB/dec).
  - In practice a resistor R_f is placed across C to limit DC gain and stop saturation by offset. It integrates properly for $f \gg 1/(2\pi R_f C)$.
- **Differentiator**: $V_o = -RC\, dv_i/dt$.
  - A square-wave input gives spikes; a triangular input gives a square wave. Behaves as a high-pass filter (+20 dB/dec).
  - Gain rises with frequency, so it amplifies **high-frequency noise** and can be unstable. In practice a resistor in series with C (and a capacitor across R_f) is added.
- **Current-to-voltage converter** (transimpedance): $V_o = -I_{in}R_f$. Used with photodiodes and DAC outputs.
- **Log amplifier** (diode or BJT in the feedback path): $V_o = -V_T \ln[v_i/(RI_s)]$. In the **antilog** amplifier the diode is in the input path: $V_o = -RI_s e^{v_i/V_T}$. Used for multiplication/division and dB compression.

- **Comparator**: an open-loop op-amp (no feedback). The output is $+V_{sat}$ if $V_+ > V_-$, otherwise $-V_{sat}$.
  - A **zero-crossing detector** converts a **sine wave into a square wave**.
  - Dedicated comparators (LM311, LM339 with open-collector output) are faster than general op-amps.
- **Schmitt trigger (regenerative comparator)**: uses **positive feedback**, giving two thresholds.
  - For the inverting type with R_1 (output to + input) and R_2 (+ input to ground), $\beta = R_2/(R_1 + R_2)$.
  - **UTP = $+\beta V_{sat}$, LTP = $-\beta V_{sat}$, hysteresis V_H = UTP − LTP = $2\beta V_{sat}$.**
  - The hysteresis gives **noise immunity**: no false triggering on a noisy input. Used for squaring circuits and in astable multivibrators.
- **Precision rectifier (super diode)**: a diode inside the feedback loop. The effective cut-in voltage becomes V_γ/A_{OL}, so it can rectify signals **smaller than 0.7 V** (mV level). Used in instrumentation and AC voltmeters. A full-wave precision rectifier is an **absolute-value circuit**.
- **Peak detector, clipper and clamper** can also be built with op-amps (precision versions).
- **Sample-and-hold (S/H)**: a switch (FET) plus a hold capacitor plus voltage-follower buffers. It holds the analog value during A/D conversion. Key parameters:
  - **Acquisition time**: time to charge the capacitor to the input value.
  - **Aperture time**: delay in opening the switch.
  - **Droop rate**: decay during hold, set by leakage.

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Use a low-leakage capacitor (polystyrene, Teflon, polypropylene). Example IC: LF398.

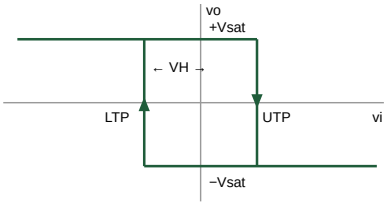

Fig. 10.2 Transfer characteristic of an inverting Schmitt trigger. The output switches to −Vsat when vi rises above UTP and back to +Vsat when vi falls below LTP. Hysteresis VH = UTP − LTP.

Active filter	Key facts
First-order LPF/HPF	f_c = **1/(2πRC)**; roll-off **20 dB/decade** (6 dB/octave); pass-band gain = 1 + R_f/R_1 (non-inverting).
Butterworth	**Maximally flat** pass band with no ripple. Roll-off = **−20n dB/decade** for order n. Response at f_c = −3 dB.
Chebyshev	**Ripple in the pass band** and a **steeper roll-off** than Butterworth of the same order.
Bessel	Linear phase (constant group delay), so pulse shape is preserved; slowest roll-off.
Elliptic (Cauer)	Ripple in both bands; the sharpest transition.
Sallen–Key (VCVS, 2nd order)	$f_c = 1/[2\pi\sqrt{(R_1R_2C_1C_2)}] = 1/(2\pi RC)$ for equal components. A Butterworth response (Q = 0.707) with equal components needs a pass-band gain of **1.586** ($R_f = 0.586R_1$). Roll-off 40 dB/dec.
Band-pass	$f_0 = \sqrt{(f_L f_H)}$; Q = f_0/BW. **Wide-band if Q < 10**, narrow-band if Q > 10.
Band-reject (notch)	A twin-T notch removes **50 Hz hum**.
All-pass	Flat gain; only the phase changes (phase shifter / delay equaliser).

- Active filters (op-amp + RC) need **no inductors**, provide gain and buffering, and are easily tuned. However, they need a power supply and are limited at high frequency by the GBW.

555 timer, VCO, PLL & regulators

- **555 timer** internals:
  - Two comparators.
  - A divider of **three 5 kΩ resistors**, which sets the thresholds at **2/3 V_{CC}** (threshold) and **1/3 V_{CC}** (trigger).
  - An SR flip-flop, a discharge transistor and an output buffer.
- 555 supply: 4.5–15 V (up to 18 V); output current ≈ 200 mA (sink/source).
- **555 pin-out**:
  - Pin 1 = ground; pin 2 = trigger; pin 3 = output; pin 4 = reset (active low).
  - Pin 5 = control voltage (bypass with 0.01 µF); pin 6 = threshold; pin 7 = discharge; pin 8 = +V_{CC}.
- **Astable** (free-running, square wave):
  - The capacitor charges through $R_A + R_B$ and discharges through R_B, swinging between $V_{CC}/3$ and $2V_{CC}/3$.
  - Duty cycle D > 50 %. Exactly 50 % needs a diode across R_B with $R_A = R_B$.
- **Monostable** (one-shot): triggered by a negative pulse on pin 2. Uses: timers, missing-pulse detection, pulse-width modulation, debouncing.
- **566 VCO** (voltage-controlled oscillator): gives **square and triangular** outputs.
  - $f_o = \mathbf{2(V_{CC} - V_C)/(R_1C_1V_{CC})}$, with V_C between ¾V_{CC} and V_{CC}.
  - Used in FM/FSK generation and function generators. Its frequency-to-voltage linearity is good.
- **PLL (565)**: phase detector → low-pass filter → amplifier → VCO, with the VCO output fed back to the phase detector.
  - Lock (tracking) range = frequency range over which the loop stays locked.
  - Capture range = range over which it can acquire lock. **Capture range < lock range**.
  - Uses: FM demodulation, frequency synthesis and multiplication, FSK decoding, clock recovery.
- **Linear IC voltage regulators**:
  - **78xx = fixed positive** (7805 = +5 V, 7812 = +12 V). **79xx = fixed negative** (7905 = −5 V).
  - 78xx rated current ≈ 1 A (with heat sink). **Dropout ≈ 2 V**, so the input must be ≥ V_o + 2 V; e.g. a 7805 needs ≥ 7 V.
  - 78xx pins: 1 = input, 2 = ground, 3 = output.
  - **LM317** = adjustable positive regulator (LM337 = negative), 1.2–37 V, 1.5 A. V_{ref} = **1.25 V** between the OUT and ADJ terminals.
  - The 723 is a general-purpose adjustable regulator (2–37 V). LDO regulators have a dropout of 0.1–0.5 V.
- **Line regulation** = $\Delta V_o/\Delta V_{in}$. **Load regulation** = $(V_{NL} - V_{FL})/V_{FL}$ × 100 %. An ideal regulator has 0 % for both.
- Linear regulators are inefficient (η ≈ V_o/V_{in}, typically 30–60 %). **SMPS** reach ~80–95 % efficiency but are noisier.

KEY FORMULAE – TIMING, SLEW & REGULATORS

- **Slew-rate limit**: $\mathbf{f_{max} = SR/(2\pi V_m)}$. Example: SR = 0.5 V/µs, V_m = 10 V gives f_{max} ≈ **7.96 kHz**. Equivalently, $V_{m(max)} = SR/(2\pi f)$.
- Closed-loop bandwidth: $f_{CL} = GBW/A_{CL}$, where A_{CL} = non-inverting noise gain $(1 + R_f/R_1)$. Example: a 741 with gain 100 has BW ≈ 10 kHz.
- Output offset (inverting/non-inverting): $V_{oo} = (1 + R_f/R_1)V_{io} + R_fI_B$, the bias-current term being removed by R_{comp}.
- 555 **astable**:
  - $t_H = 0.693(R_A + R_B)C$ and $t_L = 0.693R_BC$.
  - $T \approx 0.693(R_A + 2R_B)C$, so $\mathbf{f = 1.44/[(R_A + 2R_B)C]}$.
  - Duty cycle $\mathbf{D = (R_A + R_B)/(R_A + 2R_B)}$.
- 555 **monostable**: **T = 1.1RC** (since ln 3 = 1.0986). Example: R = 10 kΩ, C = 10 µF gives T = 0.11 s.
- LM317: $\mathbf{V_o = 1.25(1 + R_2/R_1) + I_{ADJ}R_2}$. I_{ADJ} ≈ 50–100 µA is usually neglected; R_1 ≈ 240 Ω is typical.
- Op-amp RC phase-shift oscillator: $R_f \geq 29R$. Wien bridge: $R_f \geq 2R_1$.

MEMORY TRICK

The 741's "**2-2-0.5-90-1**": gain **2** × 10^5, R_{in} **2** MΩ, SR **0.5** V/µs, CMRR **90** dB, GBW **1** MHz (plus R_o = 75 Ω). The 555 is "**1.44 and 1.1**": the astable uses $1.44/((R_A + 2R_B)C)$ and the monostable uses 1.1RC. Integrator → triangle; differentiator → spikes.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Gain of a non-inverting amplifier with R_f = 9 kΩ, R_1 = 1 kΩ? → **10**.
- Op-amp circuit with unity gain and the highest input impedance? → **Voltage follower**.
- Output of an integrator for a square-wave input? → **Triangular wave**.
- Maximum frequency for an undistorted 5 V peak sine with SR = 0.5 V/µs? → **≈ 15.9 kHz**.
- Comparator with positive feedback and hysteresis? → **Schmitt trigger**.
- 555 astable with $R_A = R_B$ = 10 kΩ, C = 0.1 µF. Frequency? → **≈ 480 Hz** (duty cycle 66.7 %).

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11. Digital Electronics & 8085 Microprocessor

NOTE

Expect 4–7 questions from this chapter. Favourite targets: 2's complement range, Gray/excess-3 codes, number of NAND/NOR gates for a function, MUX/decoder sizing, flip-flop characteristic equations, ring/Johnson counter modulus, ADC speed comparison, DAC resolution, and the 8085 (interrupt vectors, flags affected by an instruction, T-states, status signals). Numbers in bold are the usual answer options.

11.1 Number Systems & Codes ★★★

- **Radix (base)** r uses digits 0 to r − 1: binary **2**, octal **8**, decimal 10, hexadecimal **16** (0–9, A–F; A = 10, F = 15).
- Decimal → base r: **repeated division** by r for the integer part (remainders read bottom-up); **repeated multiplication** by r for the fraction (integer parts read top-down).
- Binary → octal: group **3 bits**; binary → hex: group **4 bits** (from the binary point outward). Octal ↔ hex goes via binary.
- Examples: $(25.625)_{10}$ = **$(11001.101)_2$**; $(3A)_{16}$ = **58**; $(777)_8$ = **511**; $(FF)_{16}$ = **255**; $(1101\ 0110)_2$ = **D6H** = 326_8 = 214.
- Some decimal fractions (e.g. 0.1) have **no exact binary equivalent** – non-terminating.
- Largest n-bit unsigned number = **$2^n - 1$**; number of distinct n-bit patterns = 2^n. Bits needed for decimal N = $\lceil \log_2(N+1) \rceil$.
- **1's complement** = invert every bit. **2's complement** = 1's complement + 1 (or: copy bits from the LSB up to and including the first 1, invert the rest).
- (r − 1)'s complement of N (n digits) = $(r^n - 1) - N$; r's complement = $r^n - N$. E.g. 9's complement of 3465 = **6534**; 10's complement = **6535**.
- Subtraction by 2's complement: A − B = A + (2's comp of B); **end carry is discarded**; no carry ⇒ result is negative and in 2's complement form. With 1's complement the end carry is **added back** (end-around carry).
- **Overflow** (signed addition) occurs when two numbers of the same sign give a result of opposite sign, i.e. **carry into MSB ≠ carry out of MSB** (overflow = $C_{in,MSB} \oplus C_{out,MSB}$). Adding numbers of opposite sign can never overflow.

Signed representation (n bits)	Range	Zeros	8-bit range
Sign–magnitude	$-(2^{n-1} - 1)$ to $+(2^{n-1} - 1)$	Two (+0, −0)	−127 to +127
1's complement	$-(2^{n-1} - 1)$ to $+(2^{n-1} - 1)$	Two	−127 to +127
2's complement	**-2^{n-1} to $+(2^{n-1} - 1)$**	One	**−128 to +127**

- In 2's complement, MSB is the sign bit with weight **-2^{n-1}**: 1000 0000 = −128; 1111 1111 = **−1**. 16-bit range = **−32768 to +32767**.
- **Sign extension**: copy the sign bit into the new higher bits (−3 = 1101 → 1111 1101).

Code	Type / key property
BCD (8421)	Weighted; each decimal digit → 4 bits; **6 invalid codes** (1010–1111). Needs more bits than pure binary (e.g. 99 = 1001 1001).
2421, 5211, 84–2–1	Weighted; 2421, 5211 and 84–2–1 are all **self-complementing** (weights sum to 9)
Excess-3 (XS-3)	BCD + 0011; **non-weighted**, **self-complementing** (9's complement = invert bits); 0 = 0011, 9 = 1100
Gray code	Non-weighted, **unit-distance** (successive codes differ in 1 bit), **reflected**, cyclic; used in shaft encoders and K-maps; minimises errors in A/D conversion
ASCII	**7-bit**, 128 characters; '0' = 30H (48), 'A' = **41H (65)**, 'a' = 61H (97); space = 20H
EBCDIC	8-bit code (IBM mainframes)
Unicode	16/32-bit, all scripts

- **BCD addition**: if a 4-bit sum > 9 or a carry is produced, **add 0110 (6)** to correct. Excess-3 addition: add the codes; if carry → add 0011, if no carry → subtract 0011.
- **Binary → Gray**: $G_{MSB} = B_{MSB}$; $G_i = B_{i+1} \oplus B_i$. Example 1011 → **1110**.
- **Gray → binary**: $B_{MSB} = G_{MSB}$; $B_i = B_{i+1} \oplus G_i$ (XOR with the previously obtained binary bit). Example 1110 → **1011**.
- **Parity bit**: even parity makes the total number of 1s even. Single parity detects **any odd number of errors**; cannot detect even number of errors; cannot correct.
- **Hamming code**: parity bits p for m data bits satisfy **$2^p \geq m + p + 1$**. 4 data bits → 3 parity bits → **(7,4) code**; parity bits at positions **1, 2, 4, 8 …** (powers of 2). Basic Hamming code: **single-error correction**; with an extra overall parity bit: SEC-DED.
- **Hamming distance** d_{min}: detects up to $d_{min} - 1$ errors; corrects up to $\lfloor (d_{min} - 1)/2 \rfloor$. Hamming code d_{min} = **3**.

MEMORY TRICK

"Gray goes by XOR of **neighbours**; Binary goes by XOR of the **last answer**." Excess-3 is the "**3-self**" code: excess-**3**, **self**-complementing.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Range of 8-bit 2's complement numbers? → **−128 to +127**.
- Gray code of binary 1001? → **1101**.
- Which code is self-complementing? → **Excess-3** (also 2421).
- Number of parity bits for 7 data bits (Hamming)? → **4** ($2^4 = 16 \geq 12$).
- Number of invalid states in BCD? → **6**.
- $(F2)_{16}$ in decimal? → **242**.

11.2 Boolean Algebra & Logic Gates ★★★

Law / theorem	Form (dual in brackets)
Identity / null	A + 0 = A, A + 1 = 1 (A·1 = A, A·0 = 0)
Idempotent	A + A = A (A·A = A)
Complement	A + A′ = 1 (A·A′ = 0); involution (A′)′ = A
Distributive	A(B + C) = AB + AC; **A + BC = (A + B)(A + C)** – valid only in Boolean algebra
Absorption	A + AB = A (A(A + B) = A)
Redundant literal	**A + A′B = A + B** (A(A′ + B) = AB)
Consensus	**AB + A′C + BC = AB + A′C** ((A + B)(A′ + C)(B + C) = (A + B)(A′ + C))
De Morgan	**(A + B)′ = A′B′; (AB)′ = A′ + B′**
Transposition	AB + A′C = (A + C)(A′ + B)

- **Duality**: interchange + ↔ · and 0 ↔ 1 (variables not complemented) – the dual of a valid identity is valid. Complement of a function = dual + complement every literal.
- De Morgan ⇒ **NAND = bubbled OR** (negative-OR); **NOR = bubbled AND** (negative-AND).
- Number of distinct Boolean functions of n variables = **2^{2^n}** (n = 2 → 16). Number of minterms = 2^n.
- **Minterm** m_i: product term containing all variables (value 1 for one combination) – used in **SOP**, Σm. **Maxterm** M_i: sum term (value 0 for one combination) – used in **POS**, ΠM. $M_i = m_i'$. Σm(1,3,5) of 3 variables = ΠM(0,2,4,6,7).
- SOP → **AND–OR = NAND–NAND** realisation; POS → **OR–AND = NOR–NOR** realisation.
- **Universal gates**: **NAND and NOR** – each alone can realise any Boolean function. NAND with inputs tied (or one input = 1) = NOT; NOR with one input = 0 = NOT.

Function	Min. 2-input NAND gates	Min. 2-input NOR gates
NOT	1	1
AND	**2**	3
OR	**3**	2
NOR / NAND	NOR: 4	NAND: 4
XOR	**4**	5

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XNOR	5	**4**
Half adder	5	5
Full adder	**9**	9

- **XOR**: A ⊕ B = AB′ + A′B; output 1 for **odd number of 1s** (odd function) – parity generator, controlled inverter, comparator, binary adder sum.
- XOR properties: A ⊕ 0 = **A**; A ⊕ 1 = **A′**; A ⊕ A = **0**; A ⊕ A′ = **1**; commutative & associative; A ⊕ B ⊕ A = B.
- **XNOR** (equivalence) = AB + A′B′; output 1 when inputs are **equal** – 1-bit comparator. (A ⊕ B)′ = A′ ⊕ B = A ⊕ B′.
- For 2 variables XNOR = complement of XOR; for 3 variables A ⊙ B ⊙ C = **A ⊕ B ⊕ C** (odd number of variables).
- **Positive logic**: high = 1. A positive-logic AND gate acts as an **OR** gate in negative logic; positive NAND ≡ negative NOR.

Karnaugh map

- Cells arranged in **Gray-code order** (00, 01, 11, 10) so adjacent cells differ by one variable; edges wrap around.
- Group sizes must be **powers of 2** (1, 2, 4, 8, 16); a group of 2^k cells eliminates **k variables**. 4-variable map: a pair → 3 literals, quad → 2 literals, octet → 1 literal.
- **Prime implicant**: largest possible group; **essential PI**: covers a minterm that no other PI covers – must appear in minimal SOP.
- **Don't-care (X / d)** conditions may be taken as 1 or 0, whichever gives larger groups (e.g. codes 10–15 in BCD circuits).
- K-maps practical up to **5–6 variables**; beyond that use **Quine–McCluskey (tabular)** method.

Family	t_{pd} (ns)	P_D/gate	Fan-out	Noise margin	Remark
RTL	≈12	≈12 mW	5	≈0.2 V	Oldest, poor fan-out
DTL	≈30	≈8–12 mW	8	≈0.7 V	Slow
TTL (std 74)	**≈10**	**≈10 mW**	**10**	**0.4 V**	Totem-pole o/p; supply **5 V ± 5%**; speed–power product ≈ 100 pJ
74LS (Schottky low-power)	≈9.5	≈2 mW	20	≈0.3 V	Better speed–power product (≈19 pJ) than std 74/74S
74S (Schottky)	≈3	≈19 mW	10	≈0.3 V	Schottky clamp → non-saturating
ECL	**≈1–2 (fastest)**	≈25–40 mW (highest)	≈25	≈0.2–0.3 V (lowest)	**Non saturated** (current mode) logic; supply −5.2 V; OR/NOR outputs
CMOS (4000B)	≈25–100	**≈10 nW static (lowest)**	**> 50**	**≈ 0.3–0.45 V_{DD} (best)**	Supply 3–15 V (up to 18 V); static-sensitive; $P = C V_{DD}^2 f$
I^2L	≈25–250	very low	–	low	**Highest packing density** among bipolar

- **TTL voltage levels**: $V_{IL(max)}$ = **0.8 V**, $V_{IH(min)}$ = **2.0 V**, $V_{OL(max)}$ = 0.4 V, $V_{OH(min)}$ = 2.4 V ⇒ noise margin = 2.4 − 2.0 = 0.8 − 0.4 = **0.4 V**. Input 0.8–2 V is the indeterminate region. A **floating TTL input acts as logic 1**.
- TTL input stage: **multi-emitter transistor**. Totem-pole outputs **must not be wired together**; **open-collector** outputs need an external **pull-up resistor** and permit **wired-AND**. ECL permits wired-OR.
- Fan-out = min(I_{OL}/I_{IL}, I_{OH}/I_{IH}). **Speed–power product** (figure of merit) = t_{pd} × P_D (pJ) – lower is better.
- Propagation delay $t_{pd} = (t_{PHL} + t_{PLH})/2$. CMOS power rises with **frequency** (dynamic power ∝ f), so at very high f CMOS may dissipate more than TTL.
- Basic **CMOS inverter** = PMOS (pull-up) + NMOS (pull-down) in series; only one conducts in steady state ⇒ near-zero static current.
- **Tri-state (three-state) buffer**: outputs 0, 1 and **high impedance (Hi-Z)** controlled by an enable input; lets many devices share a **common bus**.

MEMORY TRICK

"**E**CL = **E**xpress (fastest, hungry, noisy); **C**MOS = **C**heap on power (lowest power, best noise margin, highest fan-out)." NAND gate counts "**N-A-N-D = 1-2-3-4**": NOT 1, AND 2, OR 3, XOR/NOR 4.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Fastest logic family? → **ECL**; lowest power? → **CMOS**.
- Minimum NAND gates to implement XOR? → **4**; OR? → **3**.
- A + A′B simplifies to? → **A + B**.
- Number of Boolean functions of 3 variables? → **256**.
- Noise margin of standard TTL? → **0.4 V**.
- Which output allows wired-AND in TTL? → **Open-collector**.

11.3 Combinational Circuits ★★★

- **Combinational circuit**: output depends only on **present inputs**; no memory, no feedback, no clock.
- **Half adder** (2 inputs): S = **A ⊕ B**, C = **AB** → one XOR + one AND.
- **Full adder** (3 inputs): S = **A ⊕ B ⊕ C_{in}**; C_{out} = **AB + BC_{in} + AC_{in}** = AB + C_{in}(A ⊕ B). Full adder = **2 half adders + 1 OR gate**.
- **Half subtractor**: D = A ⊕ B, borrow B_o = **A′B**. **Full subtractor**: D = A ⊕ B ⊕ B_{in}; B_o = A′B + A′B_{in} + BB_{in}. (Sum and difference expressions are identical.)
- **Ripple-carry (parallel) adder**: n full adders; carry ripples through all stages ⇒ worst-case delay ∝ n (≈ **2n gate delays** for carry).
- **Carry look-ahead adder**: generate G_i = **A_iB_i**, propagate P_i = **A_i ⊕ B_i**; C_{i+1} = **G_i + P_iC_i**; all carries in parallel in ≈ 3 gate delays (independent of n) – faster but more hardware. IC **74182** (CLA generator); 4-bit adder **7483/74283**.
- **Adder–subtractor**: XOR gates on B inputs with control M; M = 1 → B inverted and C_{in} = 1 ⇒ A − B (2's complement).
- **Magnitude comparator** (IC **7485**, 4-bit): with $x_i = A_i ⊙ B_i$ (XNOR), (A = B) = $x_3x_2x_1x_0$; (A > B) = $A_3B_3' + x_3A_2B_2' + \ldots$

Circuit	Inputs → outputs	Key facts	Common IC
Multiplexer (MUX) – data selector	**2^n data inputs, n select lines** → 1 output	"Many-to-one"; parallel-to-serial conversion; **universal** logic element	74151 (8:1), 74153 (dual 4:1), 74157 (quad 2:1)
Demultiplexer – data distributor	1 input, n selects → 2^n outputs	"One-to-many"; serial-to-parallel; a decoder with enable = DEMUX	74155, 74156
Decoder	**n inputs → up to 2^n outputs**	Each output = one minterm ⇒ any function = decoder + OR gate; address decoding	74138 (3:8), 74154 (4:16), 7442 (BCD-decimal)
Encoder	2^n inputs → n outputs	Only one input active at a time	–
Priority encoder	2^n → n (+ valid bit)	Highest-priority active input encoded when several are active	74148 (8:3), 74147 (10:4, BCD)
BCD → 7-segment decoder	4 → 7	7447 (active-low, **common-anode** display), 7448 (active-high, **common-cathode**)	7447/7448

- **Implementing functions with MUX**: an n-variable function needs a **2^n:1 MUX** directly, or a **2^{n-1}:1 MUX** with n − 1 variables on select lines and the remaining variable (as 0, 1, x or x′) on data inputs.
- 2:1 MUX: Y = S′I_0 + SI_1; it realises NOT (I_0 = 1, I_1 = 0), AND (I_0 = 0, I_1 = B), OR (I_0 = B, I_1 = 1) ⇒ MUX is a **universal** element.
- Cascading: 2^n:1 MUX from 2:1 MUXes needs **2^n − 1** units (8:1 → **7**); 16:1 from 4:1 needs **5** (4 + 1); 32:1 from 8:1 needs 4 + one 4:1.
- Decoders in cascade: 4:16 from 2:4 decoders needs **5**; 5:32 from 3:8 decoders needs 4 (+ one 2:4 for enables).
- **Code converters**: binary → Gray uses (n − 1) XOR gates; BCD → excess-3 adds 0011; binary ↔ BCD.
- **Parity generator/checker**: XOR tree; 74180 (9-bit), 74280. n-bit parity needs n − 1 two-input XOR gates.
- **ROM / PLA** can also implement combinational logic (see 11.5).

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Hazard	Meaning	Cure
Static-1	Output should stay 1 but momentarily goes 0 (occurs in **SOP**/AND-OR)	Add the **redundant consensus term** (cover adjacent groups in K-map)
Static-0	Should stay 0 but momentarily goes 1 (in **POS**)	Add redundant sum term
Dynamic	Output changes 3 or more times instead of once (multi-level circuits)	Use two-level hazard-free logic
Essential	In asynchronous sequential circuits – due to unequal delays	Add delay elements

KEY FORMULAE

- Full adder: S = A ⊕ B ⊕ C, C_o = AB + BC + CA
- CLA: $C_{i+1} = G_i + P_iC_i$; $C_2 = G_1 + P_1G_0 + P_1P_0C_0$
- MUX select lines = $\log_2$(number of inputs); decoder outputs = 2^n

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Number of select lines for a 32:1 MUX? → **5**.
- Minimum 2:1 MUXes to make an 8:1 MUX? → **7**.
- Carry output of full adder? → **AB + BC + CA**.
- Static-1 hazard is removed by? → **Adding a redundant (consensus) term**.
- A 3-to-8 decoder + OR gate can implement? → **Any 3-variable function**.
- Which circuit converts parallel data to serial? → **Multiplexer** (or PISO register).

11.4 Sequential Circuits ★★★

- **Sequential circuit**: output depends on present inputs **and past state** (memory, feedback). Synchronous (clocked) vs asynchronous.
- **Latch** = **level-sensitive**, transparent while enabled; **flip-flop** = **edge-triggered** (changes only on clock edge). Basic memory cell = latch (cross-coupled NAND or NOR). Each flip-flop stores **1 bit**.
- NOR SR latch: S = R = 1 is **invalid** (both outputs 0). NAND (S′R′) latch: inputs active-low; S′ = R′ = 0 invalid.
- **JK flip-flop**: J = K = 1 → **toggle**. With level triggering, if clock pulse width t_p > propagation delay, output toggles repeatedly – **race-around condition**.
- Race-around arises when $\mathbf{t_p > t_{pd}}$; avoided by (i) **master–slave JK** (master enabled on high level, slave on low level), (ii) **edge triggering**, or (iii) keep $t_p < t_{pd} < T$.
- **D flip-flop** (delay/data): output follows D at clock edge; obtained from JK with K = J′ or SR with R = S′; used in registers. D latch = transparent latch.
- **T flip-flop** (toggle): JK with J = K = T; T = 1 → divide-by-2 (output frequency = f/2). Used in counters.
- **Preset/Clear** are **asynchronous** (direct) inputs, override clock; usually active-low.
- **Setup time** = minimum time data must be stable **before** the clock edge; **hold time** = minimum time data must remain stable **after** the edge. Violation → **metastability**.
- Max clock frequency of synchronous circuit: $f_{max} = 1/(t_{pd,FF} + t_{comb} + t_{setup})$.

FF	Characteristic equation	Truth table summary
SR	$\mathbf{Q^+ = S + R'Q}$ (SR = 0)	00 hold, 01 reset, 10 set, 11 invalid
JK	$\mathbf{Q^+ = JQ' + K'Q}$	00 hold, 01 reset, 10 set, **11 toggle**
D	$\mathbf{Q^+ = D}$	Output = D
T	$\mathbf{Q^+ = T \oplus Q}$ = TQ′ + T′Q	0 hold, 1 toggle

Q → Q⁺	S R	J K	D	T
0 → 0	0 X	0 X	0	0
0 → 1	1 0	1 X	1	1
1 → 0	0 1	X 1	0	1
1 → 1	X 0	X 0	1	0

- Excitation table (above) is used for **designing** counters; JK has the most don't-cares ⇒ simplest excitation logic.
- Conversions: JK → T (J = K = T); JK → D (J = D, K = D′); D → T (D = T ⊕ Q); SR → JK (S = JQ′, R = KQ).

Registers

Type	Entry / exit	Clocks to load n bits	Use
SISO	Serial in, serial out	n to load; output after **n** clocks (2n – 1 to read all out)	**Time delay** (delay = n × T_{clk})
SIPO	Serial in, parallel out	n	Serial → parallel
PISO	Parallel in, serial out	1 (load), then n to shift	Parallel → serial
PIPO	Parallel in, parallel out	**1**	Storage buffer (fastest)

- Shift left by one bit = **×2**; shift right = **÷2** (unsigned). Universal shift register **74194** (4-bit, bidirectional, parallel load); 7495, 74164 (SIPO), 74165 (PISO).

Counters

- Number of flip-flops for mod-N counter = $\mathbf{\lceil \log_2 N \rceil}$ (mod-10 → 4; mod-6 → 3; mod-100 → 7). n flip-flops: maximum modulus 2^n, max count $\mathbf{2^n - 1}$.
- Each flip-flop of a binary counter divides frequency by 2; output of nth stage = $\mathbf{f/2^n}$. Mod-N counter = divide-by-N.
- **Asynchronous (ripple)** counter: only first FF gets the clock; each FF clocked by the previous output. Total delay = $\mathbf{N \cdot t_{pd}}$ ⇒ $\mathbf{f_{max} = 1/(N \cdot t_{pd})}$ (plus strobe width). Glitches/decoding spikes. Simple, cheap.
- Ripple up-counter (negative-edge FFs) takes clock from **Q**; down-counter takes from **Q′** (reverse for positive-edge FFs).
- **Mod-N ripple counter** (N not power of 2): decode state N with NAND and **clear** all FFs (e.g. decade 7490: clear at 1010).
- **Synchronous** counter: all FFs clocked together; $f_{max} = 1/(t_{pd,FF} + t_{pd,gate})$, independent of number of stages; faster, no ripple glitches.
- **Ring counter**: shift register with $Q_{last} \rightarrow D_{first}$; N flip-flops → **mod-N**; one circulating 1; self-decoding (no decoder needed); must be preset; unused states = $2^N - N$.
- **Johnson (twisted-ring / switch-tail) counter**: $Q'_{last} \rightarrow D_{first}$; N flip-flops → **mod-2N**; decoding needs only **2-input AND** gates; unused states = $2^N - 2N$.
- **Lock-out**: counter enters an unused state and never returns to the valid sequence – avoid by design.
- ICs: **7490** decade (mod-2 × mod-5), 7492 (mod-12), 7493 (4-bit binary, mod-16), 74192/74193 synchronous up/down.

Counter (n FFs)	Modulus	Unused states
Binary ripple / synchronous	2^n	0
Ring	**n**	$2^n - n$
Johnson	**2n**	$2^n - 2n$

Moore machine	Mealy machine
Output depends only on **present state**	Output depends on **present state and present input**
Output changes only with clock (synchronous, glitch-free)	Output can change asynchronously with input
Needs **more states**	Needs **fewer states**; reacts one clock earlier

MEMORY TRICK

JK equation: "**J sets when Q is 0, K′ keeps when Q is 1**" → Q^+ = JQ′ + K′Q. Ring = **N**, Johnson = **2N** ("twisted = twice").

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Race-around condition occurs in? → **Level-triggered JK FF with J = K = 1**; cured by **master–slave**.
- A 4-bit Johnson counter has how many states? → **8**.
- Flip-flops needed for a mod-12 counter? → **4**.
- A 4-bit ripple counter with t_{pd} = 25 ns per FF: f_{max}? → **10 MHz** (1/100 ns).
- An 8-bit SISO register clocked at 1 MHz gives delay of? → **8 µs**.
- Characteristic equation of T flip-flop? → $\mathbf{Q^+ = T \oplus Q}$.

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11.5 A/D & D/A Converters, Schmitt Trigger, S&H, Memories ★★★

D/A converters

- **Binary-weighted resistor DAC**: resistors R, 2R, 4R … 2^{n-1}R (MSB gets smallest R); needs a **wide range of resistor values** (ratio 2^{n-1}; 8-bit → 128:1) ⇒ hard to make accurately; impractical beyond ≈ 4–8 bits.
- **R-2R ladder DAC**: only **two resistor values** (R and 2R); easy to fabricate/trim; input resistance seen by reference = R for any n; most popular. Inverted R-2R (current mode) is faster.
- **Resolution** (step size, 1 LSB) = $\mathbf{V_{ref}/2^n}$; full-scale output = $V_{ref}(1 - 2^{-n})$ = $(2^n - 1)$·step. When **full-scale output** V_{FS} is given, step = $\mathbf{V_{FS}/(2^n - 1)}$. % resolution = $100/(2^n - 1)$ % (≈ $100/2^n$). Note: textbooks use both conventions – check whether V is reference or full-scale.
- Example: 8-bit DAC, full scale 10.2 V → step = 10.2/255 = **40 mV**; 10-bit, V_{ref} = 10.24 V → 10 mV.
- **Accuracy**: deviation of actual from ideal output (typically ±½ LSB). **Settling time**: time for output to settle within ±½ LSB after a full-scale input change (ns–µs) – determines speed. **Monotonicity**: output never decreases when input increases.
- Other errors: offset, gain (scale-factor), non-linearity. Common DAC IC: **DAC0808** (8-bit, current output). Multiplying DAC: output ∝ digital input × analog V_{ref}.

A/D converters

ADC type	Conversion time (n bits)	Hardware	Key feature / use
Flash (parallel/simultaneous)	**1 clock – fastest**	**$2^n - 1$ comparators**, 2^n resistors, priority encoder (8-bit → 255 comparators)	Video, radar, digital oscilloscopes; costly, high power
Successive approximation (SAR)	**n clock cycles** (fixed, independent of input)	1 comparator, DAC, SAR logic	Most common in µP systems (e.g. **ADC0808/0809**, 8-bit, ≈100 µs)
Counter (digital ramp)	Up to **$2^n - 1$** clocks (variable, ∝ input)	Counter + DAC + comparator	Simple, slow
Tracking (servo)	Fast after initial lock	Up/down counter + DAC	Follows slowly varying input
Dual-slope (integrating)	**Slowest** (up to 2×2^n clocks; ms)	Integrator, comparator, counter	**Most accurate**, excellent **noise rejection** (mains hum averaged); independent of R, C and clock drift; used in **digital multimeters** (DVM)
Voltage-to-frequency	Slow	VCO + counter	Good noise immunity, telemetry
Sigma–delta (Σ-Δ)	Medium (oversampling)	1-bit modulator + digital filter	Very high resolution (16–24 bit), audio, precision measurement

- Speed order: **Flash > half-flash/pipeline > SAR > Σ-Δ > counter > dual-slope**. Accuracy/noise rejection: dual-slope best.
- **Quantisation error = ±½ LSB** (for rounding quantiser); quantisation noise rms = Δ/√12.
- Ideal ADC **SNR ≈ 6.02n + 1.76 dB** (sinusoidal full-scale input) ⇒ each extra bit adds ≈ **6 dB**; 8-bit ≈ 49.9 dB, 12-bit ≈ 74 dB.
- **Sampling theorem** (Nyquist): $f_s \geq \mathbf{2\, f_{max}}$; below this → aliasing (use anti-aliasing low-pass filter).
- Max input frequency for an ADC without S/H: limited because input must not change more than ½ LSB during conversion.

Sample-and-hold term	Meaning
Acquisition time	Time for hold capacitor to charge to input (within tolerance) after switching to sample mode
Aperture time	Delay between hold command and switch actually opening
Droop rate	Decay of held voltage due to capacitor leakage/bias current (dV/dt = I/C) – larger C ⇒ less droop but longer acquisition
Feed-through	Input signal leaking to output in hold mode

- S/H uses a low-leakage capacitor (polystyrene/Teflon), analog switch (FET) and buffers; IC **LF398**. Needed before SAR/flash ADC for changing inputs.

Schmitt trigger & multivibrators

- **Schmitt trigger** = regenerative comparator with **positive feedback**; two thresholds UTP and LTP; **hysteresis** V_H = UTP – LTP gives noise immunity, no chatter; converts **sine (or slow/noisy waveform) → square wave**. Inverting op-amp Schmitt: UTP/LTP = $\pm V_{sat} \cdot R_2/(R_1 + R_2)$. IC **7414** (hex Schmitt inverter).
- **Astable**: no stable state – free-running oscillator (square-wave generator, clock). **Monostable** (one-shot): one stable state – produces one pulse of fixed width per trigger (timer, pulse stretching). **Bistable**: two stable states – flip-flop (memory, frequency division).

KEY FORMULAE

- DAC step = $V_{ref}/2^n$; V_{out} = step × (decimal value of input)
- ADC: flash comparators = $2^n - 1$; SAR time = $n \cdot T_{clk}$; SNR = 6.02n + 1.76 dB
- 555 astable: $f = 1.44/[(R_A + 2R_B)C]$, duty = $(R_A + R_B)/(R_A + 2R_B)$ (> 50% without diode); 555 monostable: T = **1.1 RC**
- Op-amp Schmitt (symmetric): $V_H = 2V_{sat} \cdot R_2/(R_1 + R_2)$
- Memory: n address lines → 2^n locations; 64 KB = 2^{16} bytes

Semiconductor memories & programmable logic

Memory	Programming / erasing	Remark
Mask ROM	By manufacturer (mask)	Non-volatile; cheapest in bulk
PROM	User, **once** (fuses blown)	OTP
EPROM	Electrically programmed; erased by **UV light** (quartz window, ≈ 15–20 min) – whole chip	Floating-gate MOS (FAMOS); 2716, 2732, 2764
EEPROM / E^2PROM	Electrically erased, **byte-by-byte**, in circuit	Limited write cycles
Flash	Electrical erase in **blocks/sectors**	Fast, dense; pen drives, SSDs
SRAM	Read/write; cell = flip-flop (**6 transistors**)	Volatile; **fast, no refresh**; cache memory; costlier, less dense
DRAM	Read/write; cell = **1 transistor + 1 capacitor**	Volatile; charge leaks ⇒ **needs periodic refresh** (every few ms); dense, cheap; main memory; multiplexed row/column address (RAS/CAS)

- RAM is **volatile**; all ROM types are **non-volatile**. Memory of $2^n \times m$ needs n address lines and m data lines (e.g. 4K × 8 → 12 address, 8 data lines).
- Number of chips: total capacity / chip capacity (e.g. 16K × 8 from 1K × 4 chips = **32**).

PLD	AND array	OR array
PROM	Fixed (full decoder)	**Programmable**
PAL	**Programmable**	**Fixed**
PLA	**Programmable**	**Programmable** (most flexible, slowest)

- **CPLD**: several PAL-like blocks + interconnect. **FPGA**: array of configurable logic blocks (LUT-based) + programmable interconnect; usually **SRAM-based** (volatile configuration, re-loaded at power-up); programmed in VHDL/Verilog.

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- Fastest ADC? → **Flash**; comparators for 8-bit flash → **255**.
- ADC used in digital voltmeters? → **Dual-slope** (best noise rejection).
- Conversion time of 10-bit SAR ADC with 1 MHz clock? → **10 µs**.
- Advantage of R-2R ladder over weighted-resistor DAC? → **Only two resistor values**.
- Circuit converting sine to square wave with hysteresis? → **Schmitt trigger**.
- Memory that needs refreshing? → **DRAM**; PAL has? → **programmable AND, fixed OR**.

11.6 8085 Architecture ★★★

- Intel **8085** (1976): **8-bit** microprocessor (8-bit data bus, ALU, accumulator), **40-pin DIP**, **NMOS**, single **+5 V** supply (8080 needed +5, −5, +12 V), ≈ 6,500 transistors.
- **16-bit address bus** ⇒ addresses **2^{16} = 65,536 = 64 KB** of memory (0000H–FFFFH).
- Lower address and data lines are **multiplexed (AD_0–AD_7)**; **ALE** (Address Latch Enable) goes high in **T_1** of each machine cycle to latch A_0–A_7 into an external latch (**74LS373**). A_8–A_{15} are unidirectional address lines. Data bus is bidirectional.
- **On-chip clock generator**: crystal on X_1, X_2; internal frequency = **crystal ÷ 2** ⇒ 8085A runs at **3 MHz with a 6 MHz crystal** (T-state = 333 ns); 8085A-2 at 5 MHz (10 MHz crystal). CLK OUT supplies peripherals.
- **Registers**: Accumulator A (8-bit), flag register (8-bit, 5 flags), general-purpose **B, C, D, E, H, L** (8-bit) usable as pairs **BC, DE, HL** (16-bit); HL is the **memory pointer (M)**. **SP** and **PC** are **16-bit**. Temporary registers W, Z and instruction register are not accessible to programmer. PSW = A + flags.
- **PC** holds address of the **next** instruction (byte) to be fetched; **SP** points to top of stack.
- 8085 instruction set: **74 instructions**, **246 opcodes**. Instruction decoder, timing & control, interrupt control and serial I/O control are on chip.

D7	D6	D5	D4	D3	D2	D1	D0
S	**Z**	X	**AC**	X	**P**	X	**CY**

X = unused bit

8085 flag register: S (D7), Z (D6), AC (D4), P (D2), CY (D0).

Flag (bit)	Set (=1) when
S – Sign (D7)	D7 of result = 1 (negative in 2's complement)
Z – Zero (D6)	Result = 00H
AC – Auxiliary carry (D4)	Carry from bit D3 to D4 – used internally by **DAA**; not testable by jump instructions
P – Parity (D2)	Result has **even** number of 1s
CY – Carry (D0)	Carry out of D7 (addition) or borrow (subtraction)

- 8085 has **no overflow flag**. Conditional jumps test only **S, Z, P, CY** (JZ/JNZ, JC/JNC, JP/JM, JPE/JPO).

Machine cycles and timing

- **T-state** = one clock period. **Machine cycle** = time for one bus operation (3–6 T). **Instruction cycle** = fetch + execute = 1 to 5 machine cycles.
- **Opcode fetch** = **4 T** (6 T for some instructions: INX, DCX, PUSH, CALL, RST, PCHL, SPHL, conditional RET). **Memory read / memory write / I/O read / I/O write** = **3 T** each. Bus idle (e.g. DAD) = 3 T.
- First machine cycle of every instruction is **opcode fetch**. Max T-states for any 8085 instruction = **18** (CALL: 6 + 3 + 3 + 3 + 3).
- **READY** low ⇒ processor inserts **wait states** (for slow memory/peripherals).

Machine cycle	IO/$\overline{M}$	S1	S0	Control
Opcode fetch	0	1	1	$\overline{RD}$ = 0
Memory read	0	1	0	$\overline{RD}$ = 0
Memory write	0	0	1	$\overline{WR}$ = 0
I/O read	1	1	0	$\overline{RD}$ = 0
I/O write	1	0	1	$\overline{WR}$ = 0
Interrupt acknowledge	1	1	1	$\overline{INTA}$ = 0
Halt	Z	0	0	$\overline{RD}$, $\overline{WR}$ = Z
Hold / Reset	Z	X	X	Z

- Decoded control signals: **$\overline{MEMR}$** active when IO/$\overline{M}$ = 0 and $\overline{RD}$ = 0; $\overline{MEMW}$ when IO/$\overline{M}$ = 0 and $\overline{WR}$ = 0; **$\overline{IOR}$** when IO/$\overline{M}$ = 1 and $\overline{RD}$ = 0; $\overline{IOW}$ when IO/$\overline{M}$ = 1 and $\overline{WR}$ = 0.

Interrupts

Interrupt	Priority	Maskable?	Trigger	Vector
TRAP (pin 6)	**1 (highest)**	**Non-maskable**	**Edge and level** (rising edge + stays high)	**0024H**
RST 7.5	2	Maskable	**Rising edge** (latched)	**003CH**
RST 6.5	3	Maskable	Level (high)	**0034H**
RST 5.5	4	Maskable	Level (high)	**002CH**
INTR	5 (lowest)	Maskable (EI/DI only)	Level	**Non-vectored** – device supplies RST n / CALL via $\overline{INTA}$

- Vector address = **8 × n** (in hex): RST 5.5 → 8 × 5.5 = 44 = 2CH; TRAP behaves as "RST 4.5" → 36 = 24H.
- **EI** sets interrupt-enable flip-flop (takes effect after the next instruction); **DI** resets it. When any interrupt is accepted, IE is reset automatically ⇒ program must re-execute EI. TRAP is unaffected by EI/DI/SIM.
- **SIM** (Set Interrupt Mask) – uses accumulator: D0–D2 = masks M5.5, M6.5, M7.5 (1 = masked); **D3 = MSE** (mask set enable); D4 = R7.5 (reset RST 7.5 flip-flop); D6 = **SDE** (serial data enable); D7 = **SOD** bit.
- **RIM** (Read Interrupt Mask) – loads A: D0–D2 = current masks, D3 = IE, D4–D6 = pending I5.5, I6.5, I7.5, **D7 = SID**.
- **Serial I/O**: **SID** (serial input, pin 5) read via **RIM**; **SOD** (serial output, pin 4) written via **SIM**.
- **HOLD/HLDA**: DMA request/acknowledge; processor relinquishes buses after the current machine cycle; HOLD is generally quoted as having higher priority than even TRAP.
- **RESET IN** $\overline{}$ (active-low): PC = **0000H**, interrupt-enable and HLDA flip-flops reset; buses tri-stated during reset. **RESET OUT** resets peripherals.

Pin	Signal	Pin	Signal
1, 2	X_1, X_2	30	ALE
4 / 5	SOD / SID	34	IO/$\overline{M}$
6–10	TRAP, RST 7.5, 6.5, 5.5, INTR	35 / 36	READY / RESET IN $\overline{}$
12–19	AD_0–AD_7	38 / 39	HLDA / HOLD
20 / 40	V_{SS} (GND) / V_{CC} (+5 V)	21–28	A_8–A_{15}

MEMORY TRICK

Vectors "**24-2C-34-3C**" go up by 8: TRAP 24, 5.5 → 2C, 6.5 → 34, 7.5 → 3C. Flags from D7 down: "**S**ome **Z**ebras **X** **A**te **X** **P**lain **X** **C**arrots" = S Z X AC X P X CY.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Non-maskable interrupt of 8085? → **TRAP** (vector **0024H**).
- Signal used to demultiplex AD_0–AD_7? → **ALE**.
- Number of flags in 8085? → **5**.
- Status IO/$\overline{M}$, S1, S0 for opcode fetch? → **0, 1, 1**.
- 8085 operating frequency with 6 MHz crystal? → **3 MHz**.
- Instruction used to read SID? → **RIM**.

11.7 8085 Instruction Set, Programming & Interfacing ★★★

Addressing mode	Operand is	Examples
Immediate	In the instruction itself	MVI A, 32H; ADI 05H; LXI H, 2050H

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Register	In a register	MOV A, B; ADD C
Direct	16-bit address given in instruction (8-bit for I/O)	LDA 2050H; STA 3000H; IN 01H
Register indirect	Address in a register pair	MOV A, M; LDAX B; STAX D
Implied / implicit	Operand implied by opcode (accumulator)	CMA; RAL; STC; DAA

Size	Format	Examples
1 byte	Opcode only	MOV, ADD r, INR, CMA, RLC, PUSH, RET, XCHG, HLT
2 bytes	Opcode + 8-bit data/port	MVI, ADI, SUI, ANI, CPI, IN, OUT
3 bytes	Opcode + 16-bit data/address (**low byte first**)	LXI, LDA, STA, LHLD, SHLD, JMP, CALL

- Instruction groups: data transfer, arithmetic, logical, branch, machine control (HLT, NOP, EI, DI, SIM, RIM). Data transfer instructions **do not affect flags**.
- 8085 stores 16-bit data in memory as **little-endian** (lower byte at lower address).

Instruction	Operation	Bytes	T-states
MOV r1, r2	r1 ← r2	1	4
MOV r, M / MOV M, r	via (HL)	1	7
MVI r, data / MVI M, data	r ← data	2	7 / 10
LXI rp, data16	rp ← 16-bit data	3	10
LDA / STA addr	A ↔ (addr)	3	13
LHLD / SHLD addr	L ↔ (addr), H ↔ (addr+1)	3	16
LDAX / STAX B or D	A ↔ (BC) or (DE)	1	7
XCHG	HL ↔ DE	1	4
ADD r / ADD M / ADI	A ← A + operand	1/1/2	4 / 7 / 7
INR r / INR M	+1	1	4 / 10
INX / DCX rp	rp ± 1	1	**6**
DAD rp	HL ← HL + rp	1	10
JMP addr	PC ← addr	3	10
Jcond addr	Jump if condition	3	**10 (taken) / 7 (not)**
CALL addr	Push PC, jump	3	**18**
Ccond addr	Conditional call	3	18 / 9
RET / Rcond	Pop PC	1	10 / (12 or 6)
PUSH / POP rp	Stack	1	12 / 10
RST n	CALL 8n	1	12
IN / OUT port	A ↔ port	2	10
NOP / HLT	–	1	4 / 5

Flags affected – the most-asked table

Instruction(s)	Flags affected
MOV, MVI, LDA, STA, LXI, LHLD, XCHG, IN, OUT, PUSH, JMP, CALL	**None**
ADD, ADC, ADI, SUB, SBB, SUI, CMP, CPI, DAA	**All five** (S, Z, AC, P, CY)
INR, DCR	All **except CY**
INX, DCX	**None** (so DCX + JNZ loop needs ORA test)
DAD	**Only CY**
ANA, ANI	S, Z, P per result; **CY = 0, AC = 1**
ORA, XRA (ORI, XRI)	S, Z, P per result; **CY = 0, AC = 0**
RLC, RRC, RAL, RAR	**Only CY**
CMA	None
STC / CMC	CY = 1 / CY complemented
POP PSW	All (loaded from stack)

- **XRA A** clears A and CY in one byte (A = 00H, Z = 1, CY = 0); **ORA A** leaves A unchanged but updates flags and clears CY.
- **SUB**: done by 2's complement; CY is **set if there is a borrow** (A < operand) – i.e. CY = complement of actual carry.
- **DAA** (Decimal Adjust Accumulator): after **BCD addition** – adds 06H if low nibble > 9 or AC = 1; adds 60H if high nibble > 9 or CY = 1. Not valid after subtraction in 8085.
- **ADC** = A + operand + CY; **SBB** = A − operand − CY (multibyte arithmetic).

CMP r / CPI (A unchanged)	CY	Z
A < operand	**1**	0
A = operand	0	**1**
A > operand	0	0

Rotate	Action
RLC	Rotate A left; D7 → D0 and D7 → CY (8-bit rotate)
RAL	Rotate left **through carry**; D7 → CY, CY → D0 (9-bit rotate)
RRC	Rotate right; D0 → D7 and D0 → CY
RAR	Rotate right through carry; D0 → CY, CY → D7

- RLC multiplies by 2 (if D7 = 0); RRC divides by 2 (if D0 = 0).

Branch, stack and subroutines

- Conditional jumps: **JC/JNC** (CY), **JZ/JNZ** (Z), **JP/JM** (S = 0 plus / S = 1 minus), **JPE/JPO** (P = 1 even / P = 0 odd). Same conditions for CALL and RET.
- **Stack** = **LIFO** area in R/W memory; initialised by **LXI SP, addr**; grows **downward** (towards lower addresses).
- **PUSH rp**: high byte stored at (SP − 1), then low byte at (SP − 2) (SP = value before PUSH); net **SP ← SP − 2**. **POP rp**: low byte from (SP), high from (SP + 1), **SP ← SP + 2**. PUSH PSW saves A (high) and flags (low).
- **CALL**: saves return address (address of next instruction = PC + 3) on stack, then jumps; **RET** pops it into PC. Subroutine nesting limited only by stack size.
- **RST n** (n = 0–7): 1-byte CALL to address **8 × n** (RST 0 → 0000H, RST 1 → 0008H … RST 7 → **0038H**); software interrupts.
- **XTHL**: exchange HL with top of stack; **SPHL**: SP ← HL; **PCHL**: PC ← HL (jump to address in HL).

Delay loop calculation

KEY FORMULAE

- Delay = (total T-states) × T, where T = $1/f_{clk}$ (3 MHz → **0.333 µs**)
- Single loop: MVI C, N (7 T) ; LOOP: DCR C (4 T) ; JNZ LOOP (10/7 T)
- Total T = 7 + 14N − 3 (last JNZ not taken). For N = FFH = 255: 7 + 3570 − 3 = **3574 T ≈ 1.19 ms** at 3 MHz
- Register-pair loop: LXI D, N (10 T); LOOP: DCX D (6); MOV A, D (4); ORA E (4); JNZ LOOP (10/7) → 24 T per pass (DCX sets no flags, hence MOV/ORA test)

Sample programs

Add two numbers at 2050H, 2051H; result at 2052H	Largest of N numbers (count at 2050H, data from 2051H)

LDA 2050H
MOV B, A
LDA 2051H
ADD B
STA 2052H
HLT
	LXI H, 2050H
	MOV C, M ; count
	INX H
	MOV A, M ; first number
	DCR C
	LOOP: INX H
	CMP M
	JNC SKIP ; A ≥ M
	MOV A, M
	SKIP: DCR C
	JNZ LOOP
	STA 3000H
	HLT

- For the **smallest** number replace JNC by **JC**. For a 16-bit sum with carry use ADC / or JNC + INR for carry count.
- Swapping nibbles: four RLC (or RRC) instructions.

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Interfacing

Memory-mapped I/O	I/O-mapped (peripheral-mapped) I/O
Device treated as a memory location – **16-bit address**	Separate I/O space – **8-bit port address ⇒ 256 input + 256 output ports**
All memory instructions (LDA, STA, MOV M, arithmetic on M) usable	Only **IN** and **OUT** (data only via accumulator)
Control: $\overline{MEMR}/\overline{MEMW}$ ($IO/\overline{M}$ = 0); reduces memory space	Control: $\overline{IOR}/\overline{IOW}$ ($IO/\overline{M}$ = 1); full 64 KB memory available
Slower (13 T for LDA/STA)	IN/OUT 10 T; port address duplicated on A_8–A_{15} and AD_0–AD_7

Peripheral IC	Function / key facts
8255A PPI	Programmable Peripheral Interface: **24 I/O lines**, ports A, B, C (C split into upper/lower 4 bits). Control word D7 = 1 → **I/O mode**, D7 = 0 → **BSR** (bit set/reset of port C). **Mode 0** simple I/O; **Mode 1** strobed (handshake) I/O for A and B; **Mode 2** bidirectional – **port A only**. Address lines A_1A_0: 00 = A, 01 = B, 10 = C, 11 = control register
8253 / 8254	Programmable interval timer: **three 16-bit down-counters**, **6 modes** (0 interrupt on terminal count, 1 one-shot, 2 rate generator, **3 square-wave**, 4 software strobe, 5 hardware strobe). 8253 up to ≈2.6 MHz; 8254 up to 8 MHz (8254-2: 10 MHz) and adds read-back command
8259A PIC	Programmable interrupt controller: **8 interrupt levels**; cascade 1 master + 8 slaves → **64 levels**; fully nested, rotating priority modes
8257 (8237)	DMA controller: **4 channels**; uses HOLD/HLDA; 16-bit address, 14-bit terminal count
8279	Keyboard/display controller: 64-key keyboard scan, 8-character **FIFO**, 16-digit display RAM; debouncing
8251	USART – serial communication (synchronous/asynchronous)
8155 / 8355 / 8755	8155: 256 B RAM + I/O ports + 14-bit timer; 8355: 2 KB ROM + I/O; 8755: 2 KB EPROM + I/O
8212 / 74LS373	8-bit latch / buffer (address latch with ALE)

MEMORY TRICK

"**INX is fla(g)tless**; **DAD dates only Carry**; **INR/DCR** spare the carry." Peripheral numbers: **55**-PPI, **53**-Timer, **59**-PIC, **57**-DMA, **79**-Keyboard, **51**-USART.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Instruction that affects no flag though it increments a register pair? → **INX**.
- Number of T-states in CALL? → **18**; in MOV r, M? → **7**.
- RST 7 transfers control to? → **0038H**.
- After PUSH B, SP changes by? → **Decreases by 2**.
- Maximum I/O ports in I/O-mapped I/O? → **256 input + 256 output**.
- Mode in which port A of 8255 is bidirectional? → **Mode 2**.

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12. Power Electronics

NOTE

Power electronics usually gives **4–6 questions** in GATE/PSU-type papers. Most of them are direct formula questions (average output of converters, chopper output voltage, inverter rms values, harmonic orders, input power factor) or device facts (latching vs holding current, TRIAC modes, MOSFET vs IGBT, commutation classes). Learn the formula boxes and the comparison tables by heart. Throughout, V_m = peak supply voltage, α = firing angle, D = duty cycle, V_s = DC source voltage.

12.1 Power Semiconductor Devices ★★★

Classification

- **Uncontrolled** (on/off decided by the circuit): **power diode**.
- **Semi-controlled** (turned ON by gate, cannot be turned OFF by gate): **SCR**, TRIAC.
- **Fully controlled** (ON and OFF by control signal): **GTO, BJT, MOSFET, IGBT, IGCT, MCT, SIT**.
- **Current-controlled**: BJT, SCR, GTO. **Voltage-controlled**: **MOSFET, IGBT**, MCT, SIT (very small gate drive power).
- **Unipolar** (majority carrier only): **MOSFET, Schottky diode** → no storage time, fastest. **Bipolar** (both carriers): diode, BJT, SCR, GTO, IGBT.
- **Pulse-gate** requirement: SCR, GTO, TRIAC (a short pulse is enough). **Continuous gate signal**: BJT, MOSFET, IGBT.
- Bidirectional current devices: **TRIAC, DIAC**. Symmetrical blocking (forward + reverse): SCR, symmetrical GTO. Asymmetric (no reverse blocking): MOSFET (body diode), most IGBTs, asymmetric GTO.

Power diodes

- Power diode has a lightly doped **n^- drift region** (p^+–n^-–n^+) to block high reverse voltage; thicker drift layer → higher voltage rating but higher on-state drop.
- **Reverse recovery time t_{rr}** = time from current zero crossing until reverse current decays to 25% of peak reverse current I_{RR}; $t_{rr} = t_a + t_b$.
- **Softness factor S = t_b/t_a**; S = 1 → soft recovery; S < 1 → abrupt (snappy) recovery → voltage spikes, EMI.
- **Reverse recovery charge Q_{rr}** = area under reverse current ≈ **½ I_{RR} t_{rr}**. Q_{rr} and t_{rr} increase with forward current and temperature.
- For abrupt recovery ($t_b \approx 0$): I_{RR} = **$\sqrt{(2\,Q_{rr}\,di/dt)}$**, $t_{rr} = \sqrt{(2Q_{rr}/(di/dt))}$.
- Forward voltage drop of Si power diode ≈ **0.7–1.2 V** (higher at rated current).

Diode type	t_{rr}	Ratings	Use
General purpose (rectifier)	**≈ 25 µs** (high)	Up to several kV, several kA	Line-frequency (50 Hz) rectifiers, traction, battery charging
Fast recovery	**< 5 µs** (typ. 0.1–5 µs)	Up to few kV, ~1 kA	Choppers, inverters, freewheeling diodes
Schottky (metal–semiconductor)	**Negligible (ns)** – no minority carrier storage	Low voltage, ≤ ~100 V (SiC Schottky up to ~1.7 kV)	Low-voltage high-current SMPS outputs

- **Schottky diode**: forward drop only **0.3–0.5 V**, majority carrier device, but **higher leakage current** and low reverse voltage rating.

Thyristor (SCR)

- **SCR** = Silicon Controlled Rectifier; **4-layer p-n-p-n**, **3 junctions (J1, J2, J3)**, **3 terminals** – anode, cathode, gate. Gate is connected to the **p-layer near the cathode**.
- **Forward blocking mode**: anode +ve, J1 & J3 forward biased, **J2 reverse biased** → only small leakage current.
- **Reverse blocking mode**: anode −ve, **J1 and J3 reverse biased** (J2 forward). J3 has low breakdown (heavily doped) so J1 does most of the blocking.
- **Forward conduction**: all junctions forward biased, on-state drop ≈ **1–2 V**.
- **Two-transistor model**: SCR = pnp (Q1) + npn (Q2) with regenerative feedback. Anode current $I_A = (\alpha_2 I_G + I_{CBO1} + I_{CBO2}) / [1 - (\alpha_1 + \alpha_2)]$. Device turns on when **$\alpha_1 + \alpha_2 \to 1$**.
- **Latching current I_L** = minimum anode current required to keep SCR ON **immediately after turn-on** when gate pulse is removed.
- **Holding current I_H** = minimum anode current below which a conducting SCR **turns OFF** (returns to forward blocking).
- **$I_L > I_H$**; typically **$I_L \approx 2–3 \times I_H$**. Both are in mA range (typ. I_H few mA to ~100 mA).
- **Forward breakover voltage V_{BO}** decreases as gate current increases. With $I_G = 0$, breakover occurs at V_{BO} (maximum).
- Gate pulse width must be long enough for anode current to rise above I_L – important for **inductive loads** (current rises slowly) → use **pulse train / long pulse**.
- Once ON, **gate loses control**; SCR is turned OFF only by reducing anode current below I_H and applying reverse voltage for time ≥ t_q (**commutation**).

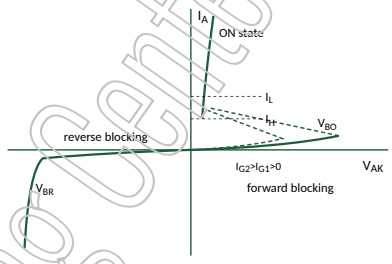

Fig 12.1 SCR static V-I characteristic: V_{BO} falls as gate current rises; $I_L > I_H$.

Turn-on methods of SCR

Method	Remark
Gate triggering	**Most common, reliable and efficient**; positive gate–cathode pulse
Forward voltage triggering ($V_{AK} > V_{BO}$)	Avalanche of J2; **may damage device**, not used
dv/dt triggering	Charging current of J2 capacitance i = C_j·dv/dt; **unwanted** (false triggering)
Temperature (thermal) triggering	Leakage rises with temperature; unwanted
Light triggering (LASCR)	Light falls on J2; used in **HVDC valves** (electrical isolation)

- Gate characteristic: operate within **$V_{g,max}$, $I_{g,max}$, $P_{g,max}$ (hyperbola)** and above minimum gate trigger voltage/current. Load line of gate source: $V_g = E_s - I_g R_s$.
- With pulse gating, allowable peak gate power $P_{gm} = P_{gav}/(\text{duty ratio}) = P_{gav}/(f \cdot T_{on})$.
- Gate should be **not** positive during reverse bias (increases reverse leakage → heating).
- **Turn-on time $t_{on} = t_d + t_r + t_p$** (delay + rise + spread), typically **1–4 µs**. Delay time measured from 90% gate current (some texts: 10%) to 10% anode current (V_{AK} falls to 90%); rise time: anode current 10% → 90% (V_{AK} 90% → 10%).
- **Turn-off time $t_q = t_{rr} + t_{gr}$** (reverse recovery + gate recovery). Converter-grade SCR **50–100 µs**; inverter-grade (fast) SCR **3–50 µs**.
- **Circuit turn-off time t_c must be > t_q** (device turn-off time); otherwise commutation failure.

Protection of SCR

Problem	Protection
dv/dt (false turn-on)	**RC snubber in parallel** with SCR
di/dt (local hot spots near gate at turn-on)	**Inductor in series** with SCR
Over-voltage (switching/lightning)	MOV, selenium suppressor, snubber
Over-current (surge)	**Fast-acting fuse** (I^2t of fuse < I^2t of SCR), circuit breaker
Over-heating	**Heat sink**, forced cooling
Gate	Resistor/zener across gate, pulse transformer or opto-isolation

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- Snubber R limits capacitor discharge current through SCR at turn-on; series L limits (di/dt) = V_s/L.
- **Sub-cycle surge current rating** and **I^2t rating** are used for fuse coordination.
- **Heat sink**: $T_j - T_a = P \times (\theta_{jc} + \theta_{cs} + \theta_{sa})$; thermal resistance in °C/W. Heat sinks are usually **aluminium** (black anodised fins for radiation). Max junction temperature of Si devices ≈ **125–150 °C**.

Other thyristor-family devices

- **TRIAC** = two SCRs in **anti-parallel** with one gate; terminals **MT1, MT2, Gate**; conducts in **both directions**; 5-layer equivalent.
- TRIAC triggering modes: **I+** (MT2 +, gate +), I− (MT2 +, gate −), III+ (MT2 −, gate +), III− (MT2 −, gate −). **Most sensitive: I+** (then III−); **least sensitive: III+**. Practical use: **I+ and III−**.
- TRIAC: low dv/dt rating, low frequency (≤ 400 Hz), used in **fan regulators, light dimmers, heater control, AC switches**.
- **DIAC** = bidirectional trigger diode, **no gate**, 3-layer; breakover ≈ **30 V** both directions; used to **trigger TRIAC** (DIAC–TRIAC fan regulator).
- **GTO** (Gate Turn-Off thyristor): turned ON by +ve gate pulse, **turned OFF by large negative gate current**. **Turn-off gain $\beta_{off} = I_A/I_{G(off)} \approx 4–5$** (low) → large negative gate current needed. No commutation circuit required; high voltage/current (≈ 6 kV, 6 kA); used in traction and large drives. Needs turn-off snubber.
- **IGCT** (Integrated Gate Commutated Thyristor) = improved GTO with integrated gate driver, turn-off gain ≈ 1, snubberless turn-off.
- **SCS** (silicon controlled switch) – two gates; **PUT** (programmable UJT) – anode-gate thyristor used in relaxation oscillators; **SUS** silicon unilateral switch; **LASCR** – light activated SCR.
- **MCT** (MOS Controlled Thyristor) – MOS gate controls thyristor; low on-state drop.

Transistor-type devices

- **Power BJT**: current-controlled, low current gain β ≈ **5–20** (use **Darlington** to increase gain), suffers **second breakdown**, negative temperature coefficient of resistance → difficult to parallel (current hogging/thermal runaway). Operated in **saturation** (ON) and **cut-off** (OFF). Safe Operating Area (FBSOA, RBSOA). Largely replaced by IGBT.
- **Power MOSFET**: **voltage-controlled** (gate insulated by SiO_2, very high input impedance), **majority carrier device**, **fastest switching** (tens of ns, up to **100 kHz–MHz**), **no second breakdown**.
- MOSFET $R_{DS(on)}$ has **positive temperature coefficient → easy paralleling**, no thermal runaway. But $R_{DS(on)}$ rises steeply with voltage rating ($\propto V^{2.5}$) → suited to **low voltage (< ~500 V), high frequency** applications (SMPS, chargers).
- Power MOSFETs are usually **enhancement-type**, **n-channel** (higher electron mobility); vertical structure (VMOS/DMOS); contain an **intrinsic (body) anti-parallel diode**. Threshold voltage $V_{GS(th)} \approx$ **2–4 V**.
- **IGBT** (Insulated Gate Bipolar Transistor): **MOSFET input (voltage-controlled gate) + BJT output (low conduction loss via conductivity modulation)**. Terminals: **Gate, Collector, Emitter**. Structure like MOSFET with extra **p^+ injecting layer** at collector.
- IGBT: medium–high power, switching frequency typically **1–50 kHz** (≈ up to 20 kHz in high power), ratings up to **6.5 kV, ~3.6 kA**; drawback – **latch-up** (parasitic thyristor) and **tail current** at turn-off. Dominant device in **VFDs, UPS, traction, solar inverters**.
- Wide band-gap devices: **SiC MOSFET, GaN HEMT** – higher voltage, temperature and frequency, lower losses (EVs, solar).

Device	Control	Typical max ratings	Switching frequency	Remarks
SCR	Current pulse, ON only	**Highest: ~10 kV, ~5 kA**	Line freq (< 1 kHz)	HVDC, rectifiers, soft starters
GTO	Current pulse, ON/OFF	~6 kV, 6 kA	≤ 1–2 kHz	Turn-off gain 4–5
TRIAC	Current pulse, ON	~1 kV, ~50 A	50/60 Hz	AC control
Power BJT	Continuous base current	~1.2 kV, ~500 A	~10 kHz	Second breakdown
Power MOSFET	**Gate voltage**	~1 kV, ~100 A (low V best)	**Highest: 100 kHz – MHz**	+ve temp coeff, easy parallel
IGBT	**Gate voltage**	~6.5 kV, ~3.6 kA	1–50 kHz	Low $V_{CE(sat)} \approx$ 1.5–3 V

MEMORY TRICK

"SCR is Strong, MOSFET is Mercurial (fast), IGBT is In-Between." Power ↑ order: MOSFET < BJT < IGBT < GTO < SCR. Frequency ↑ order is roughly the reverse, except IGBT is faster than BJT (SCR < GTO < BJT < IGBT < MOSFET).

Series and parallel operation

- **Series** connection → for **higher voltage**; **parallel** connection → for **higher current**.
- **String efficiency** = actual voltage (or current) rating of string / (n × rating of one device). **Derating factor (DRF) = 1 − string efficiency**.
- Series operation: unequal leakage currents (static) and unequal Q_{rr}/turn-on times (dynamic) cause unequal voltage sharing.
- **Static equalisation → shunt resistor R** across each SCR: $R = (nV_{bm} - V_s)/((n-1)\Delta I_b)$.
- **Dynamic equalisation → shunt capacitor C (with series R and anti-parallel diode)** across each SCR: $C = (n-1)\Delta Q/(nV_{bm} - V_s)$.
- Parallel operation: unequal current sharing due to different V-I characteristics → use **series resistance** (DC, lossy) or **magnetically coupled reactors** (dynamic) and symmetrical mounting; common heat sink.

Commutation of SCR (classes A–F)

Class	Name	Feature
A	Load commutation (self-commutation by resonating load)	Series LC load makes current naturally zero; series inverters
B	Resonant-pulse commutation (self-commutation by LC circuit)	LC circuit across SCR; current commutation
C	**Complementary commutation**	Firing one SCR turns off the other (via capacitor); both carry load current
D	**Impulse (auxiliary) commutation**	Auxiliary SCR discharges charged capacitor; **voltage commutation**; choppers
E	External pulse commutation	Pulse from external source
F	**Line / natural commutation**	AC supply reversal; **phase-controlled rectifiers, AC voltage controllers, cycloconverters**

- Forced commutation (A–E) needed in **DC circuits** (choppers, inverters); natural commutation (F) needs no extra components.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Relation between latching and holding current of SCR? → **$I_L > I_H$ (≈ 2–3 times)**.
- In forward blocking state, which junction of SCR is reverse biased? → **J2 (middle junction)**.
- dv/dt protection of SCR is provided by? → **RC snubber across the SCR**; di/dt by **series inductor**.
- Most sensitive triggering mode of TRIAC? → **I+ (MT2 positive, gate positive)**.
- Device with positive temperature coefficient that can be easily paralleled? → **Power MOSFET**.
- Natural (line) commutation is class? → **Class F**; complementary commutation → **Class C**.

12.2 Uncontrolled & Controlled Rectifiers ★★★

Uncontrolled (diode) rectifiers – quick values

Rectifier (R load)	V_{dc}	Ripple frequency	Ripple factor	PIV per diode
1-φ half-wave	**$V_m/\pi = 0.318\ V_m$**	f	**1.21**	V_m
1-φ full-wave centre-tap	**$2V_m/\pi = 0.637\ V_m$**	2f	**0.48**	**$2V_m$**
1-φ bridge	$2V_m/\pi$	2f	0.48	**V_m**
3-φ half-wave (3-pulse)	**$3\sqrt{3}V_m/2\pi = 0.827\ V_m$**	3f	0.17	$\sqrt{3}V_m$

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3-φ bridge (6-pulse)	**$3\sqrt{3}V_m/\pi = 1.654\ V_m = 1.35\ V_L$**	**6f (300 Hz)**	**0.042**	$\sqrt{3}V_m$ (= V_{mL})

- Here V_m = peak **phase** voltage; V_L = rms line voltage. TUF: half-wave **0.287**, centre-tap 0.672, bridge **0.812**, 3-φ bridge **0.954**.
- Rectification efficiency: half-wave **40.5%**, full-wave **81%**, 3-φ bridge **99.8%**.
- Higher pulse number → higher ripple frequency → **lower ripple, smaller filter**, better input pf.

Single-phase controlled rectifiers

- **Phase control**: output controlled by delaying the firing angle α measured from the point where the SCR would naturally start conducting (natural commutation, class F).
- **1-φ half-wave, R load**: conduction from α to π; $V_{dc} = (V_m/2\pi)(1 + \cos\alpha)$. Range of α: 0–180°.
- Half-wave with RL load: current continues beyond π up to extinction angle β (conduction angle $\gamma = \beta - \alpha$); output has negative portion → $V_{dc} = (V_m/2\pi)(\cos\alpha - \cos\beta)$.
- Half-wave with RLE (battery charging): SCR can fire only when $v_s > E$, i.e., $\alpha \geq \theta_1 = \sin^{-1}(E/V_m)$.
- **Freewheeling diode (FD)** across load: prevents negative output voltage, improves input pf, makes load current continuous, transfers stored energy of L to load, relieves SCR (natural commutation easier).
- **1-φ full-wave (midpoint or bridge), R load**: $V_{dc} = (V_m/\pi)(1 + \cos\alpha)$.
- **1-φ full converter (4 SCRs), highly inductive (continuous) load**: $V_{dc} = (2V_m/\pi)\cos\alpha$. For **α < 90°** → rectification ($+V_{dc}$); **α > 90°** → **inversion** ($-V_{dc}$, power flows from DC side to AC) provided a DC source exists in load (e.g., motor in regenerative braking).
- Practical inversion limit **α ≈ 150–165°** (margin for overlap and SCR turn-off time).
- **1-φ semi-converter** (2 SCRs + 2 diodes): $V_{dc} = (V_m/\pi)(1 + \cos\alpha)$ for any load; contains inherent freewheeling action; V_{dc} always positive.
- Midpoint converter: each SCR PIV = **$2V_m$**; bridge: PIV = **V_m**. Midpoint needs centre-tapped transformer (TUF lower).
- RMS output (full converter, continuous highly inductive load): $V_{or} = V_m/\sqrt{2} = V_s$ (independent of α). With R load it depends on α: $V_{or} = V_s\sqrt{[(1/\pi)(\pi - \alpha + \sin 2\alpha/2)]}$.

Feature	Semi-converter	Full converter	Dual converter
Devices (1-φ)	2 SCR + 2 diodes	4 SCR	two full converters anti-parallel
V_{dc} (1-φ, inductive)	$(V_m/\pi)(1 + \cos\alpha)$	$(2V_m/\pi)\cos\alpha$	$\pm(2V_m/\pi)\cos\alpha$
Quadrants	**One (I)**	**Two (I & IV)** – V reverses, I unidirectional	**Four**
Inversion possible	**No**	**Yes** (α > 90°)	Yes
Input pf	Better	Poorer (at same V_{dc})	–
Displacement angle	α/2	α	–

Three-phase controlled rectifiers

- In 3-φ circuits α is measured from the **natural commutation point = 30° after the zero crossing** of phase voltage (crossover of phase voltages).
- **3-φ half-wave (3-pulse)**: $V_{dc} = (3\sqrt{3}V_m/2\pi)\cos\alpha$ = **$0.827\ V_m \cos\alpha$** (continuous conduction; R load continuous up to α = 30°). Each SCR conducts **120°**. Drawback – **DC magnetisation** of transformer core.
- **3-φ full converter (6-pulse bridge)**: $V_{dc} = (3V_{mL}/\pi)\cos\alpha = (3\sqrt{6}\ V_{ph}/\pi)\cos\alpha$ = **$2.34\ V_{ph} \cos\alpha = 1.35\ V_L \cos\alpha$**.
- In the bridge, each SCR conducts **120°**; SCRs fired at **60° intervals** in sequence 1-2-3-4-5-6; two SCRs conduct at any time (one upper, one lower) → needs **double pulses or wide (> 60°) pulses** for starting.
- For R load, 3-φ full converter output is continuous up to **α = 60°**; output becomes zero at **α = 120°**. With highly inductive load, $V_{dc} = 0$ at α = 90°.
- **3-φ semi-converter**: $V_{dc} = (3V_{mL}/2\pi)(1 + \cos\alpha)$; ripple frequency **3f** for any α > 0 (6f only at α = 0).
- **Ripple frequency** of 6-pulse bridge = **6f = 300 Hz** on 50 Hz supply; of 12-pulse = **600 Hz**.

KEY FORMULAE – CONTROLLED RECTIFIERS

- 1-φ half-wave (R): $V_{dc} = (V_m/2\pi)(1 + \cos\alpha)$
- 1-φ full-wave (R) / semi-converter: $V_{dc} = (V_m/\pi)(1 + \cos\alpha)$
- 1-φ full converter (highly inductive): $V_{dc} = (2V_m/\pi)\cos\alpha = 0.9\ V_s \cos\alpha$
- 3-φ half-wave: $V_{dc} = (3\sqrt{3}\ V_m/2\pi)\cos\alpha = 1.17\ V_{ph}\cos\alpha$
- 3-φ full converter: $V_{dc} = (3\sqrt{3}\ V_m/\pi)\cos\alpha = (3\sqrt{2}/\pi)V_L \cos\alpha = 1.35\ V_L \cos\alpha$
- 3-φ semi-converter: $V_{dc} = (3V_{mL}/2\pi)(1 + \cos\alpha)$
- Dual converter: **$\alpha_1 + \alpha_2 = 180°$**

Dual converter

- Two full converters connected **back-to-back (anti-parallel)** → **four-quadrant operation** of DC motor (reversible drive).
- Condition: average voltages equal → **$\alpha_1 + \alpha_2 = 180°$** (one rectifying, other inverting).
- **Circulating-current mode**: both converters on; instantaneous voltages differ → circulating current limited by **intergroup (circulating current) reactor**; fast response, smooth reversal.
- **Non-circulating current mode**: only one converter conducts at a time; no reactor; small dead time (≈ 10–20 ms) during changeover; more common.

Effect of source inductance

- Source (leakage) inductance L_s prevents instantaneous current transfer → **overlap (commutation) angle μ**, during which incoming and outgoing SCRs conduct simultaneously.
- During overlap output voltage = average of the two phase voltages → **reduction in average output voltage** (acts as a non-dissipative "resistance" R = $3\omega L_s/\pi$ for 3-φ bridge).
- Also causes notches in supply voltage; reduces maximum inverting angle; increases effective firing delay (extinction angle).

KEY FORMULAE – SOURCE INDUCTANCE

- 1-φ full converter: $V_{dc} = (2V_m/\pi)\cos\alpha$ – **$(2\omega L_s I_d/\pi)$** = $(2V_m/\pi)\cos\alpha - 4fL_sI_d$; $\cos\alpha - \cos(\alpha + \mu) = 2\omega L_s I_d/V_m$
- 3-φ full converter: $V_{dc} = (3V_{mL}/\pi)\cos\alpha$ – **$(3\omega L_s I_d/\pi)$** = $(3V_{mL}/\pi)\cos\alpha - 6fL_sI_d$
- 3-φ bridge overlap: $V_{dc} = (3V_{mL}/2\pi)[\cos\alpha + \cos(\alpha + \mu)]$; $\cos\alpha - \cos(\alpha + \mu) = 2\omega L_s I_d/V_{mL}$

Firing circuits

Circuit	Range of α	Remarks
R firing	**0 to 90°** only	Simplest; gate voltage in phase with supply; temperature sensitive
RC firing (half/full wave)	**0 to 180°**	Capacitor provides phase shift
UJT relaxation oscillator	Wide; sharp pulses	Synchronised with supply via zener-clipped rectified voltage; pulse transformer isolates
Ramp / cosine-comparator, IC (TCA 785), digital (microcontroller)	0–180°	Cosine firing gives linear V_{dc} vs control voltage

- **UJT** (unijunction transistor): 3 terminals E, B1, B2; one p-n junction; exhibits **negative resistance** region; **intrinsic stand-off ratio $\eta = R_{B1}/(R_{B1} + R_{B2})$ ≈ 0.5–0.8**.
- Peak point voltage $V_P = \eta V_{BB} + V_D$ ($V_D \approx 0.7$ V).
- UJT oscillator period **$T = RC \ln[1/(1 - \eta)]$**; for η = 0.63, T ≈ RC.
- Pulse transformer / opto-coupler provides **electrical isolation** between control and power circuits.

MEMORY TRICK

"Half the waves, half the π." R-load forms carry (1 + cosα) and a divisor of 2π (half-wave) or π (full-wave); continuous inductive full converters carry **2 × peak × cosα / π**. Three-phase bridge: **"1.35 V_L"**; 3-φ half-wave: **"1.17 V_{ph}"**.

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- Average output of 1-φ full converter with highly inductive load at α = 60°, V_s = 230 V? → $(2\sqrt{2} \times 230/\pi) \times 0.5 \approx$ **103.5 V**.
- Range of α for inverting operation of a full converter? → **90° < α < 180°** (practically up to ~160°).
- Ripple frequency of a 3-φ fully controlled bridge on 50 Hz? → **300 Hz**.
- Condition for dual converter operation? → $\alpha_1 + \alpha_2 = 180°$.
- Maximum firing angle obtainable with simple R triggering? → **90°**.
- Effect of source inductance on a controlled rectifier? → **Reduces average output voltage (overlap)**.

12.3 Power Factor, Harmonics & AC–DC Voltage Source Converters ★★★

Input power factor of converters

- For a sinusoidal supply voltage and non-sinusoidal current, only the **fundamental** current produces real power: $P = V_s I_{s1} \cos\varphi_1$.
- **Input pf = $(I_{s1}/I_s) \times \cos\varphi_1$ = Distortion factor × Displacement factor**.
- **Distortion factor (CDF) g = $I_{s1}/I_{s,rms}$** (≤ 1). **Displacement factor (DPF) = $\cos\varphi_1$**, φ_1 = angle between supply voltage and fundamental current.
- For a full converter with continuous current, $\varphi_1 = \alpha$; for a semi-converter, $\varphi_1 = \alpha/2$ (hence better pf).
- **THD = $\sqrt{(I_s^2 - I_{s1}^2)}/I_{s1} = \sqrt{(1/g^2 - 1)}$**. Also $g = 1/\sqrt{(1 + THD^2)}$.
- Crest factor = I_{peak}/I_{rms} ($\sqrt{2}$ for sine).

Converter (highly inductive load, ripple-free I_d)	Input current waveform	Distortion factor	Input pf	THD
1-φ full converter	Square wave $\pm I_d$	**$2\sqrt{2}/\pi$ = 0.9**	**0.9 cosα**	**48.3%**
3-φ full converter	Quasi-square, 120° blocks	**$3/\pi$ = 0.955**	**0.955 cosα** (= $(3/\pi)\cos\alpha$)	**31.1%**
1-φ semi-converter	Blocks of width $(\pi - \alpha)$	$2\sqrt{2}\cos(\alpha/2)/\sqrt{(\pi(\pi - \alpha))}$	DF × $\cos(\alpha/2)$	–

- Input pf of phase-controlled converters **falls as α increases** (poor pf at low output voltage) → reason to use PWM/active front-end rectifiers.
- RMS of square-wave current = I_d; fundamental rms = $0.9 I_d$; nth harmonic rms = $0.9 I_d/n$ (odd n only).
- For 3-φ bridge, $I_{s,rms} = \sqrt{(2/3)}\, I_d = 0.816\, I_d$; $I_{s1} = (\sqrt{6}/\pi)\, I_d = 0.78\, I_d$.

KEY FORMULAE – POWER FACTOR & HARMONICS

- Input pf = g × $\cos\varphi_1$; $g = I_{s1}/I_s$
- THD = $\sqrt{(I_s^2 - I_{s1}^2)}/I_{s1}$
- Characteristic harmonics of p-pulse converter: h = **kp ± 1** (AC side); **kp** (DC side)
- Ideal harmonic magnitude: $I_h = I_1/h$

Harmonic orders

Converter	AC-side current harmonics	DC-side voltage harmonics
1-φ full-wave (2-pulse)	**3, 5, 7, 9 …** (all odd)	2, 4, 6 …
3-φ bridge (6-pulse)	**6k ± 1 → 5, 7, 11, 13, 17, 19 …**	6, 12, 18 …
12-pulse	**12k ± 1 → 11, 13, 23, 25 …**	12, 24 …
24-pulse	23, 25, 47, 49 …	24, 48 …

- Balanced 3-φ 3-wire systems carry **no triplen harmonics** (3, 9, 15 …) in line current. Triplen harmonics are **zero sequence** → add up in **neutral** of 4-wire systems; trapped in delta windings.
- Sequence: 5th → **negative sequence** (6k − 1), 7th → **positive sequence** (6k + 1), 3rd → zero sequence.
- **12-pulse** = two 6-pulse bridges fed from **Y–Y and Y–Δ transformers (30° phase shift)** → 5th & 7th cancel. Used in **HVDC** converters.
- Even harmonics are absent with half-wave symmetry.

Effects of line current harmonics

- Additional **copper and iron losses** → heating of transformers, motors, cables (transformers need **K-factor** rating / derating).
- **Neutral overloading** by triplen harmonics (neutral current can reach ≈ 1.73 × phase current).
- **Resonance** with pf correction capacitors; capacitor overloading and failure.
- Voltage distortion (flat-topped voltage), notching, malfunction of electronic equipment, nuisance tripping, metering errors, **communication interference (telephone interference factor, TIF)**.
- Pulsating torques in motors (5th harmonic produces reverse rotating field).
- Poor **true power factor** → higher kVA demand.

Standards & mitigation

- **IEEE 519** (Recommended Practice for Harmonic Control) – limits apply at the **Point of Common Coupling (PCC)**. Voltage THD limits: **8% (≤ 1 kV)**, **5% (1–69 kV)**, **2.5% (69–161 kV)**, **1.5% (> 161 kV)**; individual harmonic ≤ 5%, 3%, 1.5%, 1% respectively.
- Current limits expressed as **TDD (Total Demand Distortion)** relative to maximum demand load current I_L; limit depends on **I_{SC}/I_L** ratio – e.g. TDD **5%** for I_{SC}/I_L < 20, rising to 20% for > 1000 (stiffer system → larger limit).
- **IEC 61000-3-2** – harmonic limits for equipment ≤ 16 A per phase; **IEC 61000-3-12** for 16–75 A.
- **Passive filters**: tuned shunt LC filters (single-tuned at 5th, 7th, 11th …) + high-pass filter; also provide reactive power. Risk – resonance, fixed tuning.
- **Active power filter (APF)**: VSI injects equal and opposite harmonic current; adapts to changing loads. Hybrid filters combine both.
- Other remedies: **line reactors / DC chokes** (3–5% impedance), **multipulse (12/18/24-pulse)** converters, phase-shifting transformers, PWM (active front end) rectifiers.

CODE UPDATE

The current edition of IEEE 519 is **IEEE 519-2022** (replacing 519-2014); it retains the PCC-based voltage THD limits (8/5/2.5/1.5%) and TDD-based current limits, and adds limits/guidance for even harmonics and inter-harmonics.

Power-factor correction (PFC) and PWM rectifiers

- **Diode bridge + capacitor filter** draws narrow peaky current → pf ≈ **0.5–0.7**, THD > 100%.
- **PFC boost rectifier**: diode bridge followed by **boost converter** controlled so that input current follows rectified sine voltage → pf ≈ **0.99**, THD < 5%. Output DC voltage must exceed peak AC (e.g. ≈ **380–400 V** DC for 230 V AC). Modes: CCM (average current control), CrM/BCM, DCM.
- **Boost** is preferred for PFC because inductor is in series with input → continuous input current.
- **PWM rectifier / Active Front End (AFE)**: IGBT bridge (VSI operated as rectifier) with line inductors; DC-link voltage > peak line voltage (boost type).
- AFE features: **near-sinusoidal input current, unity (or controllable leading/lagging) pf, bidirectional power flow → regeneration to grid**, controllable DC voltage; used in regenerative VFDs, cranes, lifts, traction, grid-tied inverters, STATCOM.
- Control typically in **d-q (synchronous) reference frame** with PLL; d-axis current → active power/DC voltage, q-axis current → reactive power.
- **Vienna rectifier** – 3-level unidirectional boost-type PFC rectifier (telecom, EV chargers).

Feature	Phase-controlled (thyristor) rectifier	PWM (active front end) rectifier
Devices	SCR, line commutated	IGBT/MOSFET, self commutated
Input pf	Poor, ∝ cosα	**≈ Unity** (adjustable)
Input current	Rich in 5th, 7th …	Near sinusoidal; harmonics at switching frequency
Power flow reversal	Reverse DC voltage (α > 90°)	**Reverse DC current** (V_{dc} polarity fixed)
Reactive power	Always absorbs (lagging)	Can absorb or supply

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PREVIOUSLY ASKED / LIKELY QUESTIONS

- Input pf of 1-φ full converter with ripple-free current at firing angle α? → **0.9 cosα**.
- THD of a square-wave input current? → **48.3%**.
- Lowest-order harmonics in line current of 6-pulse converter? → **5th and 7th**.
- Characteristic harmonics of a 12-pulse converter? → **11, 13, 23, 25 (12k ± 1)**.
- Input pf = ? → **Distortion factor × Displacement factor**.
- Converter used for PFC in SMPS front end? → **Boost converter**.

12.4 DC–DC Converters (Choppers) ★★★

Principle and control

- **Chopper** = static DC-to-DC converter (DC equivalent of an AC transformer with continuously variable turns ratio). Switch: MOSFET/IGBT (earlier SCR with forced commutation).
- **Duty cycle D (α) = $T_{on}/T = T_{on}\cdot f$**, $0 \le D \le 1$; $T = T_{on} + T_{off}$.
- Basic step-down chopper (R load): $V_o = DV_s$; $V_{or} = \sqrt{D}\cdot V_s$; ripple factor = $\sqrt{((1 - D)/D)}$.
- **Time Ratio Control (TRC)**: (a) **constant frequency (PWM)** – T fixed, T_{on} varied → **preferred** (easy filter design, predictable harmonics); (b) **variable frequency (frequency modulation)** – T_{on} or T_{off} fixed, T varied → large frequency range makes filter design difficult, interference.
- **Current Limit Control (CLC)** / hysteresis control: switch toggled when load current reaches upper or lower limit (I_{max}, I_{min}) → variable frequency.

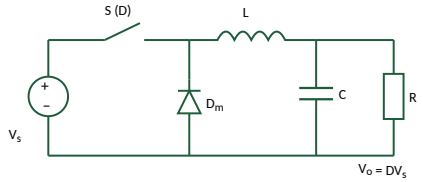

Fig 12.2 Buck (step-down) converter: switch S, freewheeling diode D_m, LC filter. $V_o = DV_s$ in CCM.

Basic non-isolated converters (ideal, CCM)

Converter	V_o/V_s	Output polarity	Input current	Output current
Buck (step-down)	**D**	Same	Pulsating	Continuous
Boost (step-up)	**1/(1 – D)**	Same	Continuous	Pulsating
Buck-boost	**–D/(1 – D)**	**Inverted**	Pulsating	Pulsating
Ćuk	**–D/(1 – D)**	**Inverted**	Continuous	Continuous
SEPIC	D/(1 – D)	Same	Continuous	Pulsating

- Buck-boost/Ćuk: **D < 0.5 → step-down**, **D > 0.5 → step-up**. D = 0.5 → $|V_o| = V_s$.
- **Ćuk converter** transfers energy through a **series capacitor** (not inductor); two inductors → low ripple at both input and output.
- Ideal (lossless): $P_{in} = P_{out}$ → $I_s/I_o = V_o/V_s$. Boost input current $I_s = I_o/(1 - D)$.
- Boost converter: theoretically $V_o \to \infty$ as $D \to 1$; practically limited (gain ≈ 4–5) by inductor/switch resistances. At no load, output rises dangerously.
- In all converters, average voltage across inductor = 0 and average capacitor current = 0 in steady state (**volt-second balance** & charge balance).
- Switch voltage stress: buck = V_s; boost = V_o; buck-boost = $V_s + |V_o|$.

KEY FORMULAE – CHOPPERS

- Buck: $V_o = DV_s$; $\Delta I_L = \mathbf{V_sD(1 - D)/(fL)} = V_o(1 - D)/(fL)$; maximum ripple at **D = 0.5**: $\Delta I_{max} = V_s/(4fL)$
- Buck output ripple: $\Delta V_o = \Delta I_L/(8fC)$; $\Delta V_o/V_o = (1 - D)/(8LCf^2)$
- Buck critical inductance (CCM/DCM boundary): $L_c = (1 - D)R/(2f)$
- Boost: $V_o = V_s/(1 - D)$; $\Delta I_L = V_sD/(fL)$; $\Delta V_o = I_oD/(fC)$; $L_c = D(1 - D)^2R/(2f)$
- Buck-boost: $V_o = -DV_s/(1 - D)$; $\Delta I_L = V_sD/(fL)$; $\Delta V_o = I_oD/(fC)$
- Chopper-fed load current ripple (RL load, constant f) is maximum at **D = 0.5**

CCM vs DCM

- **CCM** (continuous conduction mode): inductor current never falls to zero; V_o depends only on D (load-independent).
- **DCM**: inductor current falls to zero for part of the cycle (light load, small L, low f); V_o depends on D, L, f and load → **voltage gain higher** than CCM value.
- Boundary when $\Delta I_L = 2I_{L,avg}$. To ensure CCM at light load, $L > L_c$.
- Higher switching frequency → smaller L and C, but higher switching losses ($P_{sw} \propto f$).

Quadrant classification of choppers

Class	Quadrant(s)	V_o	I_o	Application
A	**I**	+	+	Motoring (step-down)
B	**II**	+	–	**Regenerative braking** (step-up action)
C	**I & II**	+	±	Motoring + regeneration (current reversible)
D	**I & IV**	±	+	Voltage reversible
E	**All four**	±	±	Reversible regenerative DC drive (H-bridge)

- Class B chopper: V_s (battery/line) receives energy from load EMF E; acts like boost converter (load side → supply).
- Class C = Class A + Class B; Class E = two Class C choppers (full H-bridge).
- SCR choppers used forced commutation: **voltage commutated (class D – impulse)**, current commutated (class B – resonant), load commutated; **Jones chopper** and **Morgan chopper** are classic SCR chopper circuits.
- Applications of choppers: **battery-operated vehicles, DC traction (metro, trolley buses), DC motor speed control with regenerative braking, SMPS, solar MPPT, LED drivers**.

Isolated converters and SMPS

- **Flyback converter**: derived from **buck-boost**; coupled inductor stores energy when switch ON, delivers when OFF. $V_o = V_s(N_2/N_1)\cdot D/(1 - D)$. Simplest, fewest components; for **low power (< ~150 W)**, multiple outputs (chargers, TV/PC auxiliary supplies).
- **Forward converter**: derived from **buck**; true transformer action (energy transferred during ON). $V_o = V_s(N_2/N_1)\cdot D$. Needs **core reset (demagnetising) winding**; with $N_3 = N_1$, $\mathbf{D_{max} = 0.5}$. Medium power (~100–500 W).
- **Push-pull, half-bridge, full-bridge** converters: core excited in both directions (better utilisation); full bridge for **high power (kW)**. Full bridge: $V_o = 2V_s(N_2/N_1)D$.
- Isolated converters provide **galvanic isolation**, multiple outputs, and high step-up/step-down via turns ratio. High-frequency transformer → **ferrite core**.

Feature	Linear power supply	SMPS
Operation	Series pass transistor in **active region**	Switch in **saturation/cut-off**
Efficiency	**30–60%**	**70–95%**
Transformer	Line frequency (50 Hz) – heavy, bulky	High frequency (20 kHz–MHz) ferrite – small
Size/weight	Large	Small, light (high power density)
Output ripple & noise	**Very low**	Higher; EMI/RFI problems
Transient response	Fast	Slower
Output	Step-down only	Step-up/down/invert, multiple outputs

MEMORY TRICK

"Buck = D, Boost = 1/(1–D), Buck-Boost = their product (D × 1/(1–D)) with a minus." Flyback is the isolated buck-boost; Forward is the isolated buck.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Boost converter with V_s = 12 V and D = 0.6, V_o = ? → 12/0.4 = **30 V**.
- Buck-boost with D = 0.25 and V_s = 24 V? → –24 × 0.25/0.75 = **–8 V**.
- Duty cycle at which inductor ripple of buck converter is maximum? → **D = 0.5**.
- Chopper operating in first and second quadrants? → **Class C**.
- Preferred control strategy for choppers? → **Constant frequency PWM (TRC)**.
- Isolated converter derived from buck-boost? → **Flyback**.

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12.5 Inverters ★★★

VSI vs CSI

Feature	Voltage Source Inverter (VSI)	Current Source Inverter (CSI)
DC link	**Large capacitor** (stiff voltage)	**Large series inductor** (stiff current)
Output	Voltage waveform fixed (square/PWM); current depends on load	Current waveform fixed (quasi-square); voltage depends on load
Switches	Need **anti-parallel feedback diodes** (reactive energy)	Need **reverse-voltage blocking** (series diodes), no feedback diodes
Short circuit	Dangerous (shoot-through) – needs fast protection, **dead time**	**Inherently short-circuit proof**; open circuit is dangerous
Regeneration	Needs extra converter (AFE)	**Easy** (reverse V_{dc} of front-end rectifier)
Typical use	VFDs, UPS, solar inverters (most common)	Large synchronous/induction motor drives (load-commutated inverter, LCI), induction heating

- Feedback diodes return reactive (inductive load) energy to the DC source; they conduct when load current and voltage have opposite polarity.
- **Dead time (blanking time)** between switching off one switch and on the complementary switch in the same leg prevents **shoot-through**.
- **Series resonant inverter** (class A load commutation) – sine-like output, used in **induction heating**, fluorescent lighting.

Single-phase square-wave inverters

- **Half-bridge**: 2 switches + split DC capacitors; output $\pm V_s/2$; $V_{o,rms}$ = **$V_s/2$**; switch blocking voltage V_s.
- **Full-bridge**: 4 switches; output $\pm V_s$; $V_{o,rms}$ = **V_s**; for same V_s, output voltage and power are **double/four times** those of half-bridge.
- Fourier series of square wave (full bridge): $v_o = \Sigma_{n=1,3,5...} (4V_s/n\pi) \sin n\omega t$ → only **odd harmonics**, amplitude ∝ 1/n.
- Fundamental peak = $4V_s/\pi = 1.273V_s$; **fundamental rms = $2\sqrt{2}V_s/\pi = 0.9V_s$** (half-bridge: **$0.45V_s$**).
- THD of square wave = **48.3%**; lowest harmonic = **3rd** (33.3% of fundamental).
- **Quasi-square / single-pulse modulation** (full bridge with phase-shift control, pulse width 2d): $V_n = (4V_s/n\pi) \sin nd$; $V_{o,rms} = V_s\sqrt{(2d/\pi)}$.
- **Harmonic elimination**: nth harmonic is zero when $\sin nd = 0$ → **$2d = 2\pi/n$**. Pulse width **$2d = 120° (2\pi/3)$ eliminates the 3rd harmonic** (and all triplens); 2d = 144° eliminates 5th.

Three-phase bridge inverter (6 switches)

Quantity (Y-connected R load, DC input V_s)	180° conduction	120° conduction
Switches ON at any instant	**3**	**2**
Phase voltage waveform	Six-step ($V_s/3$, $2V_s/3$)	Quasi-square ($\pm V_s/2$, 120° blocks)
Line voltage waveform	Quasi-square ($\pm V_s$, 120° blocks)	Six-step
RMS phase voltage	**$\sqrt{2}V_s/3 = 0.471\ V_s$**	$V_s/\sqrt{6} = 0.408\ V_s$
RMS line voltage	**$\sqrt{(2/3)}V_s = 0.816\ V_s$**	$V_s/\sqrt{2} = 0.707\ V_s$
Fundamental line voltage (rms)	$(\sqrt{6}/\pi)V_s = 0.78\ V_s$	–
Fundamental phase voltage (rms)	$(\sqrt{2}/\pi)V_s = 0.45\ V_s$	–
Shoot-through risk	Higher (needs dead time)	**Lower** (60° gap between devices of a leg)
Utilisation	**Better** (more output)	Poorer

- In 180° mode each device conducts **180°**, switching every **60°**; six-step output contains harmonics **6k ± 1 (5, 7, 11, 13 ...)**, no triplen harmonics in line voltage.
- Phase voltage in 180° mode = 1/3 of DC voltage steps ($V_s/3$, $2V_s/3$).

KEY FORMULAE – INVERTERS

- Half-bridge: $V_o = V_s/2$; $V_{1,rms} = 0.45\ V_s$; full-bridge: $V_o = V_s$; $V_{1,rms} = 0.9\ V_s$
- Square wave nth harmonic: $V_n = V_1/n$ (n odd)
- Single-pulse: $V_{n,peak} = (4V_s/n\pi) \sin(nd)$; harmonic n eliminated when $2d = 2\pi/n$
- 3-φ 180°: $V_L = 0.816\ V_s$; $V_{ph} = 0.471\ V_s$; 120°: $V_L = 0.707\ V_s$, $V_{ph} = 0.408\ V_s$
- SPWM: $m_a = V_r/V_c$; $m_f = f_c/f_r$; $V_{1,peak}$ (half-bridge/phase) = $m_aV_s/2$ for $m_a \le 1$
- 3-φ SPWM line voltage (rms, fundamental) = $0.612\ m_aV_s$; SVPWM max line peak = V_s (vs $0.866\ V_s$ for SPWM)

PWM techniques

- PWM controls output voltage **within the inverter** (no separate DC voltage control) and reduces **low-order harmonics** (pushed to high frequency → easy filtering).
- **Single-pulse PWM**: one pulse per half cycle; width controls voltage; high low-order harmonics.
- **Multiple-pulse (uniform/symmetric) PWM**: several equal-width pulses per half cycle (triangular carrier vs square reference); lower harmonics reduced, switching losses increase.
- **Sinusoidal PWM (SPWM)**: sinusoidal reference (f_r, V_r) compared with triangular carrier (f_c, V_c); pulse width varies sinusoidally → **most widely used**.
- **Amplitude modulation index $m_a = V_r/V_c$** (peak reference / peak carrier). **Linear region: $m_a \le 1$** → fundamental ∝ m_a.
- **Over-modulation ($m_a > 1$)**: fundamental no longer linear, **low-order harmonics reappear**; at large m_a becomes square wave (max fundamental $4/\pi \times V_s/2$).
- **Frequency modulation ratio $m_f = f_c/f_r$**. Harmonics appear as sidebands around **m_f, $2m_f$, $3m_f$ ...** (e.g. m_f, $m_f \pm 2$; $2m_f \pm 1$).
- m_f should be an **odd integer** (synchronous PWM; avoids even harmonics/subharmonics); for 3-φ inverters an **odd multiple of 3** (triplen-centred harmonics cancel in line voltage). For $m_f > 21$, asynchronous PWM acceptable.
- **Unipolar SPWM** (full bridge, output 0, $\pm V_s$) – harmonics shifted to **$2m_f$**, effectively doubles switching frequency; **bipolar SPWM** ($\pm V_s$) – harmonics at m_f.
- **Third-harmonic injection PWM** – adds 3rd harmonic to reference; increases fundamental by **15.5%** (m_a up to 1.155).
- **Space Vector PWM (SVPWM)**: 8 switching states → **6 active vectors + 2 zero vectors** forming a hexagon; reference vector synthesised by adjacent active vectors and zero vectors. Gives **15.5% higher DC bus utilisation** than SPWM ($2/\sqrt{3} = 1.155$), lower THD, fewer switchings; easily implemented digitally.
- **Selective harmonic elimination (SHE)**: notch angles pre-computed to eliminate chosen harmonics (e.g., 5th and 7th).
- **Hysteresis (current-controlled) PWM** – fast response, variable switching frequency.

Other inverter topics

- **McMurray inverter**: SCR inverter with **auxiliary-commutated (impulse) commutation** – each main SCR has an auxiliary SCR that fires an LC resonant circuit to turn it off. **McMurray–Bedford** inverter uses **complementary commutation** (firing one main SCR turns off the other).
- **Multilevel inverters**: output staircase of several voltage levels → lower THD, lower dv/dt, lower device voltage stress; used in **medium-voltage drives, STATCOM, HVDC (MMC)**.
  - **Diode-clamped (NPC)**: m-level → (m − 1) DC capacitors, 2(m − 1) switches/leg, (m − 1)(m − 2) clamping diodes per leg; 3-level NPC most common.
  - **Flying-capacitor** (capacitor clamped).
  - **Cascaded H-bridge** with separate DC sources: s H-bridges per phase → **2s + 1 levels** (phase voltage).
  - **Modular Multilevel Converter (MMC)** – sub-modules in series; dominant in VSC-HVDC.

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UPS

Type	Normal operation	Transfer time	Remarks
Offline (standby)	Load fed directly from mains; inverter OFF, charger keeps battery charged	**2–10 ms** (switch-over)	Cheap, for PCs; no conditioning
Line-interactive	Mains via AVR/tap changing transformer; inverter connected in parallel	Small (~2–4 ms)	Voltage regulation
Online (double conversion)	AC → rectifier → DC (battery float) → inverter → load, **always**	**Zero** (no break)	**Best protection**; critical loads (data centres, hospitals); lower efficiency (~90–95%), static bypass switch

- Batteries for UPS: VRLA (sealed lead-acid), Li-ion. Static transfer switch = anti-parallel SCRs.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- RMS fundamental output of a single-phase full-bridge square-wave inverter fed from V_s? → **0.9 V_s**.
- RMS line voltage of 3-φ 180° mode inverter? → **0.816 V_s**; phase → **0.471 V_s**.
- Pulse width to eliminate 3rd harmonic in single-pulse modulation? → **120° (2π/3)**.
- Linear range of SPWM? → **$m_a \le 1$**; beyond it is **over-modulation**.
- DC-bus utilisation advantage of SVPWM over SPWM? → **15.5%**.
- Inverter that is inherently short-circuit proof? → **CSI**.

12.6 AC Voltage Controllers, Cycloconverters & Drives ★★☆

AC voltage controllers (AC regulators)

- Convert fixed AC to **variable-voltage AC at the same frequency**; use **anti-parallel SCRs or a TRIAC**; natural (line) commutation.
- **Phase (angle) control**: SCR fired at α in each half cycle → fine voltage control but introduces **harmonics and poor pf** (lagging even with R load).
- 1-φ full-wave AC controller with R load: **$V_{o,rms} = V_s\sqrt{[(1/\pi)(\pi - \alpha + \sin 2\alpha/2)]}$**; α range 0–180°.
- With RL load (load angle φ): control range **$\varphi \le \alpha \le \pi$**; for $\alpha < \varphi$ the output is full sine wave (no control); needs **continuous/long gate pulse** when $\alpha < \varphi$.
- Unidirectional (half-wave) controller (one SCR + one diode) – introduces **DC component** → not used.
- **Integral cycle (on-off / burst firing) control**: switch ON for n cycles, OFF for m cycles at zero crossing → **$V_{o,rms} = V_s\sqrt{k}$**, $k = n/(n + m)$; input pf = √k; no low-order harmonics/RFI, but **sub-harmonics** → suitable for loads with **large thermal time constant** (heating, ovens) – **not for lighting** (flicker) or motors.
- **Applications**: **fan regulators**, **light dimmers**, **industrial heating**, induction motor **soft starters**, **static VAR compensator (TCR)**, transformer tap changers.
- 3-φ AC controllers: fully controlled (6 SCRs) or half-controlled (3 SCRs + 3 diodes); used in **soft starters** for induction motors.

Feature	Phase control	Integral cycle control
Switching	At α within each half cycle	At voltage zero crossing
Harmonics	Low-order (3, 5, 7...) harmonics	Sub-harmonics, no low-order
RFI/EMI	High	Low
Applications	Lamp dimmers, fan regulators	Heaters, ovens, furnaces

Cycloconverters & matrix converters

- **Cycloconverter** = direct **AC-to-AC frequency changer** without DC link; uses **naturally commutated** (line commutated) SCRs → output frequency normally **lower** than input (**step-down cycloconverter**).
- Practical output frequency ≤ **1/3 to 1/2 of supply frequency** (harmonics grow at higher f_o).
- 1-φ to 1-φ: two full converters back-to-back (8 SCRs for bridge type, 4 SCRs for midpoint type). 3-φ to 3-φ (6-pulse bridge type): **36 SCRs**; 3-pulse type: **18 SCRs**.
- **Step-up cycloconverter** needs forced commutation (rare).
- **Applications**: **large low-speed, high-power drives** – cement and ball mills, rolling mills, mine hoists, ship propulsion, gearless drives; also VSCF (variable-speed constant-frequency) generation.
- Drawbacks: large number of devices, complex control, poor input pf, harmonics.
- **Matrix converter**: array of **9 bidirectional switches** (3×3) connecting each input phase to each output phase; direct AC–AC with **any output frequency** (up or down), sinusoidal input current, unity input pf, regeneration, no bulky DC-link capacitor. Limitation: maximum voltage transfer ratio **0.866 (√3/2)**, complex commutation.

MEMORY TRICK

"Cyclo = Cement, Crushers, Cranes (mine hoists) – slow & heavy." Same frequency, variable voltage → AC voltage controller; variable frequency directly → cycloconverter/matrix converter; via DC link → rectifier + inverter (VFD).

Converter summary

Conversion	Converter
AC → DC (fixed)	Diode rectifier
AC → DC (variable)	**Phase-controlled rectifier** / PWM rectifier
DC → DC	**Chopper**
DC → AC	**Inverter**
AC → AC (same f, variable V)	**AC voltage controller**
AC → AC (variable f)	**Cycloconverter**, matrix converter

DC motor drives

- Speed of separately excited DC motor: $N \propto (V - I_aR_a)/\varphi$. **Armature voltage control** → below base speed (**constant torque**); **field weakening** → above base speed (**constant power**).
- **Converter-fed (phase-controlled) DC drives** – from AC supply:
  - 1-φ half-wave / semi-converter drive → **one quadrant** (up to ~15 kW).
  - 1-φ / 3-φ full converter drive → **two quadrant (I & IV)**; regenerative braking possible only with reversed EMF (e.g. overhauling hoist load) or field/armature reversal.
  - **Dual converter drive → four quadrant** (reversible regenerative drive, rolling mills).
  - Armature: $V_a = (2V_m/\pi)\cos\alpha$ (1-φ full) ; $E_b = V_a - I_aR_a = K_m\omega$.
- **Chopper-fed DC drives** – from DC supply (battery vehicles, DC traction): Class A → motoring; **Class B → regenerative braking**; Class C → two quadrant; **Class E → four quadrant**. Advantages: high efficiency, fast response, smooth control, regenerative braking down to low speed.
- Chopper drive: $V_a = DV_s$ (motoring); in regeneration $E_b = (1 - D)V_s$.
- **Dynamic (rheostatic) braking** – energy dissipated in resistor; **regenerative braking** – energy returned to supply; **plugging** – supply reversed (highest braking torque, most losses).

Induction motor drives

Method	Principle	Remarks
Stator voltage control	AC voltage controller (soft starter)	Narrow speed range, poor efficiency; fans/pumps
V/f control (VFD)	Rectifier + DC link + PWM inverter; **V/f kept constant** → constant air-gap flux → constant max torque	**Most common**; below base speed constant torque; above base speed (V fixed) field weakening → constant power
Rotor resistance control	Chopper-controlled resistance in slip-ring rotor	Inefficient (slip power wasted)
Slip power recovery	**Static Kramer** (slip power fed back to mains – subsynchronous); **Static Scherbius** (bidirectional – sub & super-synchronous)	Slip-ring motors; large pumps/fans; DFIG wind generators
Vector (field-oriented) control	Decouples flux (i_d) & torque (i_q) like DC motor	High dynamic performance

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Direct Torque Control (DTC)	Hysteresis control of flux and torque	Fast torque response

- At low frequency, V/f control needs **voltage boost (IR compensation)** to overcome stator resistance drop.
- VFD benefits: **energy saving in fans/pumps (power ∝ N^3, affinity law)**, soft start (low starting current), speed control, improved pf at supply. Drawbacks: harmonics, bearing currents, **high dv/dt** stress on motor insulation with long cables (use dv/dt or sine filters).
- Synchronous motor drives: LCI (load-commutated inverter, CSI) for large MW drives; PMSM/BLDC drives for EVs, fans.

Utility & other applications

- **HVDC**: **LCC (line-commutated converters)** use **thyristor valves** (12-pulse), light-triggered/electrically triggered; converter absorbs reactive power ≈ **50–60% of DC power** → needs filters and capacitor banks. Rectifier controlled by α, inverter by **extinction angle γ** (≥ ~15–18°). **VSC-HVDC** (IGBT, MMC) – independent P & Q control, black start, no commutation failure, suitable for offshore wind and weak grids.
- **FACTS**: SVC (**TCR + TSC**, thyristor based), **STATCOM** (VSI based, shunt), TCSC (series), SSSC, **UPFC** (shunt + series VSCs – controls V, impedance and angle). STATCOM delivers rated current even at low voltage (better than SVC whose output ∝ V^2).
- **Solar PV integration**: **MPPT** via DC-DC converter (boost) using **Perturb & Observe (P&O)** or **Incremental Conductance** algorithm; grid-tie inverter synchronised by PLL, anti-islanding protection. Inverter types: central, string, micro-inverters. Typical inverter efficiency **97–99%**.
- **Wind**: DFIG with back-to-back converter rated ≈ **25–30%** of machine rating; full-converter PMSG turbines.
- **Electric traction**: modern locomotives (e.g. Indian Railways WAP-7/WAG-9 class) use **25 kV AC → 4-quadrant (PWM) line converter → DC link → VSI (IGBT/GTO) → 3-φ induction motors** with regenerative braking; metros use 750 V DC third rail or 25 kV AC OHE; earlier DC series motors with tap-changer/ thyristor converter control.
- Others: **induction heating** (resonant inverter, high frequency), electronic ballasts, EV chargers, battery energy storage (bidirectional converters), SMPS.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Integral cycle control is suitable for? → **Heating loads with large thermal time constant**.
- Cycloconverter converts? → **AC at one frequency to AC at lower frequency directly** (no DC link).
- Number of SCRs in a 3-φ to 3-φ bridge-type cycloconverter? → **36**.
- In VFD of induction motor, V/f is kept constant to maintain? → **Constant air-gap flux**.
- Chopper used for regenerative braking of DC motor? → **Class B** (second quadrant).
- Algorithm commonly used for MPPT in solar PV? → **Perturb & Observe** (or Incremental Conductance).

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13. Last-Day Revision (Formula & Value Sheet)

Topic	Must-remember facts
Network theorems	Max power (DC) $R_L = R_{th}$, $P_{max} = V_{th}^2/4R_{th}$, η = 50%; AC: $Z_L = Z_{th}^*$; pure-R load → $R_L = \|Z_{th}\|$; superposition not valid for power; star from delta $R_a = R_{ab}R_{ca}/\Sigma R$ (equal: R/3)
Transients	τ = L/R, τ = RC; 63.2% in one τ; ~99% in 5τ; inductor current and capacitor voltage cannot change instantly; series RLC ζ = (R/2)√(C/L)
AC & resonance	Sine: form factor 1.11, peak factor 1.414; square 1, 1; triangle 1.155, 1.732; $f_0 = 1/2\pi\sqrt{LC}$; series Q = (1/R)√(L/C), BW = f_0/Q; parallel tank Z_{dyn} = L/CR; S = VI*
Two-port & 3-phase	Reciprocal: $Z_{12} = Z_{21}$, AD − BC = 1, $h_{12} = -h_{21}$; symmetric: $Z_{11} = Z_{22}$, A = D; star $V_L = \sqrt{3}V_{ph}$; $P = \sqrt{3}V_L I_L \cos\varphi$; two-wattmeter $\tan\varphi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)$; one reading negative when pf < 0.5
EM fields	$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m; $\mu_0 = 4\pi \times 10^{-7}$ H/m; line charge $E = \rho_L/2\pi\varepsilon_0 r$; sheet $\rho_s/2\varepsilon_0$; C coax = 2πεL/ln(b/a); H wire = I/2πr; loop centre I/2a; solenoid nI; η_0 = 377 Ω; skin depth $\delta = 1/\sqrt{(\pi f \mu \sigma)}$; ∇·B = 0
Signals	Nyquist rate = $2f_m$; product of signals → bandwidths add; BIBO stable ⇔ $\int\|h(t)\|dt < \infty$; causal + stable DT ⇔ poles inside unit circle; half-wave symmetry → odd harmonics only; Gibbs overshoot ≈ 9%; δ(at) = δ(t)/|a|
Transformer	$E = 4.44 f N \Phi_m$; max η when Cu loss = core loss, load fraction $x = \sqrt{(P_i/P_{cu,fl})}$; regulation ≈ $\varepsilon_r\cos\varphi \pm \varepsilon_x\sin\varphi$ (max when φ = θ); OC test on LV side → core loss; SC test on HV side → Cu loss; V-V capacity 57.7% of Δ-Δ; parallel operation needs same phase shift (vector group); Dy1 and Dy11 only after reconnection
DC machines	$E = P\Phi ZN/60A$ (lap A = P, wave A = 2); $E_b = V - I_aR_a$; $N \propto E_b/\Phi$; series motor never started on no load; max power when $E_b = V/2$; field control → above base speed; armature control → below base speed; Swinburne's test for shunt, Hopkinson's back-to-back
Induction motor	$N_s = 120f/P$; rotor frequency sf; $P_g : P_{cu2} : P_m = 1 : s : (1 - s)$; $s_m = R_2/X_2$; T_{max} independent of R_2; $T \propto V^2$; star–delta starting → I_{st} and T_{st} = 1/3 of DOL; autotransformer → x^2; V/f constant for speed control
Synchronous machines	$E = 4.44 f \Phi T K_p K_d$; $K_p = \cos(\alpha/2)$; $P = EV\sin\delta/X_s$, max at δ = 90°; salient: $X_d > X_q$; EMF method pessimistic, MMF optimistic; over-excited motor → leading pf (synchronous condenser); V-curves; motor not self-starting (damper winding)
Generation & lines	Hydro $P = 9.81\eta QH$ kW; wind $P = \frac{1}{2}\rho AV^3$, Betz 59.3%; short line < 80 km, medium 80–250 km, long > 250 km; Ferranti effect at light load; SIL = V^2/Z_c (Z_c ≈ 400 Ω line, 40–50 Ω cable); sag = $wl^2/8T$; GMR r′ = 0.7788r; copper $\propto 1/V^2$
Insulators & cables	String efficiency = $V/(n \times V_{\text{unit nearest line}})$; unit nearest the line most stressed; guard ring equalises; cable max stress at conductor surface; economic core r = R/e; insulation resistance ∝ ln(R/r)/l
Load flow & per unit	Slack bus: |V|, δ given; PV: P, |V|; PQ: P, Q; GS – linear convergence, acceleration ≈ 1.6; NR – quadratic, 3–5 iterations; $Z_{pu,new} = Z_{pu,old}(MVA_{new}/MVA_{old})(kV_{old}/kV_{new})^2$; Y_{bus} sparse and symmetric
Faults	a = 1∠120°, 1 + a + a² = 0; LG: $I_f = 3E/(Z_1 + Z_2 + Z_0 + 3Z_f)$; LL: $\sqrt{3}E/(Z_1 + Z_2)$, no zero sequence; LG most common (~70–80%), 3-phase usually most severe; X″ < X′ < X_d; Petersen coil L = $1/(3\omega^2 C)$
Protection & CBs	Standard IDMT t = 0.14 TMS/$(PSM^{0.02} - 1)$; mho relay inherently directional (long lines); reactance relay for short lines (arc resistance); transformer differential with 2nd-harmonic restraint for inrush; Buchholz for incipient faults; SF_6 electronegative; vacuum CB for distribution; making current = 2.55 × breaking current; RCCB 30 mA
Stability	Swing equation $M\,d^2\delta/dt^2 = P_m - P_e$; H in MJ/MVA; equal-area criterion for transient stability; steady-state limit $P_{max} = EV/X$; improved by fast CBs, auto-reclosing, series capacitors, higher voltage
Control – time response	$\omega_d = \omega_n\sqrt{(1 - \zeta^2)}$; $t_p = \pi/\omega_d$; $M_p = e^{(-\pi\zeta/\sqrt{(1-\zeta^2)})}$; $t_s = 4/\zeta\omega_n$ (2%); step $e_{ss} = 1/(1 + K_p)$, ramp $1/K_v$, parabolic $1/K_a$; type-1 → zero step error
Control – stability	Routh: sign changes = RHP poles; root locus starts at OL poles, ends at zeros; asymptote angle (2q + 1)180°/(P − Z); centroid (Σp − Σz)/(P − Z); stable if GM > 0 and PM > 0; Nyquist Z = P − N (N = anticlockwise encirclements of −1); $M_r = 1/(2\zeta\sqrt{(1 - \zeta^2)})$
Compensators & state space	Lead → better transient, more BW; lag → better steady state; I eliminates e_{ss}; D adds damping; $\sin\varphi_m = (1 - \alpha)/(1 + \alpha)$; $\varphi(t) = e^{At} = L^{-1}[(sI - A)^{-1}]$; $G(s) = C(sI - A)^{-1}B + D$; controllable if rank[B AB …] = n
Measurements	PMMC – DC, average, linear scale; MI – AC/DC, rms, square-law; shunt $R_{sh} = R_m/(m - 1)$; multiplier $R_s = R_m(m - 1)$; Kelvin double bridge for low R; Hay's for high Q; Maxwell for medium Q; Schering for capacitance/tan δ; Wien for frequency; never open-circuit a CT secondary; 3½-digit DVM = 1999 counts; Lissajous f_y/f_x = horizontal/vertical tangencies
Diodes & rectifiers	V_T = 26 mV at 300 K; HWR: $V_{dc} = V_m/\pi$, ripple 1.21, η 40.6%, PIV V_m; FWR: $2V_m/\pi$, ripple 0.482, η 81.2%, PIV centre-tap $2V_m$, bridge V_m; Zener < 5 V negative temperature coefficient
BJT, FET & amplifiers	β = α/(1 − α); CE 180° phase shift; CC = emitter follower (buffer); voltage-divider bias most stable; JFET $I_D = I_{DSS}(1 - V_{GS}/V_P)^2$; class A 25% (50% transformer-coupled), class B 78.5%; Miller $C_{in} = C(1 + \|A_v\|)$; negative feedback: BW × (1 + Aβ)
Oscillators & op-amps	Barkhausen |Aβ| = 1, phase 0°/360°; RC phase shift f = 1/(2πRC√6), gain ≥ 29; Wien f = 1/2πRC, gain ≥ 3; inverting $-R_f/R_1$, non-inverting $1 + R_f/R_1$; $f_{max} = SR/2\pi V_{in}$; 741 slew rate 0.5 V/µs; 555 astable f = 1.44/((R_A + 2R_B)C); monostable T = 1.1RC
Digital	NAND/NOR universal; n-bit 2's complement range -2^{n-1} to $2^{n-1} - 1$; Gray code unit distance; excess-3 self-complementing; JK race-around → master–slave; ring counter mod-N, Johnson mod-2N; flash ADC $2^n - 1$ comparators (fastest), dual-slope most accurate; SAR n clocks; ECL fastest family
8085	8-bit, 40 pins, 16 address lines (64 KB), ALE demultiplexes AD_0–AD_7; 5 flags (S, Z, AC, P, CY); TRAP non-maskable 0024H; RST 7.5 → 003CH, 6.5 → 0034H, 5.5 → 002CH; opcode fetch 4 T-states; INR/DCR do not affect CY; INX/DCX affect no flags; RST n → 8n; 256 I/O ports
Power devices	SCR latching current > holding current; snubber RC for dv/dt, series L for di/dt; TRIAC bidirectional; GTO turn-off by negative gate; MOSFET fastest, positive temperature coefficient; IGBT = MOSFET input + BJT output
Converters	1-φ full converter (inductive) $V_{dc} = (2V_m/\pi)\cos\alpha$, inverts for α > 90°; 3-φ bridge $1.35V_L\cos\alpha$; 6-pulse harmonics 6k ± 1; buck $V_o = DV_s$; boost $V_s/(1 - D)$; buck-boost $-DV_s/(1 - D)$; input pf = distortion factor × displacement factor; SPWM linear for $m_a \le 1$; full-bridge square-wave fundamental rms = $0.9V_s$

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14. 500 Most Expected Questions & Answers

NOTE

Rapid-fire revision: 500 one-line questions chosen for the CWC AM (Electrical) paper, in the same order as the chapters. Cover the answer, attempt, then check. Numericals carry a short working hint.

14.1 Electric Circuits (Q 1–50)

1. SI unit of conductance? → **Siemens (S), also called mho**
2. 1 kWh equals how many joules? → **3.6×10^6 J (3.6 MJ)**
3. Energy stored in a 10 µF capacitor charged to 100 V? → **0.05 J ($\frac{1}{2}CV^2 = \frac{1}{2} \times 10^{-5} \times 10^4$)**
4. Internal resistance of an ideal voltage source? → **Zero**
5. 12 V source in series with 4 Ω; equivalent current source? → **3 A in parallel with 4 Ω (I = V/R)**
6. Three equal 6 Ω resistors in star; equivalent delta arm? → **18 Ω ($R_\Delta = 3R_Y$)**
7. Number of trees of a complete graph with 5 nodes? → **125 ($n^{n-2} = 5^3$)**
8. A graph has 7 nodes and 12 branches. Number of twigs? → **6 (n − 1)**
9. Rank of the complete incidence matrix of an n-node connected graph? → **n − 1**
10. Effect of interchanging any two supply lines of a 3-phase motor? → **Phase sequence and rotation direction reverse**
11. Nodal analysis is based on which law? → **KCL (node voltages as unknowns)**
12. 4 Ω and 6 Ω in series across 20 V. Voltage across 6 Ω? → **12 V (20 × 6/10)**
13. When is a supermesh formed? → **When a current source is common to two meshes**
14. Dual of a series RLC circuit fed by a voltage source? → **Parallel GCL circuit fed by a current source**
15. V_{th} = 10 V, R_{th} = 2 Ω, R_L = 3 Ω. Load current? → **2 A (10/(2 + 3))**
16. V_{th} = 24 V, R_{th} = 6 Ω. Norton current? → **4 A ($I_N = V_{th}/R_{th}$)**
17. In superposition, an ideal current source is replaced by? → **Open circuit**
18. Theorem used to find neutral shift in an unbalanced star load? → **Millman's theorem**
19. Theorem not applicable to networks with dependent sources? → **Reciprocity theorem**
20. V_{th} = 12 V, R_{th} = 3 Ω. Maximum power to load? → **12 W ($V^2/4R = 144/12$)**
21. Fixed load, variable source resistance R_s: R_s for maximum load power? → **$R_s = 0$**
22. Time constant of an RC circuit with R = 1 kΩ, C = 100 µF? → **0.1 s (τ = RC)**
23. Capacitor discharging: voltage remaining after one time constant? → **36.8% of initial value**
24. A first-order transient is practically over after how many time constants? → **5τ (99.3%)**
25. An uncharged inductor at $t = 0^+$ behaves as? → **Open circuit**
26. L_1 = 2 H, L_2 = 8 H, M = 2 H. Coefficient of coupling? → **0.5 ($M/\sqrt{(L_1L_2)} = 2/4$)**
27. Series RLC with R = 4 Ω, L = 1 H, C = 1 F. Nature of response? → **Overdamped ($\zeta = (R/2)\sqrt{(C/L)} = 2$)**
28. 10%–90% rise time of a first-order circuit? → **2.2τ**
29. RMS value of a symmetrical square wave of peak V_m? → **V_m**
30. RMS value of $v = 3 + 4\sqrt{2}\sin\omega t$? → **5 V ($\sqrt{(3^2 + 4^2)}$)**
31. Peak (crest) factor of a triangular wave? → **$\sqrt{3} = 1.732$**
32. Average value of a half-wave rectified sine of peak 314 V? → **≈ 100 V (V_m/π)**
33. Reactance of a 0.1 H inductor at 50 Hz? → **31.4 Ω ($2\pi fL$)**
34. A 20 kVA load at 0.6 pf. Active power? → **12 kW (S cos φ)**
35. Expression for complex power in terms of phasors? → **$S = V I^* = P + jQ$**
36. An over-excited synchronous motor on no load used for pf correction is called? → **Synchronous condenser**
37. Resonant frequency of series RLC with L = 10 mH, C = 1 µF? → **≈ 1.59 kHz ($1/(2\pi\sqrt{(LC)})$)**
38. Relation of f_0 to half-power frequencies f_1, f_2? → **$f_0 = \sqrt{(f_1 f_2)}$ (geometric mean)**
39. Bandwidth of series RLC with R = 10 Ω, L = 0.1 H? → **≈ 15.9 Hz ($R/(2\pi L)$)**
40. Series resonance: Q = 20, supply 10 V. Voltage across C? → **200 V ($V_C = QV$)**
41. Below its resonant frequency a parallel tank circuit behaves as? → **Inductive**
42. Condition under which a practical tank circuit cannot resonate? → **$R > \sqrt{(L/C)}$ (coil resistance)**
43. Cut-off frequency of RC low-pass filter, R = 1 kΩ, C = 1 µF? → **≈ 159 Hz ($1/(2\pi RC)$)**
44. 1 neper equals how many decibels? → **8.686 dB**
45. Z-parameters are also called? → **Open-circuit impedance parameters**
46. Condition for symmetry in Z-parameters? → **$Z_{11} = Z_{22}$**
47. For series–series connected two-ports, which parameters add? → **Z-parameters**
48. Balanced 3-phase load: V_L = 400 V, I_L = 10 A, pf 0.8. Power? → **≈ 5.54 kW ($\sqrt{3}\, V_L I_L \cos\varphi$)**
49. Two-wattmeter readings 3 kW and 1 kW. Reactive power? → **≈ 3.46 kVAR ($\sqrt{3}(W_1 - W_2)$)**
50. L_1 = 3 H, L_2 = 5 H, M = 1 H in series aiding. L_{eq}? → **10 H ($L_1 + L_2 + 2M$)**

14.2 Electromagnetic Fields (Q 51–85)

51. Unit of magnetic vector potential A? → **Wb/m**
52. Divergence theorem converts a closed-surface integral into? → **Volume integral of divergence**
53. Curl of the gradient of any scalar field? → **Zero ($\nabla\times\nabla V = 0$)**
54. Differential surface element on a sphere (normal a_r)? → **$r^2 \sin\theta\, d\theta\, d\varphi$**
55. E due to an infinite line charge varies with distance as? → **$1/\rho$ ($\rho_L/2\pi\varepsilon_0\rho$)**
56. Force between two 1 µC charges 1 m apart in air? → **9 mN ($9\times10^9\times10^{-12}/1^2$)**
57. E at 1 m from a 1 nC point charge in air? → **9 V/m (kQ/r^2)**
58. Curie temperature of iron? → **≈ 770 °C**
59. Potential at 0.1 m from a 1 nC point charge in air? → **90 V ($9\times10^9\times10^{-9}/0.1$)**
60. Relation between E and potential V? → **$E = -\nabla V$**
61. Poisson's equation? → **$\nabla^2 V = -\rho_v/\varepsilon$**
62. Number of images for a charge between two perpendicular conducting planes? → **3 ($360°/90° - 1$)**
63. Point form of Ohm's law? → **$J = \sigma E$**
64. Point form of the continuity equation? → **$\nabla\cdot J = -\partial\rho_v/\partial t$**
65. Approximate relaxation time of copper? → **≈ 1.5×10^{-19} s (ε_0/σ)**
66. Relative permittivity of distilled water? → **≈ 80**
67. Tangential E at the surface of a conductor? → **Zero**

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68. Air in a parallel-plate capacitor replaced by dielectric $\varepsilon_r = 5$; capacitance becomes? → **5 times**

69. Capacitance of an isolated sphere of radius 9 cm in air? → **10 pF ($4\pi\varepsilon_0 a = 0.09/9\times10^9$)**

70. Capacitance of the Earth treated as an isolated sphere? → **≈ 711 µF**

71. Hysteresis loss when frequency is doubled at constant B_m? → **Doubles ($P_h \propto f$)**

72. Force on 1 m conductor carrying 10 A perpendicular to 0.5 T? → **5 N (F = BIL)**

73. Force per metre between two conductors 1 m apart, 1 A each? → **2×10^{-7} N/m**

74. H at 1 m from an infinite wire carrying 2π A? → **1 A/m (I/2πr)**

75. H inside a long solenoid, 1000 turns/m, 2 A? → **2000 A/m (H = nI)**

76. 1 tesla equals how many gauss? → **10^4 gauss**

77. Radius of path of charge q moving ⊥ to uniform B? → **r = mv/(qB)**

78. Maxwell's equation (point form) from Faraday's law? → **$\nabla\times E = -\partial B/\partial t$**

79. Displacement current density? → **$J_d = \partial D/\partial t$**

80. Velocity of EM wave in a medium with $\varepsilon_r = 4$, $\mu_r = 1$? → **1.5×10^8 m/s ($c/\sqrt{\varepsilon_r}$)**

81. Intrinsic impedance of a medium with $\varepsilon_r = 9$, $\mu_r = 1$? → **40π ≈ 125.7 Ω (377/3)**

82. Skin depth of copper at 50 Hz? → **≈ 9.3 mm (66/√50 mm)**

83. AT needed for a 1 mm air gap at B = 1 T? → **≈ 796 AT ($B\,l_g/\mu_0$)**

84. Energy stored in a 2 H inductor carrying 3 A? → **9 J (½LI²)**

85. Mutual inductance for $L_1 = 4$ H, $L_2 = 9$ H, k = 0.5? → **3 H ($k\sqrt{L_1L_2}$) = 0.5 × 6)**

14.3 Signals & Systems (Q 86–115)

86. Fundamental period of x(t) = sin(2πt/3) + cos(πt/2)? → **12 s (LCM of 3 s and 4 s)**

87. Fundamental period of x[n] = cos(πn/6)? → **N = 12 ($\omega_0/2\pi = 1/12$)**

88. Fundamental period of x[n] = sin(4πn/7)? → **N = 7 ($\omega_0/2\pi = 2/7$)**

89. Average power of x(t) = 3 + 5 cos 10t? → **21.5 W (9 + 25/2)**

90. Energy of a rectangular pulse of amplitude 2 V and width 3 s? → **12 J ($A^2\tau = 4 \times 3$)**

91. Even part of $x(t) = e^t$? → **cosh t ($[e^t + e^{-t}]/2$)**

92. Value of $\int_{-\infty}^{\infty} (t^2 + 1)\,\delta(t - 2)\,dt$? → **5 (sifting: $2^2 + 1$)**

93. x(t) is non-zero only for $-1 \le t \le 3$; x(t/2) is non-zero over? → **$-2 \le t \le 6$ (expansion by 2)**

94. Is the system $y[n] = x[n^2]$ time-invariant? → **No – time-variant (also non-causal)**

95. LTI system $h[n] = (0.5)^n\,u[n + 2]$ is? → **Non-causal but stable**

96. x(t) exists on [0, 2], h(t) on [1, 4]; y = x * h exists on? → **[1, 6] (add the limits)**

97. {1, 1, 1} * {1, −1} = ? → **{1, 0, 0, −1}**

98. Step response $(1 - e^{-2t})u(t)$; impulse response? → **$2e^{-2t}u(t)$ (h = ds/dt)**

99. Dirichlet conditions for Fourier series are? → **Sufficient, not necessary**

100. FS of a signal with odd and half-wave symmetry contains? → **Odd-harmonic sine terms only**

101. DC value of full-wave rectified sine of peak 10 V? → **6.37 V (2A/π)**

102. RMS value of x(t) = 2 + 4 sin ωt? → **3.46 (√(4 + 16/2) = √12)**

103. Fourier transform of $e^{-2|t|}$? → **$4/(4 + \omega^2)$ ($2a/(a^2 + \omega^2)$)**

104. FT of x(2t) in terms of X(ω)? → **½ X(ω/2)**

105. First spectral null of a rectangular pulse of width 1 ms? → **1 kHz (f = 1/τ)**

106. Frequency resolution of a 512-point DFT at $f_s = 10.24$ kHz? → **20 Hz (f_s/N)**

107. Complex multiplications in a 16-point radix-2 FFT? → **32 ($(N/2)\log_2 N = 8 \times 4$)**

108. Nyquist rate of x(t) = cos(400πt)·cos(200πt)? → **600 Hz (highest component 300 Hz)**

109. A 9 kHz tone sampled at 12 kHz appears at? → **3 kHz (|9 − 12|)**

110. Laplace transform of $e^{-3t}\cos 4t\,u(t)$? → **$(s + 3)/[(s + 3)^2 + 16]$**

111. Final value of x(t) for $X(s) = 10/[s(s^2 + 3s + 2)]$? → **5 (sX(s) at s = 0: 10/2)**

112. Initial value $x(0^+)$ for $X(s) = (3s + 5)/(s^2 + 4s + 8)$? → **3 ($\lim_{s\to\infty} sX(s)$)**

113. Z-transform of $(-0.8)^n u[n]$ and its ROC? → **$z/(z + 0.8)$, $|z| > 0.8$**

114. Causal $H(z) = 1/(1 - 1.2z^{-1})$ is stable? → **No – pole at z = 1.2 outside unit circle**

115. Final value of x[n] for $X(z) = z/[(z - 1)(z - 0.5)]$? → **2 ((z − 1)X(z) at z = 1: 1/0.5)**

14.4 Transformers & DC Machines (Q 116–165)

116. EMF per turn of a 50 Hz transformer with $\Phi_m = 0.01$ Wb? → **2.22 V (4.44 × 50 × 0.01)**

117. Maximum core flux of a 2200 V, 50 Hz primary with 1100 turns? → **≈ 9 mWb (V/4.44fN)**

118. Steinmetz exponent of B_m in the hysteresis-loss formula? → **1.6**

119. Eddy current loss varies with lamination thickness t as? → **t^2**

120. Core material used in modern power transformers? → **CRGO silicon steel (≈ 3% Si)**

121. 200/400 V transformer, $R_2 = 0.4$ Ω (HV). Value referred to LV side? → **0.1 Ω ($0.4/2^2$)**

122. Short-circuit current of a transformer with 4% impedance (infinite bus)? → **25 × rated current (100/4)**

123. The magnetising component I_μ of no-load current is in phase with? → **Core flux Φ (lags V_1 by 90°)**

124. Magnetising inrush current is rich in which harmonic? → **2nd harmonic**

125. Meaning of the cooling code ONAN? → **Oil Natural, Air Natural**

126. Main function of the conservator? → **Allows oil expansion/contraction; keeps tank full**

127. The upper float of a Buchholz relay operates to give? → **Alarm (slow gas accumulation, incipient fault)**

128. SC test at half rated current reads 200 W. Full-load Cu loss? → **800 W (Cu loss ∝ I^2)**

129. OC test is normally performed on which side? → **LV side at rated voltage, HV open**

130. 100 kVA transformer, $P_i = 1$ kW, $P_{cu,fl} = 1.5$ kW. Efficiency at full load, UPF? → **≈ 97.6% (100/102.5)**

131. 500 kVA transformer, $P_i = 1.8$ kW, $P_{cu,fl} = 3.2$ kW. kVA at maximum efficiency? → **375 kVA (500 × √(1.8/3.2))**

132. % regulation with $\varepsilon_r = 2\%$, $\varepsilon_x = 4\%$ at 0.8 pf lagging? → **4% (2×0.8 + 4×0.6)**

133. $\varepsilon_r = 3\%$, $\varepsilon_x = 4\%$. Maximum regulation and pf at which it occurs? → **5%, at 0.6 pf lagging (cos φ = R/Z)**

134. The value of maximum efficiency of a transformer is highest at which pf? → **Unity pf**

135. With V/f constant, frequency doubled. Eddy current loss becomes? → **4 times ($P_e \propto f^2$ at constant B_m)**

136. Supply voltage constant, frequency increased. Eddy current loss? → **Unchanged ($P_e \propto V^2$)**

137. In Sumpner's test, the wattmeter in the primary circuit reads? → **$2P_i$ (iron loss of both transformers)**

138. Phase displacement in a Dyn11 transformer? → **LV leads HV by 30°**

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139. Usual vector group of 11 kV/433 V distribution transformers? → **Dyn11**

140. Paralleling transformers with unequal voltage ratios causes? → **Circulating current even at no load**

141. Two 100 kVA transformers (Z = 4% and 6%, same X/R) share 150 kVA. Loads? → **90 kVA and 60 kVA (inverse to Z)**

142. Capacity of two 50 kVA single-phase units in open delta? → **86.6 kVA (√3 × 50)**

143. In V-V connection at UPF load, each transformer works at pf? → **0.866 (cos 30°)**

144. Copper saving of a 250/200 V autotransformer over a two-winding one? → **80% (k = 200/250)**

145. 240/200 V step-down autotransformer supplying 12 kVA. kVA transferred inductively? → **2 kVA ((1 − k) × 12, k = 5/6)**

146. Main disadvantage of an autotransformer? → **No electrical isolation between primary and secondary**

147. Tap changers are normally provided on which winding? → **HV winding (lower current)**

148. Transition element used in a Jansen-type OLTC? → **High-speed transition resistors**

149. Number of parallel paths in a 6-pole simplex wave winding? → **2 (independent of poles)**

150. 6-pole wave-wound generator, Z = 300, Φ = 0.02 Wb, 1000 rpm. EMF? → **300 V (6×0.02×300×1000/120)**

151. DC generator speed raised 20% and flux reduced 10%. Change in EMF? → **Rises 8% (E ∝ ΦN = 1.2 × 0.9)**

152. Shunt generator: V = 220 V, R_{sh} = 110 Ω, I_L = 48 A, R_a = 0.1 Ω. E_g? → **225 V (I_a = 50 A, drop 5 V)**

153. R_c = 200 Ω at 1000 rpm; shunt field resistance 150 Ω. Critical speed? → **750 rpm (1000 × 150/200)**

154. Dummy coils are used in which armature winding? → **Wave winding (mechanical balance)**

155. Interpole windings are connected in? → **Series with the armature**

156. DC motor: E_b = 200 V, I_a = 50 A, N = 1000 rpm. Armature torque? → **95.5 N·m (9.55 E_bI_a/N)**

157. 220 V shunt motor, R_a = 0.2 Ω, 1000 rpm at I_a = 50 A. Speed at I_a = 100 A? → **≈ 952 rpm (1000 × 200/210)**

158. Unsaturated DC series motor: current doubled. Torque becomes? → **4 times (T ∝ I_a^2)**

159. 220 V motor, R_a = 0.5 Ω. External resistance to limit starting current to 40 A? → **5 Ω (220/40 − 0.5)**

160. Field circuit of a lightly loaded DC shunt motor opens. Result? → **Speed rises dangerously (runaway)**

161. Armature resistance control of a DC shunt motor suits which load type? → **Constant torque, below base speed**

162. How does the overload release of a DC starter trip the motor? → **Short-circuits the NVR coil, releasing the handle**

163. Hopkinson's (back-to-back) test needs? → **Two identical shunt machines**

164. Field's test is used for testing? → **Two identical series motors (traction motors)**

165. Approximate armature current at the instant of plugging? → **(V + E_b)/R ≈ 2V/R**

14.5 Induction & Synchronous Machines (Q 166–215)

166. Slip of an 8-pole, 50 Hz induction motor running at 720 rpm? → **4% (N_s = 750; 30/750)**

167. Rotor frequency of a 4-pole, 50 Hz motor running at 1470 rpm? → **1 Hz (s = 0.02; f_r = sf)**

168. Synchronous speed of a 6-pole motor on 60 Hz? → **1200 rpm (120 × 60/6)**

169. Rotor Cu loss is 300 W at 3% slip. Air-gap power? → **10 kW ($P_g = P_{c2}/s$)**

170. Gross mechanical power when air-gap power is 20 kW and s = 5%? → **19 kW ((1 − s)P_g)**

171. Poles and full-load slip of a 50 Hz motor rated 725 rpm? → **8 poles; s ≈ 3.3% (N_s = 750)**

172. Torque of a 4-pole, 50 Hz motor with air-gap power 31.4 kW? → **≈ 200 N·m (P_g/ω_s, ω_s = 157.1)**

173. Slip at maximum torque if R_2 = 0.06 Ω and X_2 = 0.4 Ω? → **0.15 ($s_m = R_2/X_2$)**

174. Supply voltage falls by 20%. Torque at a given slip becomes? → **64% of original (T ∝ V^2)**

175. T_{st}/T_{max} when s_m = 0.25? → **≈ 0.47 ($2s_m/(1 + s_m^2)$)**

176. Rotor efficiency (mechanical loss ignored) at 4% slip? → **96% (1 − s)**

177. Motor running at s = 0.04 is plugged. Slip at that instant? → **1.96 (2 − s)**

178. In a double-cage rotor, the outer cage has? → **High resistance, low reactance**

179. Typical no-load power factor of a 3-phase induction motor? → **≈ 0.1–0.2 lagging**

180. How is friction & windage loss separated in the no-load test? → **Extrapolate P_0 vs V curve to V = 0**

181. The circle diagram of an induction motor is also called? → **Heyland diagram**

182. Typical voltage applied in the blocked-rotor test? → **≈ 10–20% of rated (to circulate rated current)**

183. DOL starting current is 6 I_{FL}. Line current with a star–delta starter? → **2 I_{FL} (one-third of DOL)**

184. Starting torque with a 60% autotransformer tap, as a fraction of DOL? → **0.36 (x^2)**

185. DOL starting torque if I_{sc} = 5 I_{FL} and full-load slip = 4%? → **1.0 T_{FL} (25 × 0.04)**

186. Power devices used in a soft starter? → **Anti-parallel thyristors (SCRs)**

187. Which slip-power recovery scheme allows both sub- and super-synchronous speeds? → **Static Scherbius drive**

188. Differential cascade speed of 8-pole and 4-pole motors on 50 Hz? → **1500 rpm (6000/(8 − 4))**

189. Pole amplitude modulation (PAM) gives what kind of speed ratios? → **Close ratios (e.g. 8/10 poles), single winding**

190. Operating region of a V/f drive above base speed? → **Constant-power (field-weakening) region**

191. Rotor-side converter rating of a DFIG, as share of machine rating? → **≈ 25–30%**

192. Two requirements for voltage build-up of a self-excited induction generator? → **Residual magnetism; capacitance above critical value**

193. By double revolving field theory, each rotating component of a pulsating flux Φ_m is? → **$\Phi_m/2$**

194. Centrifugal switch of a split-phase motor opens at about? → **70–80% of synchronous speed**

195. Phase angle between main and auxiliary currents in a capacitor-start motor? → **≈ 80–90°**

196. Typical capacitor of a ceiling fan (PSC) motor? → **≈ 2–4 µF**

197. Single-phase motor with the lowest efficiency and starting torque? → **Shaded-pole motor**

198. Step angle of a 3-phase VR stepper with 8 rotor teeth? → **15° (360°/(m N_r) = 360/24)**

199. Steps per revolution of a 1.8° stepper motor? → **200 (360/1.8)**

200. Frequency of a 10-pole alternator at 600 rpm? → **50 Hz (PN/120)**

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201. Distribution factor for 2 slots/pole/phase, slot angle 30°? → **0.966 (sin 30°/(2 sin 15°))**

202. Pitch factor of a 2/3-pitch coil? → **0.866 (α = 60°, cos 30°)**

203. Coil span to eliminate the 7th harmonic? → **6/7 of pole pitch**

204. An alternator has SCR = 0.5. Its saturated X_s? → **2 pu (X_s = 1/SCR)**

205. Most accurate method of alternator regulation? → **ZPF (Potier triangle) method**

206. In the slip test, X_q equals? → **V_{min}/I_{max}**

207. Reluctance power of a salient-pole machine is maximum at δ =? → **45°**

208. P_{max} for E = 1.2 pu, V = 1.0 pu, X_s = 0.8 pu (cylindrical)? → **1.5 pu (EV/X_s)**

209. Synchroscope pointer rotating clockwise indicates? → **Incoming machine running fast**

210. Over-excited alternator on an infinite bus supplies? → **Lagging VAr (operates at lagging pf)**

211. Power factor of an under-excited synchronous motor? → **Lagging**

212. Relay protecting an alternator against motoring on prime-mover failure? → **Reverse power relay (ANSI 32)**

213. Maximum temperature of Class H insulation? → **180 °C**

214. Synchronous speed of a LIM with pole pitch 0.25 m on 50 Hz? → **25 m/s (2τf) = 90 km/h**

215. Constant loss 1.8 kW, full-load Cu loss 3.2 kW. Load at η_{max}? → **75% of full load (√(1.8/3.2))**

14.6 Power Systems I – Generation, Transmission & Load Flow (Q 216–265)

216. Overall efficiency of a thermal plant with heat rate 2580 kcal/kWh? → **≈ 33.3% (860/2580)**

217. Hydro plant: Q = 100 m³/s, net head 50 m, η = 0.9. Output? → **≈ 44.1 MW (9.81 × 0.9 × 100 × 50 kW)**

218. Most suitable turbine for a head of about 20 m? → **Kaplan (axial-flow reaction turbine)**

219. Upper limit of "small hydro" as per MNRE? → **25 MW**

220. Critical point of water (above which a boiler is supercritical)? → **221.2 bar, 374.15 °C**

221. Energy released per fission of U-235? → **≈ 200 MeV**

222. Fertile Th-232 is converted into which fissile material? → **U-233 (basis of India's Stage III)**

223. Wind speed rises from 6 m/s to 9 m/s. Power becomes? → **3.375 times ($P \propto V^3$; 1.5^3)**

224. Swept area 100 m², V = 10 m/s, ρ = 1.2 kg/m³. Maximum extractable power? → **≈ 35.6 kW (Betz $0.593 \times \frac{1}{2}\rho A V^3 \approx 60$ kW)**

225. Standard Test Conditions (STC) for PV modules? → **1000 W/m², 25 °C cell temperature, AM 1.5**

226. Maximum demand 100 MW, annual load factor 0.6. Annual energy? → **525.6 GWh (100 × 0.6 × 8760 MWh)**

227. Plant capacity 120 MW, maximum demand 100 MW, load factor 0.6. Capacity factor? → **0.5 (LF × UF = 0.6 × 100/120)**

228. Connected load 200 kW, maximum demand 120 kW. Demand factor? → **0.6 (MD/connected load)**

229. The three-part tariff is also called? → **Doherty tariff**

230. Date on which India's grid became one synchronous national grid? → **31 December 2013 (Southern grid synchronised)**

231. India's non-fossil installed capacity target for 2030? → **500 GW**

232. Highest AC transmission voltage in commercial use in India? → **765 kV**

233. India's first VSC-based HVDC link? → **Pugalur–Thrissur, ±320 kV**

234. 6-pulse bridge, V_{LL} = 100 kV, α = 60°. DC output voltage? → **67.5 kV (1.35 × 100 × cos 60°)**

235. Lowest characteristic AC-side harmonics of a 12-pulse converter? → **11th and 13th (12k ± 1)**

236. Break-even distance of HVDC for overhead lines? → **≈ 500–800 km (cables ≈ 30–50 km)**

237. Number of strands in a 3-layer stranded conductor (centre strand counted as layer 1)? → **19 ($3n^2 - 3n + 1$, n = 3)**

238. Transposed 3-φ line with spacings 2 m, 2 m and 4 m. Equivalent GMD? → **≈ 2.52 m ($(2 \times 2 \times 4)^{1/3}$)**

239. Lossless line, L = 1 mH/km, C = 0.01 μF/km. Surge impedance? → **≈ 316 Ω (√(L/C) = √10^5)**

240. SIL of a 400 kV line with Z_c = 400 Ω? → **400 MW ($400^2/400$)**

241. $V_S = V_R$ = 400 kV, X = 100 Ω, δ = 30°. Power transferred? → **800 MW (400 × 400 × 0.5/100)**

242. Constant C of a nominal-π medium line? → **Y(1 + YZ/4)**

243. Length of a quarter-wavelength line at 50 Hz? → **≈ 1500 km (λ ≈ 6000 km)**

244. Air density factor at 73 cm Hg and 20 °C? → **≈ 0.977 (3.92 × 73/293)**

245. Span 200 m, conductor weight 1 kg/m, tension 1000 kg. Sag? → **5 m ($wL^2/8T$ = 40000/8000)**

246. String voltage 30 kV, 3 discs, voltage across disc nearest line 12 kV. String efficiency? → **83.3% (30/(3 × 12))**

247. 3-disc string with m = 0.1. Ratio V_3/V_1 (line disc to top disc)? → **1.31 ($1 + 3m + m^2$)**

248. Single-core cable, 50 kV to sheath, permissible g_{max} = 50 kV/cm. Most economical conductor diameter? → **2 cm ($d = 2V/g_{max}$)**

249. Cable with r = 1 cm and sheath radius R = 2.5 cm. g_{max}/g_{min}? → **2.5 (= R/r)**

250. 3-core cable, C_s = 0.3 μF, C_c = 0.2 μF. Capacitance per phase to neutral? → **0.9 μF ($C_s + 3C_c$)**

251. Continuous and short-circuit temperature ratings of XLPE cable? → **90 °C and 250 °C**

252. kVAR needed to raise the pf of a 100 kW load from 0.6 to 0.8 lagging? → **≈ 58.3 kVAR (100 × (1.333 − 0.75))**

253. Tariff ₹100/kVA/yr, capacitor cost ₹60/kVAR/yr. Most economical pf? → **0.8 lagging (sin φ_2 = y/x = 0.6)**

254. Area with β = 250 MW/Hz, load rises 50 MW, primary control only. Δf_{ss}? → **−0.2 Hz ($-\Delta P_D/\beta$)**

255. 40% series compensation of a line. Maximum power transfer increases by? → **≈ 1.67 times (1/(1 − 0.4))**

256. Most versatile FACTS controller (controls V, X and δ)? → **UPFC**

257. Uniformly loaded distributor: drop 8 V when fed at one end. Drop if fed at both ends at equal voltage? → **2 V (one-fourth; $irl^2/8$ vs $irl^2/2$)**

258. X = 0.1 pu on a 10 kV base. Value on an 11 kV base (same MVA)? → **≈ 0.0826 pu ($0.1 \times (10/11)^2$)**

259. Base impedance for 132 kV, 100 MVA? → **174.24 Ω ($132^2/100$)**

260. Base current for 100 MVA, 11 kV (3-φ)? → **≈ 5.25 kA (100 MVA/(√3 × 11 kV))**

261. Line of impedance j0.2 pu between buses 1 and 2. Y_{12} of Y_{bus}? → **+j5 pu ($-y_{12}$; $y_{12} = -j5$)**

262. 10-bus system with 1 slack and 3 PV buses. Order of polar NR Jacobian? → **15 (2(n − 1) − m = 18 − 3)**

263. Typical acceleration factor in Gauss–Seidel load flow? → **≈ 1.6 (range 1.4–1.6)**

264. DC load flow: $\theta_1 - \theta_2$ = 0.1 rad, x_{12} = 0.2 pu. Power flow 1→2? → **0.5 pu ($(\theta_1 - \theta_2)/x_{12}$)**

265. Why is fast decoupled load flow fast per iteration? → **B′ and B″ are constant and factorised only once**

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14.7 Power Systems II – Faults, Protection & Stability (Q 266–310)

266. $I_a = I_b = I_c = 10\angle 0°$ A. Positive-sequence current I_{a1}? → **0 (only zero sequence, I_{a0} = 10 A)**

267. Zero-sequence current I_{a0} = 50 A. Neutral current? → **150 A ($I_n = 3I_{a0}$)**

268. Value of $a + a^2$ ($a = 1\angle 120°$)? → **−1 (since $1 + a + a^2 = 0$)**

269. Typical ratio Z_0/Z_1 of an overhead transmission line? → **≈ 3 (range 2–3.5)**

270. Fault MVA of a 20 MVA, 8% reactance transformer (infinite source)? → **250 MVA (20 × 100/8)**

271. Fault current at an 11 kV bus with fault level 500 MVA? → **≈ 26.2 kA (500/(√3 × 11))**

272. X = 0.1 pu on 50 MVA. Value on 100 MVA base, same kV? → **0.2 pu (× 100/50)**

273. Base impedance for 11 kV, 100 MVA base? → **1.21 Ω (kV^2/MVA)**

274. Multiplying factor for momentary (first-cycle) current over symmetrical rms? → **≈ 1.6**

275. Typical subtransient reactance X_d'' of a turbo-alternator? → **0.1–0.25 pu**

276. E = 1 pu, $X_1 = X_2 = 0.25$, $X_0 = 0.1$ (solid). Bolted LG fault current? → **5 pu (3/0.6)**

277. Same machine ($X_1 = X_2 = 0.25$). Bolted LL fault current? → **≈ 3.46 pu (√3/0.5)**

278. Same machine ($X_1 = 0.25$). 3-phase fault current? → **4 pu (1/0.25)**

279. $X_1 = 0.25$, $X_0 = 0.1$. Neutral reactance making $I_{LG} = I_{3\varphi}$? → **0.05 pu ($(X_1 - X_0)/3$)**

280. $Z_1 = Z_2 = Z_0 = j0.1$ pu, E = 1 pu. Bolted LG fault current? → **10 pu (I_{a1} = 1/0.3 = 3.33; ×3)**

281. Connection of sequence networks for an LLG fault? → **All three in parallel ($Z_1 + Z_2 \parallel Z_0$)**

282. Ungrounded system, normal charging current 10 A/phase. LG fault (capacitive) current? → **30 A (3 × per-phase charging current)**

283. Petersen coil inductance for C = 1 µF/phase at 50 Hz? → **≈ 3.38 H ($L = 1/(3\omega^2 C)$)**

284. Maximum coefficient of earthing for an effectively earthed system? → **80% (earth-fault factor ≤ 1.4)**

285. CT 200/1 A, plug setting 100%, fault current 4000 A. PSM? → **20 (relay current 20 A ÷ 1 A)**

286. Normal-inverse IDMT relay, PSM 10, TMS 0.5. Operating time? → **≈ 1.49 s (2.97 × 0.5)**

287. Very-inverse relay, PSM 4, TMS 0.3. Operating time? → **1.35 s (13.5/3 × 0.3)**

288. Extremely-inverse relay, PSM 5, TMS 1. Operating time? → **≈ 3.33 s (80/(25 − 1))**

289. ANSI device numbers 21, 50, 51, 87? → **Distance, instantaneous OC, inverse-time OC, differential**

290. Typical grading margin between electromechanical IDMT relays? → **≈ 0.4 s (0.25–0.3 s numerical)**

291. Current in the restraining coil of a biased differential relay? → **$(I_1 + I_2)/2$**

292. Harmonic used to block a transformer differential relay during over-excitation? → **5th harmonic**

293. Protection for earth faults near the neutral of a star winding? → **Restricted earth fault (REF) protection**

294. Operation of the Buchholz upper float on slow gas accumulation? → **Alarm (lower float/flap trips)**

295. Stator winding covered by a conventional generator earth-fault scheme? → **≈ 95% (100% needs 3rd-harmonic scheme)**

296. Reach and time of Zone 1 of a distance relay? → **80–90% of line, instantaneous**

297. Frequency range of power-line carrier (PLCC)? → **50–500 kHz**

298. Natural frequency of TRV with L = 1 mH, C = 0.01 µF? → **≈ 50.3 kHz ($1/(2\pi\sqrt{LC})$)**

299. Maximum RRRV for V_m = 100 kV, L = 1 mH, C = 0.01 µF? → **≈ 31.6 kV/µs ($V_m/\sqrt{LC}$)**

300. Current of 10 A chopped; L = 2 H, C = 0.02 µF. Overvoltage? → **100 kV ($i\sqrt{(L/C)}$)**

301. Symmetrical breaking capacity of a 33 kV CB rated 20 kA? → **≈ 1143 MVA (√3 × 33 × 20)**

302. Shunt resistance for critical damping, L = 4 mH, C = 0.01 µF? → **≈ 316 Ω ($0.5\sqrt{(L/C)}$)**

303. Standard lightning impulse wave shape? → **1.2/50 µs**

304. Kinetic energy stored in a 100 MVA machine with H = 5 MJ/MVA? → **500 MJ (H × S)**

305. H = 4 MJ/MVA on 200 MVA rating. H on 100 MVA base? → **8 MJ/MVA (4 × 200/100)**

306. E = 1.2 pu, V = 1 pu, X = 0.6 pu, P = 1 pu. Load angle? → **30° (P_{max} = 2; sin δ = 0.5)**

307. Synchronising power coefficient for the above case? → **≈ 1.732 pu/rad (2 cos 30°)**

308. Above case with H = 5 s, 50 Hz. Natural frequency of oscillation? → **≈ 7.38 rad/s ≈ 1.17 Hz ($\sqrt{(\omega_s P_s/2H)}$)**

309. Maximum input suddenly applicable to an unloaded machine on an infinite bus? → **≈ 0.725 P_{max} (equal-area criterion)**

310. Fault type most severe for transient stability? → **3-phase (LLL) fault**

14.8 Control Systems (Q 311–355)

311. Type and order of G(s)H(s) = 5(s+3)/[s(s+2)(s+4)]? → **Type 1, order 3**

312. DC gain of the closed loop for G = 10/(s+1) with unity negative feedback? → **10/11 ≈ 0.91 (T = 10/(s+11))**

313. G = 9/(1+s) with unity feedback: closed-loop time constant? → **0.1 s ($\tau/(1+K) = 1/10$)**

314. In the force–voltage analogy, spring stiffness K is analogous to? → **1/C (elastance)**

315. Mass–spring–damper with M = 1 kg, B = 4 N·s/m, K = 16 N/m: damping ratio? → **0.5 (ω_n = 4; ζ = B/2√(KM))**

316. An AC servomotor is basically a? → **Two-phase induction motor with high rotor R/X**

317. Step angle of a 4-phase stepper motor with 50 rotor teeth? → **1.8° (360°/(4 × 50))**

318. Step angle of a VR stepper with 8 stator and 6 rotor teeth? → **15° ((8 − 6) × 360°/48)**

319. SFG: one forward path of gain 10 touching two non-touching loops of gains −2 and −3. Overall gain? → **0.833 (Δ = 1 + 5 + 6 = 12; 10/12)**

320. Laplace transform of the unit parabolic input $t^2/2$? → **$1/s^3$**

321. Settling time (2%) of a first-order system 1/(1 + 2s)? → **8 s (4T)**

322. Rise time (10–90%) of a first-order system with T = 0.5 s? → **1.1 s (2.2T)**

323. Peak time for G = 100/[s(s+10)] with unity feedback? → **0.363 s (ω_d = 8.66; π/ω_d)**

324. Settling time (2%) for G = 64/[s(s+8)] with unity feedback? → **1 s ($\zeta\omega_n$ = 4; 4/4)**

325. Damping ratio that gives 16.3% peak overshoot? → **0.5**

326. Logarithmic decrement for ζ = 0.5? → **3.63 ($2\pi\zeta/\sqrt{(1-\zeta^2)}$)**

327. Damped frequency for ω_n = 10 rad/s, ζ = 0.6? → **8 rad/s ($10\sqrt{(1 - 0.36)}$)**

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328. Unit-step response is $1 - e^{-2t}$. Impulse response? → **$2e^{-2t}$ (derivative of step response)**

329. G = K/[s(s+4)], unity feedback: K for ζ = 0.5? → **16 ($2\zeta\omega_n = 4 \rightarrow \omega_n = 4$)**

330. Unit-ramp e_{ss} for G = 50/[s(s+5)(s+10)]? → **1 ($K_v = 50/50 = 1$)**

331. Unit-step e_{ss} for G = 10/[(s+2)(s+5)]? → **0.5 ($K_p = 1$; $1/(1+K_p)$)**

332. Unit-parabolic e_{ss} for G = 20(s+2)/[s^2(s+10)]? → **0.25 ($K_a = 40/10 = 4$)**

333. Range of K for stability of $s^3 + 2s^2 + 4s + K = 0$? → **$0 < K < 8$ ($a_1a_2 > a_0a_3$)**

334. Number of RHP roots of $s^3 - 4s^2 + s + 6 = 0$? → **2 (roots 2, 3, −1; two sign changes)**

335. G = K/[s(s+2)(s+3)]: marginal K and oscillation frequency? → **K = 30, ω = √6 ≈ 2.45 rad/s**

336. Asymptote centroid and angles for G = K/[s(s+2)(s+4)(s+6)]? → **−3; ±45°, ±135° (−12/4)**

337. Breakaway point of the root locus for G = K/[s(s+6)]? → **s = −3 (−a/2)**

338. Centroid of asymptotes for G = K(s+1)/[s(s+2)(s+3)]? → **−2 ((−5 + 1)/2), asymptotes ±90°**

339. Number of root-locus branches for G = K(s+1)/[s^2(s+4)(s+5)]? → **4 (number of OL poles)**

340. Resonant peak M_r for ζ = 0.4? → **1.36 ($1/(2\zeta\sqrt{(1-\zeta^2)})$)**

341. Resonant frequency for ω_n = 10 rad/s, ζ = 0.5? → **7.07 rad/s ($\omega_n\sqrt{(1 - 2\zeta^2)}$)**

342. |GH| = 0.1 at the phase crossover frequency. Gain margin? → **20 dB (−20 log 0.1)**

343. ∠GH = −135° at the gain crossover frequency. Phase margin? → **45° (180° + ∠GH)**

344. Bode initial −20 dB/dec line of a type-1 system cuts 0 dB at 5 rad/s. K_v? → **5 s^{-1}**

345. OL has one RHP pole (P = 1). Encirclements of −1 + j0 needed for CL stability? → **One anticlockwise (N = P − Z, Z = 0)**

346. Centre and radius of the constant-M circle for M = 2? → **(−4/3, 0); radius 2/3**

347. Maximum phase lead of a lead network with α = 0.1? → **≈ 54.9° (sin φ_m = 0.9/1.1)**

348. Lead network with T = 0.1 s, α = 0.25: frequency of maximum lead? → **20 rad/s (1/(T√α))**

349. Z–N ultimate-gain tuning with K_{cr} = 10, P_{cr} = 2 s: PID settings? → **$K_p = 6$, $T_i = 1$ s, $T_d = 0.25$ s**

350. TF for A = [[0, 1], [−6, −5]], B = [0, 1]^T, C = [1, 0], D = 0? → **$1/(s^2 + 5s + 6)$; poles −2, −3**

351. State transition matrix for A = diag(−1, −3)? → **diag(e^{-t}, e^{-3t})**

352. Is A = [[0, 1], [−2, −3]], B = [0, 1]^T controllable? → **Yes (Q_c = [[0, 1], [1, −3]], det = −1 ≠ 0)**

353. Number of state variables for a network with 2 independent inductors and 1 capacitor? → **3 (number of energy-storage elements)**

354. Phase of the dead time $e^{-0.1s}$ at ω = 10 rad/s? → **−1 rad ≈ −57.3° (−ωT)**

355. Polar plot crosses the negative real axis at −0.25. Gain margin? → **4 ≈ 12 dB (1/0.25)**

14.9 Electrical & Electronic Measurements (Q 356–400)

356. A 0–200 V voltmeter (±1% FSD) reads 50 V. Limiting error? → **±4% (±2 V ÷ 50 V)**

357. % error in $P = I^2R$ if I is ±2% and R is ±1%? → **±5% (2 × 2 + 1)**

358. Fraction of readings within ±3σ (Gaussian)? → **99.73%**

359. Standard deviation formula for fewer than 20 readings? → **$s = \sqrt{(\Sigma d^2/(n - 1))}$**

360. Relation between static correction and static error? → **Equal and opposite ($\delta C = -\delta A$)**

361. Example of an absolute instrument? → **Tangent galvanometer (also Rayleigh current balance)**

362. An energy meter is which class of instrument? → **Integrating instrument**

363. Shunt to extend a 1 mA, 100 Ω meter to 1 A? → **≈ 0.1001 Ω (100/999)**

364. Sensitivity of a voltmeter using a 100 µA FSD movement? → **10 kΩ/V ($1/I_{fs}$)**

365. Deflecting torque of a moving-iron instrument? → **$T_d = ½ I^2 dL/d\theta$**

366. Material of control springs? → **Phosphor bronze**

367. Scale of a gravity-controlled instrument? → **Cramped/non-uniform ($T_c \propto \sin\theta$)**

368. Damping used in PMMC instruments? → **Eddy-current damping (aluminium former)**

369. Damping used in electrostatic instruments? → **Fluid (oil) friction damping**

370. Frequency error of an MI voltmeter is compensated by? → **Capacitor across the series (swamping) resistance**

371. Material of the swamping resistor for PMMC temperature compensation? → **Manganin**

372. Two-wattmeter readings 4 kW and 2 kW (balanced load). Power factor? → **0.866 lag (tan φ = √3 × 2/6; φ = 30°)**

373. Two-wattmeter readings 5 kW and 3 kW. Reactive power? → **≈ 3.46 kVAR ($\sqrt{3}(W_1 - W_2)$)**

374. Wattmeters needed for an n-wire system (Blondel)? → **n − 1**

375. Effect of PC inductance on wattmeter at lagging pf? → **Reads high; error serious at low pf**

376. LPF wattmeter is used in which transformer test? → **Open-circuit test**

377. Meter constant 1200 rev/kWh; 1 kW load for 30 min. Revolutions? → **600 rev (0.5 kWh × 1200)**

378. Voltage at which an energy-meter creep test is done? → **110% of rated voltage, no load**

379. Effect of moving brake magnet towards disc centre? → **Braking torque falls; disc runs faster**

380. Light-load (friction) adjustment of energy meter is done at? → **≈ 5–10% of full load, unity pf**

381. Vibrating-reed frequency meter works on? → **Mechanical resonance of tuned reeds**

382. Q of the coil in Hay's bridge? → **$Q = 1/(\omega C_4 R_4)$**

383. Range of coil Q suited to Maxwell L–C bridge? → **Medium Q (1 < Q < 10)**

384. Wien bridge with R = 10 kΩ, C = 0.01 µF in both arms. Balance frequency? → **≈ 1.59 kHz ($1/(2\pi RC)$)**

385. Bridges for mutual inductance? → **Heaviside-Campbell, Campbell, Carey-Foster (Heydweiller)**

386. Detector for AC bridges at power frequency? → **Vibration galvanometer**

387. Range of medium resistance and the usual method? → **1 Ω – 100 kΩ; Wheatstone bridge**

388. Loss-of-charge method formula for R? → **$R = t/[C \ln(V/v)] = 0.4343t/[C \log_{10}(V/v)]$**

389. Drysdale-Tinsley AC potentiometer is which type? → **Polar type (magnitude and phase)**

390. Maximum burden impedance of a 10 VA, 1 A CT? → **10 Ω (VA/I_s^2)**

391. Percentage ratio error of an instrument transformer? → **$(K_n - R)/R \times 100$**

392. Knee-point voltage of a CT is defined where? → **10% rise in voltage gives 50% rise in exciting current**

393. Turns compensation in a CT corrects? → **Ratio error only (secondary reduced by 1–2 turns)**

394. A PT normally operates close to? → **Open circuit; secondary must never be shorted**

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395. Comparators needed for a 6-bit flash ADC? → **63 ($2^n - 1$)**

396. Resolution of a 4½-digit DVM on 1 V range? → **0.1 mV (1 V × 10^{-4})**

397. Counter with 1 s gate measures 500 Hz. ±1-count error? → **±0.2% (1/500)**

398. Rise time of a 50 MHz bandwidth oscilloscope? → **7 ns (0.35/BW)**

399. Lissajous: 3 horizontal tangencies, 1 vertical; f_x = 50 Hz. f_y? → **150 Hz (f_y/f_x = 3/1)**

400. Resistance of a Pt100 RTD at 100 °C (α = 0.00385)? → **138.5 Ω (100(1 + 0.385))**

14.10 Analog Electronics (Q 401–435)

401. Energy gap of silicon at 300 K? → **≈ 1.1 eV (1.12 eV)**

402. Mass-action law for a semiconductor in thermal equilibrium? → **$n \cdot p = n_i^2$**

403. Einstein relation between diffusion constant and mobility? → **$D/\mu = V_T = kT/q$**

404. Junction capacitance that dominates under reverse bias? → **Transition (depletion) capacitance $C_T \propto 1/\sqrt{V_R}$**

405. Diode used for electronic tuning as a voltage-variable capacitor? → **Varactor (varicap), reverse-biased**

406. Temperature coefficient of Zener (tunnelling) breakdown? → **Negative**

407. Maximum rectification efficiency of a half-wave rectifier? → **40.6 % ($4/\pi^2$)**

408. Transformer utilisation factor (TUF) of a half-wave rectifier? → **0.287**

409. DC output of an ideal full-wave rectifier with V_m = 100 V? → **≈ 63.7 V ($2V_m/\pi$)**

410. Ripple frequency of a half-wave rectifier on 50 Hz mains? → **50 Hz (equal to supply frequency)**

411. Output of an ideal positive clamper for a ±10 V sine input? → **0 to +20 V (shifted up by V_m)**

412. Most heavily doped region of a BJT? → **Emitter**

413. α of a transistor with β = 19? → **0.95 (β/(1 + β) = 19/20)**

414. I_{CBO} = 10 µA, β = 99. I_{CEO} = ? → **1 mA ($(1 + \beta)I_{CBO}$)**

415. BJT configuration with the highest power gain? → **Common emitter (CE)**

416. Overall current gain of a Darlington pair with $\beta_1 = \beta_2$ = 100? → **≈ 10 000 ($\beta_1\beta_2$)**

417. Junction biasing in the active region of a BJT? → **E–B forward, C–B reverse**

418. JFET with I_{DSS} = 16 mA at $V_{GS} = V_P/2$. I_D = ? → **4 mA ($I_{DSS}/4$)**

419. E-MOSFET: k = 0.5 mA/V², V_T = 2 V, V_{GS} = 4 V (saturation). I_D = ? → **2 mA ($k(V_{GS} - V_T)^2$)**

420. Why can power MOSFETs be paralleled easily? → **Positive temperature coefficient of $R_{DS(on)}$**

421. Transconductance of a BJT at I_C = 2 mA (300 K)? → **≈ 77 mS (I_C/V_T = 2 mA/26 mV)**

422. Three identical non-interacting stages, each f_H = 1 MHz. Overall f_H? → **≈ 0.51 MHz ($f_H\sqrt{(2^{1/3} - 1)}$)**

423. Conduction angle of a Class C amplifier? → **Less than 180° (typically 90°–150°)**

424. Harmonics cancelled by a push-pull output stage? → **Even harmonics**

425. A = 500, β = 0.02 with negative feedback. Closed-loop gain? → **≈ 45.5 (500/(1 + 10))**

426. Amplifier with f_H = 20 kHz gets negative feedback with Aβ = 9. New f_H? → **200 kHz ($f_H(1 + A\beta)$)**

427. Minimum amplifier gain for a Wien-bridge oscillator? → **3 (β = 1/3)**

428. RC phase-shift oscillator, R = 10 kΩ, C = 10 nF. Frequency? → **≈ 650 Hz ($1/(2\pi RC\sqrt{6})$)**

429. Colpitts oscillator, $C_1 = C_2$ = 2 nF, L = 1 mH. Frequency? → **≈ 159 kHz (C_{eq} = 1 nF)**

430. Typical CMRR of the µA741 op-amp? → **90 dB**

431. Inverting amplifier, R_1 = 10 kΩ, R_f = 100 kΩ, v_i = 0.5 V. Output? → **−5 V ($-R_f/R_1 \times v_i$)**

432. Schmitt trigger, $\pm V_{sat}$ = ±12 V, $\beta = R_2/(R_1 + R_2)$ = 0.1. Hysteresis? → **2.4 V ($2\beta V_{sat}$)**

433. 555 monostable pulse width with R = 100 kΩ, C = 10 µF? → **1.1 s (T = 1.1RC)**

434. Relation between capture range and lock range of a PLL? → **Capture range < lock range**

435. LM317 output with R_1 = 240 Ω, R_2 = 720 Ω (I_{ADJ} neglected)? → **5 V (1.25(1 + R_2/R_1))**

14.11 Digital Electronics & 8085 (Q 436–465)

436. 8-bit 2's complement of 0101 1000? → **1010 1000 (invert, add 1)**

437. $(247)_{10}$ in hexadecimal? → **F7H (15 × 16 + 7 = 247)**

438. Excess-3 code of decimal 5? → **1000 (0101 + 0011)**

439. Gray code of binary 0111? → **0100 (XOR adjacent bits)**

440. Minimum Hamming distance needed to correct 2-bit errors? → **5 ($d_{min} \geq 2t + 1$)**

441. AB + A′C + BC simplifies to? → **AB + A′C (consensus theorem)**

442. Minimum 2-input NOR gates to realise an AND gate? → **3**

443. Minimum 2-input NAND gates for a full adder? → **9**

444. Standard TTL $V_{IL(max)}$ and $V_{IH(min)}$? → **0.8 V and 2.0 V**

445. Speed–power product of standard 74 TTL? → **≈100 pJ (10 ns × 10 mW)**

446. A floating (unconnected) TTL input behaves as? → **Logic 1 (HIGH)**

447. Number of 4:1 MUXes needed to build a 16:1 MUX? → **5 (4 + 1)**

448. Borrow output of a half subtractor? → **$B_o = A'B$**

449. Generate and propagate terms in a carry look-ahead adder? → **$G_i = A_iB_i$, $P_i = A_i \oplus B_i$**

450. Static-1 hazard occurs in which realisation? → **SOP (AND–OR) circuits**

451. Characteristic equation of JK flip-flop? → **$Q^+ = JQ' + K'Q$**

452. Unused states in a 4-bit ring counter? → **12 ($2^4 - 4$)**

453. 4-stage binary counter clocked at 32 kHz: output frequency of last stage? → **2 kHz (32 kHz ÷ 2^4)**

454. Minimum time data must be stable before the clock edge is called? → **Setup time**

455. Resolution of 8-bit DAC with V_{ref} = 5.12 V? → **20 mV ($5.12/2^8$)**

456. Number of comparators in a 4-bit flash ADC? → **15 ($2^n - 1$)**

457. Ideal SNR of a 10-bit ADC? → **≈ 62 dB (6.02n + 1.76)**

458. Maximum quantisation error of an ideal (rounding) ADC? → **±½ LSB**

459. Address lines needed for an 8K × 8 memory? → **13 (2^{13} = 8192)**

460. PLD with both AND and OR arrays programmable? → **PLA**

461. Vector address of RST 6.5 in 8085? → **0034H (8 × 6.5 = 52)**

462. Execution time of STA at 3 MHz clock? → **≈ 4.33 µs (13 T × 0.333 µs)**

463. Flag(s) affected by 8085 DAD instruction? → **Only CY**

464. Status of CY and AC after an ANA instruction? → **CY = 0, AC = 1**

465. 8255 control word with D7 = 0 selects which mode? → **BSR (bit set/reset of port C)**

14.12 Power Electronics (Q 466–500)

466. Number of layers and junctions in an SCR? → **4 layers (p-n-p-n), 3 junctions**

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467. Gate terminal of an SCR is connected to which layer? → **p-layer nearest the cathode**

468. Junctions reverse biased when SCR is in reverse blocking mode? → **J1 and J3**

469. Components of SCR turn-on time t_{on}? → **Delay + rise + spread time ($t_d + t_r + t_p$)**

470. Condition for successful commutation of an SCR? → **Circuit turn-off time t_c > device turn-off time t_q**

471. Series inductance to limit di/dt to 20 A/µs on a 200 V supply? → **10 µH ($L = V_s/(di/dt)$)**

472. GTO with turn-off gain 5 carries 1000 A. Negative gate current needed? → **200 A (I_A/β_{off})**

473. Power device with no storage time (majority carrier), hence fastest? → **Power MOSFET**

474. Three terminals of an IGBT? → **Gate, collector, emitter**

475. Failure phenomenon in IGBT caused by its parasitic thyristor? → **Latch-up**

476. Typical forward voltage drop of a Schottky diode? → **0.3–0.5 V**

477. Five 1000 V SCRs in series operate a 4000 V string. String efficiency? → **80% (4000/(5 × 1000)); DRF = 0.2**

478. UJT with η = 0.6, V_{BB} = 20 V, V_D = 0.7 V. Peak point voltage? → **12.7 V ($\eta V_{BB} + V_D$)**

479. Typical breakover voltage of a DIAC? → **≈ 30 V (both directions)**

480. 1-φ half-wave controlled rectifier, R load, V_m = 325 V, α = 90°. V_{dc}? → **≈ 51.7 V ($(V_m/2\pi)(1 + \cos\alpha)$)**

481. 3-φ full converter, V_L = 415 V, α = 60°, continuous current. V_{dc}? → **≈ 280 V (1.35 × 415 × 0.5)**

482. Input pf of 3-φ full converter (ripple-free I_d) at α = 30°? → **≈ 0.827 ($(3/\pi)\cos30°$)**

483. Displacement factor of a 1-φ semi-converter at firing angle α? → **$\cos(\alpha/2)$**

484. Main effect of a freewheeling diode on a controlled rectifier with RL load? → **No negative output voltage; improved input pf**

485. Phase sequence of the 3rd harmonic in a balanced 3-φ system? → **Zero sequence (flows in neutral)**

486. IEEE 519 voltage THD limit at PCC for systems ≤ 1 kV? → **8%**

487. Rectification efficiency of a 1-φ half-wave diode rectifier? → **40.5%**

488. Transformer utilisation factor of a 1-φ bridge rectifier? → **0.812**

489. Buck converter: V_s = 100 V, D = 0.5, f = 20 kHz, L = 1 mH. Peak-to-peak ΔI_L? → **1.25 A ($V_sD(1 - D)/fL$)**

490. Element through which a Ćuk converter transfers energy? → **Series (coupling) capacitor**

491. Maximum duty ratio of forward converter with $N_3 = N_1$ reset winding? → **0.5**

492. Step-down chopper, V_s = 200 V, f = 10 kHz, T_{on} = 30 µs. V_o? → **60 V (D = 30 µs × 10 kHz = 0.3)**

493. Class B chopper regenerating into 220 V supply at D = 0.4. Motor EMF (R_a neglected)? → **132 V ($E = (1 - D)V_s$)**

494. Half-bridge square-wave inverter, V_s = 200 V. RMS fundamental output? → **90 V ($0.45V_s$)**

495. 3-φ 180° inverter, V_s = 600 V. RMS fundamental line voltage? → **≈ 468 V ($(\sqrt{6}/\pi)V_s = 0.78V_s$)**

496. Purpose of dead (blanking) time in a VSI leg? → **Prevent shoot-through (DC bus short circuit)**

497. Preferred frequency modulation ratio m_f for 3-φ SPWM inverter? → **Odd multiple of 3**

498. Cascaded H-bridge inverter with 3 H-bridges per phase gives how many phase-voltage levels? → **7 (2s + 1)**

499. Integral cycle control: 3 cycles ON, 1 OFF, V_s = 230 V. Output rms voltage? → **≈ 199 V ($230\sqrt{(3/4)}$)**

500. Transfer time of an online (double-conversion) UPS? → **Zero (no break)**

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